Beta-Limits of the Resistive Wall Mode with Kinetic Effects

Bo Hu and Riccardo Betti

Depts. of Mech. Eng. and Physics & Astronomy,
Laboratory for Laser Energetics

University of Rochester, Rochester, NY 14623

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Trapped particles are stabilizing

- Kinetic effects are stabilizing for the RWM

- For a small equilibrium electric fields, the trapped particle contribution improves the beta limits by decreasing the fluid instability drive

- For large equilibrium electric fields, the trapped particle contribution is still large but dissipative. This causes the RWM to rotate leading to AC wall stabilization.

- For extra-large equilibrium electric fields, the trapped particle contribution is small.

The trapped particle compressibility contributes to the total δW

- **RWM marginal stability condition**

$$\delta W_{tot} = \delta W_{Fluid} + \delta W^{\infty}_{vacuum} + \delta W^{trapped}_{kinetic} < 0$$

- **The effects of trapped particles enters through the kinetic component of the perturbed perpendicular pressure**

$$\delta W^{trapped}_{kin} = -\int d^3 r (\tilde{p}^K_\perp)(\kappa \bullet \xi_\perp)$$

The trapped particle compressibility is stabilizing for low frequency modes and small equilibrium electric field

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- **Trapped particle contribution**

$$\delta W_K(\omega) \sim -\in^{3/2} \delta W_f \frac{\omega - \omega_{E\times B} - \omega_*}{\omega - \omega_{E\times B} - \omega_D}$$

- **The kinetic δW is positive and stabilizing for small equilibrium electric fields and low frequency modes**

$$\omega_{\mathrm{E\times B}} < \omega_{\mathrm{D}}, \qquad \omega < \omega_{\mathrm{D}}, \qquad \frac{\omega_*}{\omega_{\mathrm{D}}} \sim \frac{1}{\in}$$

$$\delta W_K(\omega = 0) \sim -\sqrt{\in}\,\delta W_f$$

Small electric field → Slow rotation

- **The E-field is given by the ion force balance**

$$\omega_{E\times B}\,\mathrm{sgn}(I_\phi) = \Omega_{rot} - \omega_{*i}\,\mathrm{sgn}(I_\phi)$$

