CONTROL OF LINEAR AND NONLINEAR RESISTIVE WALL MODES

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LANL

OUTLINE

- Resistive wall modes: when MHD modes are wall stabilized, they can persist as *resistive wall modes.* They can be stabilized by rotation, but too much rotation is required.

- Model: reduced resistive MHD in a slab, $0 < x < L_x,\ 0 < y < L_y$. Sensor at resistive wall ($y = L_y$), control at outer wall $y = W$: flux specified.

- Complex gain: $\psi(x, y = W) = -G\psi(x - \delta, y = L_y)$: $Ge^{-ik\delta} = G_r + iG_i$.

Outline, continued

- Equivalence of G_r to a closer outer wall (caveat - single k).
- Equivalence of G_i to rotation of the resistive wall (caveat - single k).
- Two walls – G_i corresponds to rotation of the outer wall – differential rotation. 'Fake' rotating wall.
- Nonlinear simulations with G_r, G_i.

Linear stabilization.

Limiting the nonlinear saturation amplitude – just below the β for linear stabilization with PC wall, very large G_r is required for stabilization, but much smaller gain is required for low level saturation.

MODEL: On $0 < x < L_x$, $0 < y < L_y$ Slab with curvature (cylinder)

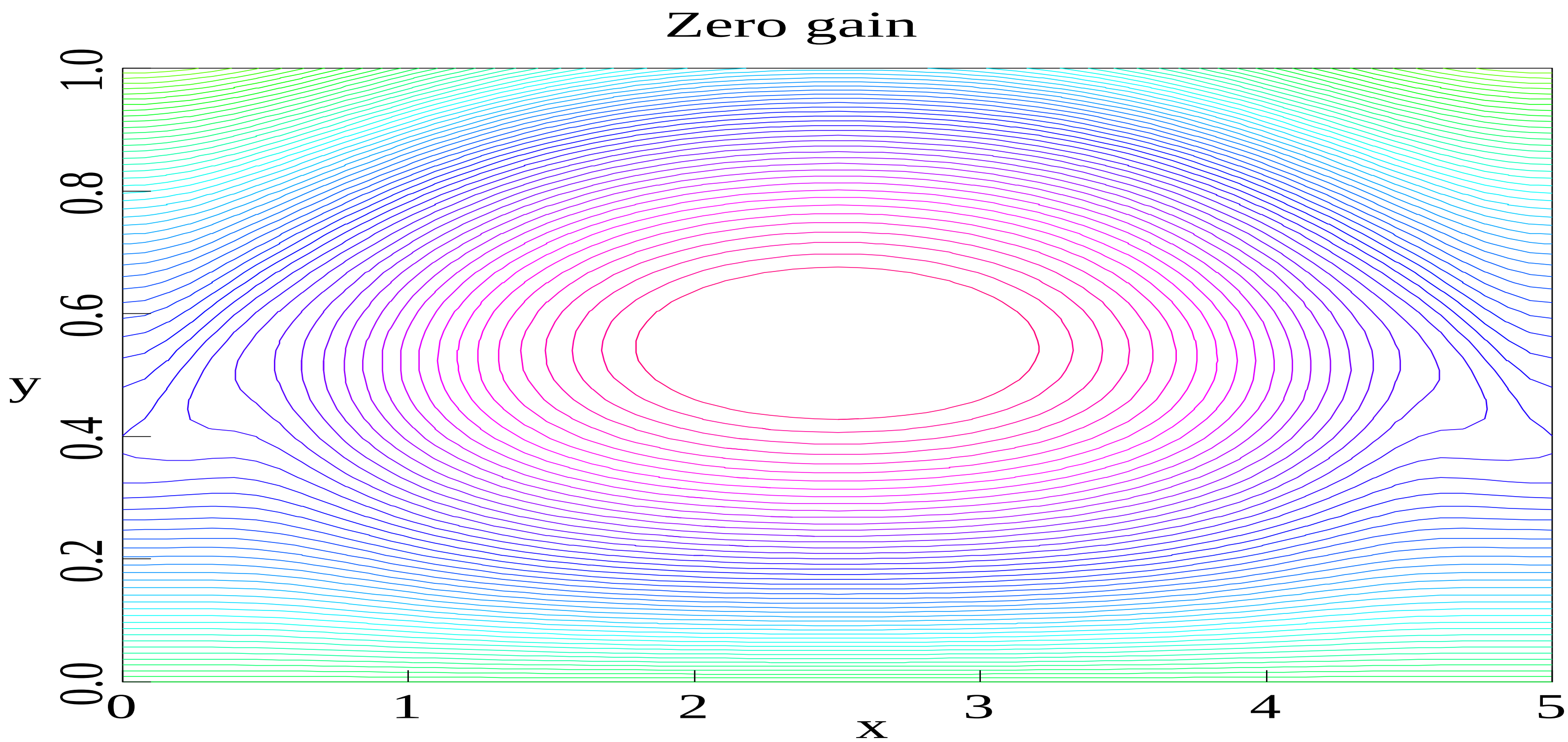

Figure 1: Large amplitude saturated island w/ resistive wall.

Equations

$$\overrightarrow{B} = \nabla\psi \times \widehat{z} + B_0\widehat{z}, \qquad \overrightarrow{v} = \nabla\phi \times \widehat{z} + v_{||}\widehat{z}$$

$$\left(\frac{\partial}{\partial t} + \nabla\phi \times \widehat{z} \cdot \nabla\right)\omega = \overrightarrow{B} \cdot \nabla j - \kappa T \partial n / \partial x + \mu \nabla^2 \omega$$

$$\frac{\partial}{\partial t}\psi - \overrightarrow{B} \cdot \nabla\phi = \eta \nabla^2 \psi + E(y)$$

$$\nabla^2\phi = -\omega, \qquad j = -\nabla^2\psi$$

$$\left(\frac{\partial}{\partial t} + \nabla\phi \times \widehat{z} \cdot \nabla\right)n = -n(\overrightarrow{B}/B_0) \cdot \nabla v_{||} + D\nabla^2 n$$

$$\left(\frac{\partial}{\partial t} + \nabla\phi \times \widehat{z} \cdot \nabla\right)v_{||} = -\frac{c_s^2}{n}(\overrightarrow{B}/B_0) \cdot \nabla n + \mu_{||}\nabla^2 v_{||}$$

RESISTIVE WALL BOUNDARY CONDITION AND MATCHING TO VACUUM

$$\frac{\tau_w}{L_y}\frac{\partial}{\partial t}\psi(x, y = L_y) = \left[\frac{\partial \psi}{\partial y}\right]_{y=L_y}$$

$$\tau_w = L_y \Delta / \eta_{wall}$$

Thin wall boundary condition.

Vacuum ($\nabla^2 \psi = 0$) for $0 < x < L_x$, $L_y < y < W$

RESISTIVE WALL AND VACUUM

Vacuum ($\widetilde{\psi}_{k,vac} \sim e^{\pm ky}$) and feedback boundary condition:

$$\psi(x, W) = -G\psi(x - \delta, L_y),$$

$$\widetilde{\psi}_k(W) = -Ge^{-ik\delta}\widetilde{\psi}_k(L_y) = -(G_r + iG_i)\widetilde{\psi}_k(L_y) \quad \Rightarrow$$

.

$$\frac{\tau_w}{L_y}\frac{\partial}{\partial t}\widetilde{\psi}_k(y = L_y) = -k\widetilde{\psi}_k(L_y)\left[\coth k(W - L_y) + \frac{G}{\sinh k(W - L_y)}\right] - (\frac{\partial \widetilde{\psi}_k}{\partial y})_{pl}$$

REAL GAIN

Proportional gain – real G: exactly equivalent to a wall closer, at $y = W'$ for a fixed k:

$$\coth k(W - L_y) + G_r/\sinh k(W - L_y) = \coth k(W' - L_y)$$

for one specific k, i.e. $W' = W'(G_r, W, k)$.

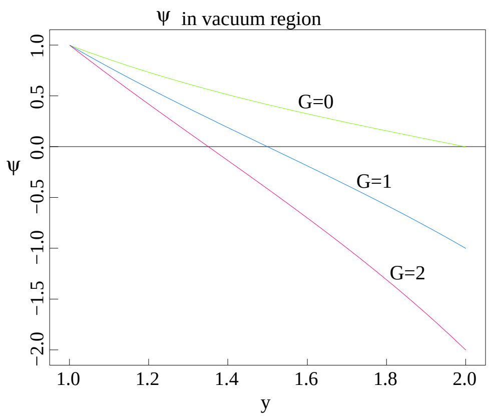

Figure 2: Real gain and equivalent wall position

TOROIDAL GEOMETRY

This equivalence works too for toroidal and nonlinear, except for the spectrum of k.

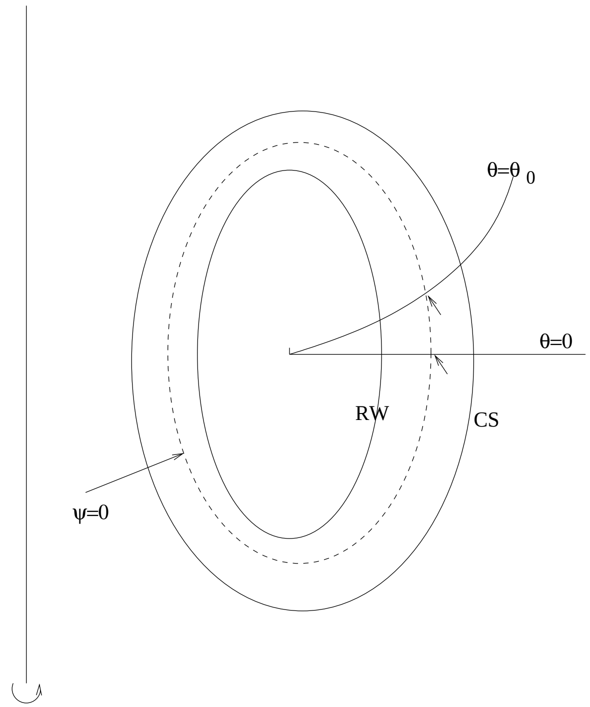

Figure 3: Effective conducting wall in a torus.

IMAGINARY GAIN

Stationary resistive wall with imaginary gain:

$$\frac{\tau_w}{L_y}\frac{\partial}{\partial t}\widetilde{\psi}_k(y=L_y) = -ik\psi_k(L_y)\left[\coth k(W-L_y) + i\frac{G_i}{\sinh k(W-L_y)}\right] - \left(\frac{\partial\widetilde{\psi}_k}{\partial y}\right)_{pl}$$

Rotating wall with no gain:

$$\frac{\tau_w}{L_y}\frac{\partial}{\partial t}\widetilde{\psi}_k(y=L_y) + \frac{ikv_0\tau}{L_y}\widetilde{\psi}_k(L_y) = -k\widetilde{\psi}_k(L_y)\left[\coth k(W-L_y)\right] - \left(\frac{\partial\widetilde{\psi}_k}{\partial y}\right)_{pl}$$

IMAGINARY GAIN, cont'd

Exact equivalence for single k:

$$\frac{ikv_0\tau_w}{L_y}\widetilde{\psi}(L_y) = ik\widetilde{\psi}(L_y)\frac{G_i}{\sinh k(W-L_y)}$$

–

$$v_0 = v_0(G_i, W, k) = \frac{G_i}{\sinh k(W-L_y)}$$

This equivalence holds nonlinearly too, except for the spectrum of k.

Complex gain is equivalent to a closer outside wall *-plus-* rotation of the RW.

But remember, rotational stabilization has hysteresis (locking-unlocking).

TWO RESISTIVE WALLS PLUS G_i

- Outer wall has effective rotation
- Differential rotation of the two walls, with the plasma rotation, can stabilize completely.

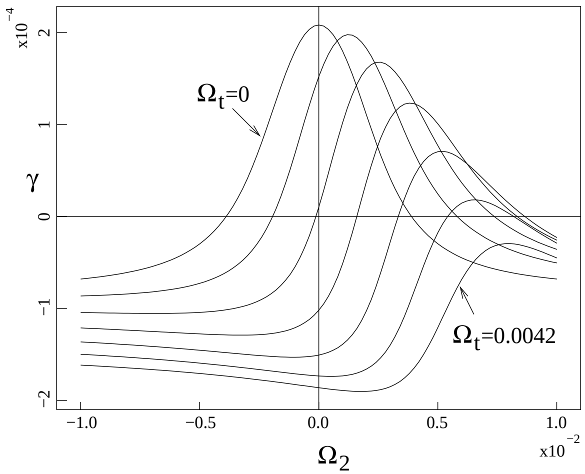

Figure 4: γ vs. Ω_2 for various Ω_t with $\Omega_1 = 0$ WLG.

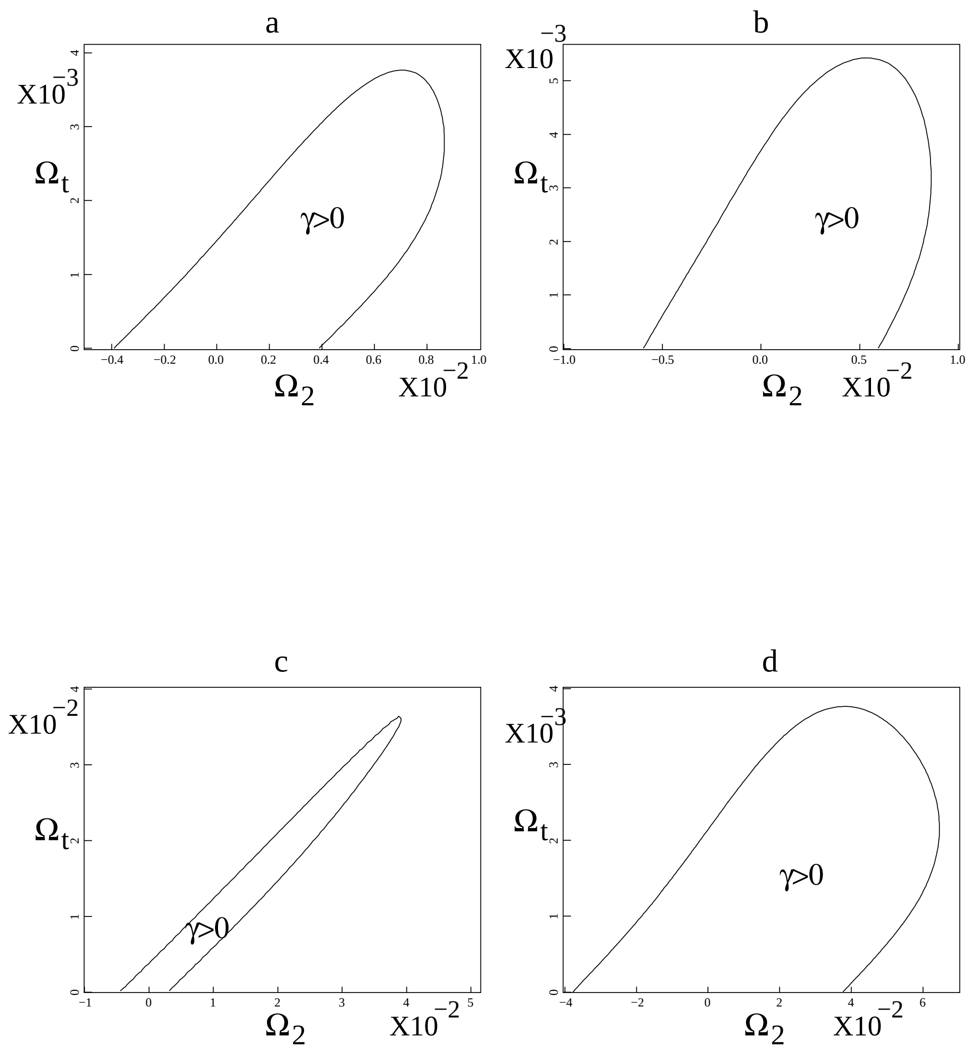

Figure 5: Region with $\gamma > 0$ for various τ_t, τ_1, τ_2.

MODE CONTROL EQUIVALENCE

[Simple] mode control: flux at control proportional to *plasma* currents, i.e. with $\widetilde{\psi}_{coil}(L_y)$ subtracted:

$$\widetilde{\psi}_{coil}(y) = \widetilde{\psi}(W)e^{-k|W-y|}$$

is the flux due to the control currents at $y = W$.

$$\widetilde{\psi}(W) = -G\left(\widetilde{\psi}(L_y) - \widetilde{\psi}(W)e^{-k(W-L_y)}\right)$$

$$\widetilde{\psi}(W) = -\frac{G\widetilde{\psi}(L_y)}{1 - Ge^{-k(W-L_y)}}$$

$$G' = \frac{G}{1 - Ge^{-k(W-L_y)}}$$

Equvalent to larger gain without “mode control”. *Full MC* - account for currents in RW: ’equivalence’ but with γ dependence.

Sensing the poloidal field inside the resistive wall

$$\gamma\tau_w \widetilde{\psi}_k(y=1) = \widetilde{\psi}'_k(1+) - \widetilde{\psi}'_k(1-) \qquad \widetilde{\psi}'_k(1+) = -A_k\widetilde{\psi}_k(1) + B_k\widetilde{\psi}_k(W)$$

$$(\gamma\tau_w + A_k)\widetilde{\psi}_k(1) = B_k\widetilde{\psi}_k(W) - \widetilde{\psi}'_k(1-)$$

Recall radial field: $\widetilde{\psi}_k(W) = -G\widetilde{\psi}_k(1)$

$$\widetilde{\psi}_k(1) = -\frac{1}{\gamma\tau_w + A_k + B_k G}\widetilde{\psi}'_k(1-)$$

Poloidal field inside RW: $\widetilde{\psi}_k(W) = K\widetilde{\psi}'_k(1-)$ [NOTE 90^o PHASE SHIFT] – $\widetilde{\psi}_k(1)$ can be *zero*.

$$\widetilde{\psi}_k(1) = -\frac{1 - B_k K}{\gamma\tau_w + A_k}\widetilde{\psi}'_k(1-)$$

LINEAR THEORY $\lambda = 0.5,\ \eta = \mu = D = \mu_{||} = 10^{-4},\ c_s/v_A = 0.25,\ L_y = 1,\ L_x = 5,\ W = 2.5,\ \tau_w = 1000$

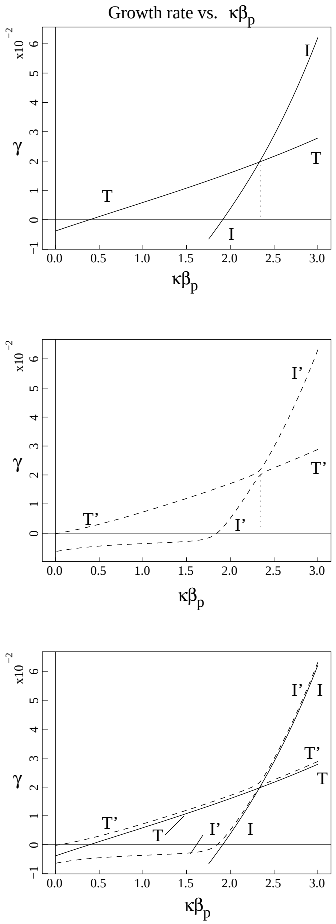

Figure 6: γ vs. $\kappa\beta_p$ with PC and resistive wall; $B_x(y) = \tanh[(y - 1/2)/\lambda]$

Linear theory, continued

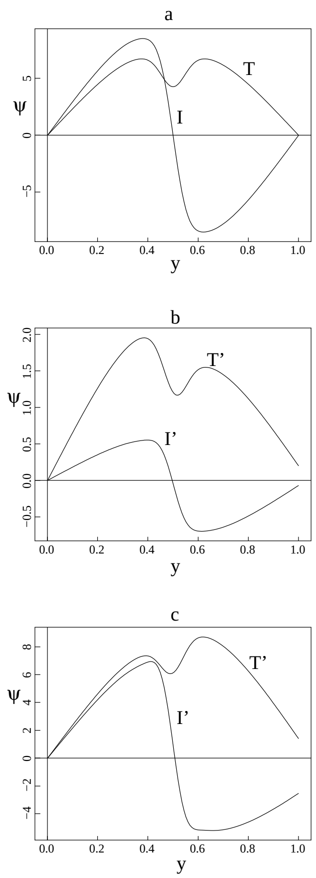

Figure 7: Mode structure (RW) with $\kappa\beta_p$ above crossing, just below crossing, and at marginal stab. for I'

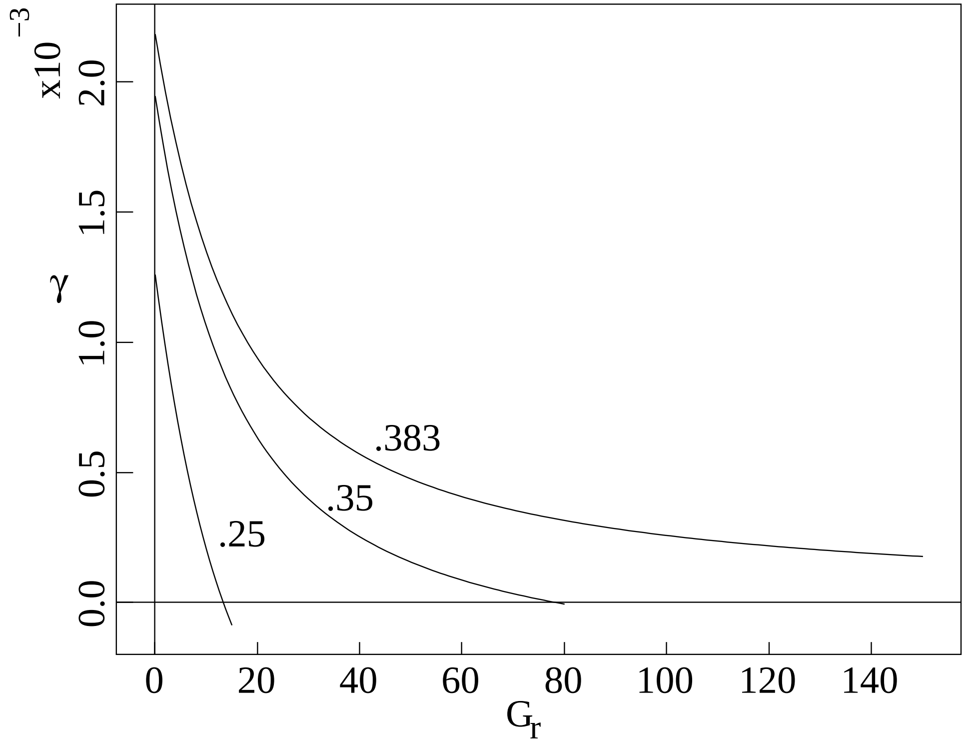

Figure 8: γ vs G_r for $\kappa\beta_p = 0.25, 0.35,$ and 0.383 (marginal with PC wall).

NONLINEAR SIMULATIONS

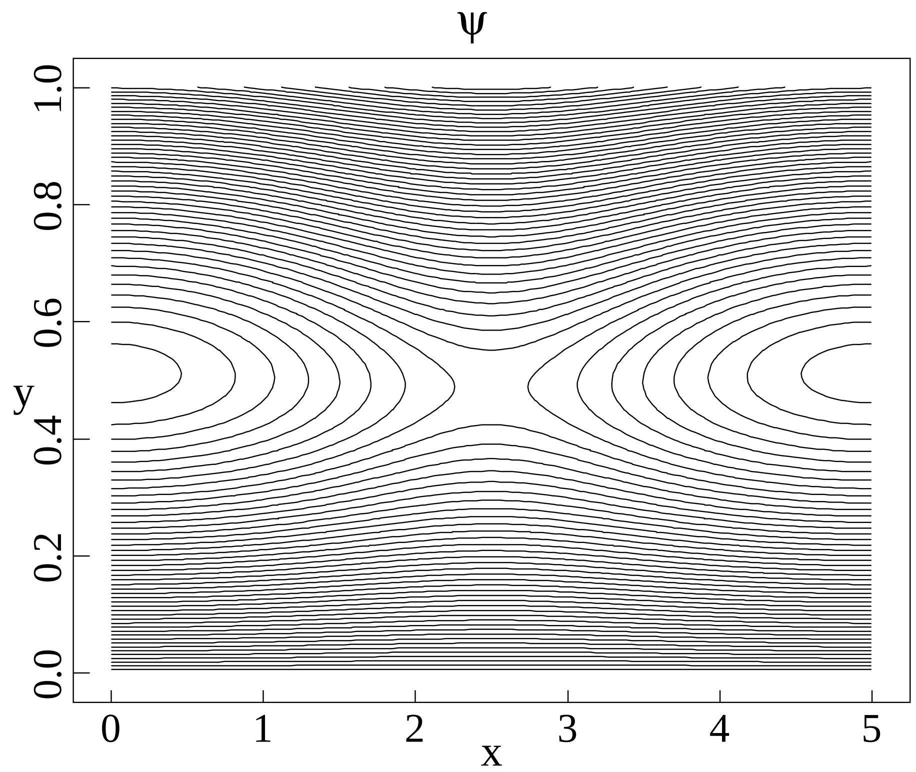

Figure 9: The case with zero gain (RW) is mixed tearing-interchange and has an island at saturation.

Table 1: Saturation amplitude $\|\widetilde{\psi}\|$ vs. G_r or equivalent wall position W'.

G_r or W'	γ	$\|\widetilde{\psi}\|$ with G_r	W'
Base case $(0.0,\ W = 2.5)$	1.95×10^{-3}	0.0165	—
$G_r = 5.0,\ \ W' = 1.3225$	1.44×10^{-3}	0.00608	0.00604
$10.0,\ \ 1.1953$	1.06×10^{-3}	0.00382	0.00377
$20.0,\ \ 1.1102$	6.3×10^{-4}	0.00226	0.00221
$30.0,\ \ 1.077$	4.0×10^{-4}	$0.00162*$	0.00156

Table 2: Saturation amplitude $\|\widetilde{\psi}\|$ as a function of G_i or wall velocity v_0.

G_i or v_0	γ	$\|\widetilde{\psi}\|$ with G_i	v_{wall}
$G_i = 5.0,\ \ v_0 = 1.55 \times 10^{-3}$	1.83×10^{-3}	0.0164	0.01685
$10.0,\ \ 3.11 \times 10^{-3}$	1.46×10^{-3}	0.0165	0.0165
$15.0,\ \ 4.66 \times 10^{-3}$	8.7×10^{-4}	$0.0165*$	0.0165
$20.0,\ \ 6.21 \times 10^{-3}$	3.2×10^{-4}	0.0165	$0.0167*$
$80.0,\ \ 2.48 \times 10^{-4}$	$0*$	$0.016*$	$0.016*$
$100.0,\ \ 3.11 \times 10^{-2}$	0	0	0

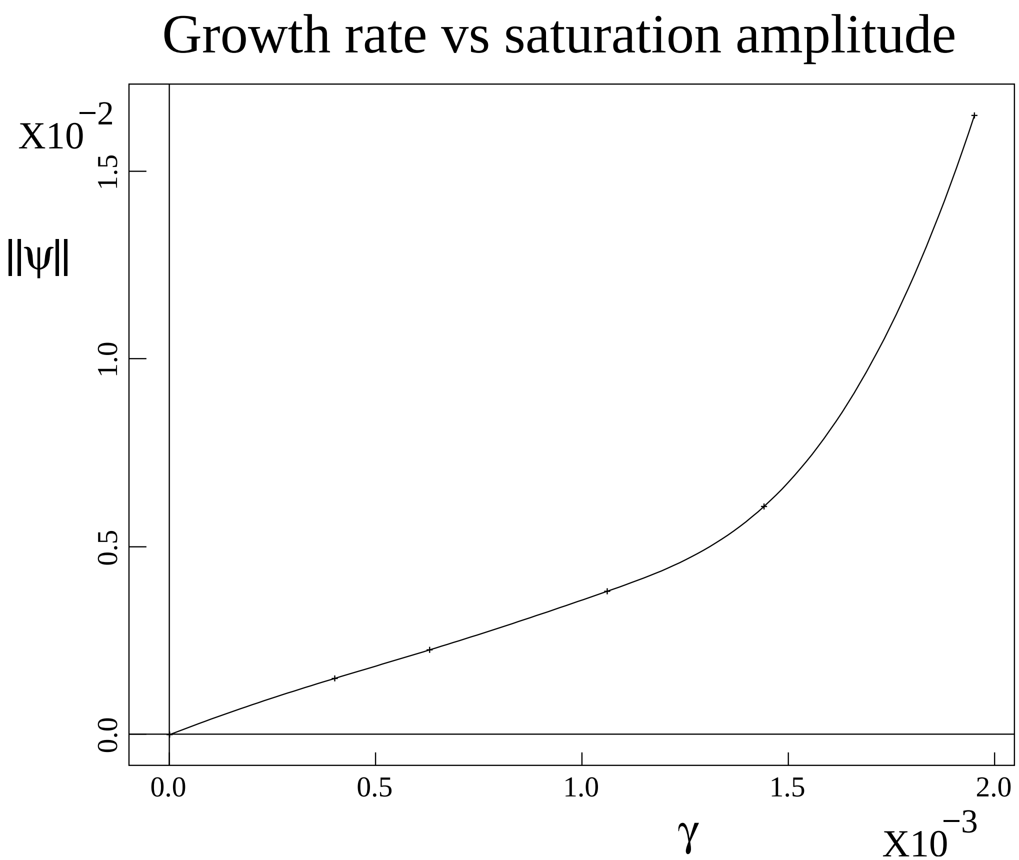

Figure 10: $\kappa\beta_p = 0.35$; for small γ, the saturated $||\widetilde{\psi}||$ is linear and small.

CONCLUSIONS

- Real (proportional) gain is equivalent to a closer perfectly conducting wall for each k.

- Imaginary gain is equivalent to rotation of the resistive wall, which is equivalent to rotating the plasma in the opposite direction.

- Rotational stabilization (G_i) has hysteresis, which might be dangerous, i.e. allow locking for finite perturbation even if RWM is linearly stable. Two resistive walls with G_i can stabilize linearly for any plasma rotation; probably there is no locking (hysteresis).

- β must be below the resistive-plasma, ideal wall marginal point, above the resistive-plasma, no-wall limit. 'Tearing' and 'interchange' cross near marginal stability.

CONCLUSIONS, cont'd

- The linear equivalences work pretty well in nonlinear simulations.

- Just below the resistive-plasma, ideal wall marginal point, very large gain is required for linear stabilization, but **much** smaller gain is required for benign saturation.