$$\omega_{E\times B} = -\frac{qE_r}{rB}$$

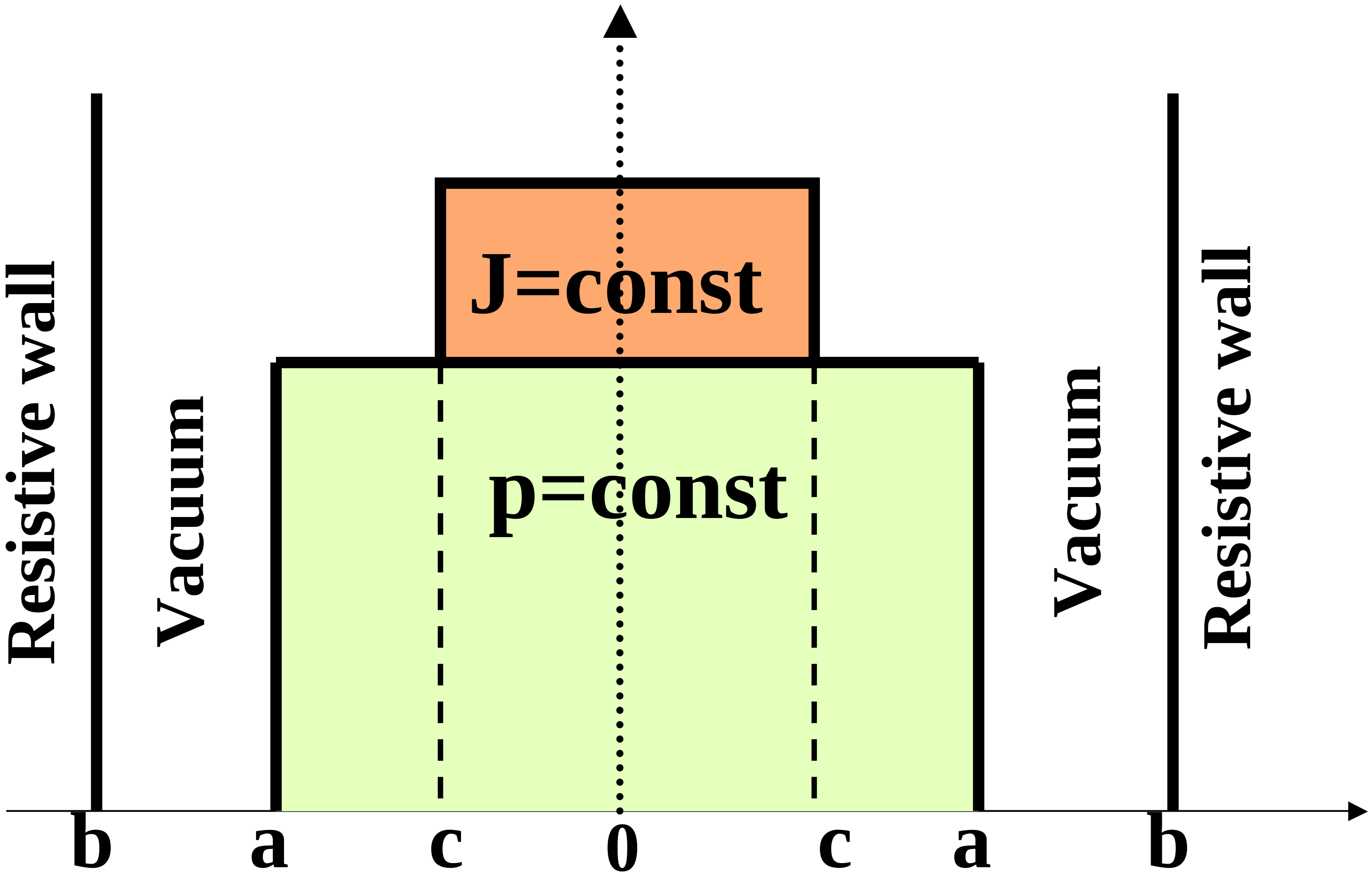
- **A small EXB drift ($\omega_{EXB} < \omega_D \sim \epsilon\, \omega_{*i}$) requires**

$$\Omega_{rot} \approx \omega_{*i}\,\mathrm{sgn}(I_\phi)$$

- **For DIII-D at the q=2 surface**

$\omega_D \approx 4\,\mathrm{kHz}$ $\qquad$ $\omega_{*i} \approx 12\,\mathrm{kHz}$

A simplified sharp boundary equilibrium is used to calculate the kinetic effects on the RWM



The stability problem is reduced to the simple fluid theory in the plasma core.

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- **The kinetic pressure enters in the linearized momentum equation**

$$-\rho\omega^2\vec{\xi} - (\vec{\mathbf{B}}\bullet\nabla \mathrm{B})\vec{\mathbf{b}} + \nabla\left(p^F + \frac{B^2}{2} + \tilde{p}_\perp^K \right) + \vec{\kappa}\left(\tilde{p}_\perp^K + B^2 \right) = 0$$

- **The kinetic pressure vanishes in the plasma core because the equilibrium density and pressure are flat**

$$\tilde{p}_\perp^K \sim \frac{\partial f}{\partial r} = 0 \text{ inside the plasma } (r < a)$$

The fluid solution in the plasma core is determined using the small *a*/*R* expansion

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- **The perturbed magnetic flux follows simple power laws of *r***

$$\tilde{\psi}_m(r \le c) = \psi_m^0 \left(\frac{r}{c}\right)^{|m|} \qquad \tilde{\psi}_m(c \le r \le a) = \psi_{1m}\left(\frac{r}{c}\right)^{|m|} + \psi_{2m}\left(\frac{c}{r}\right)^{|m|}$$

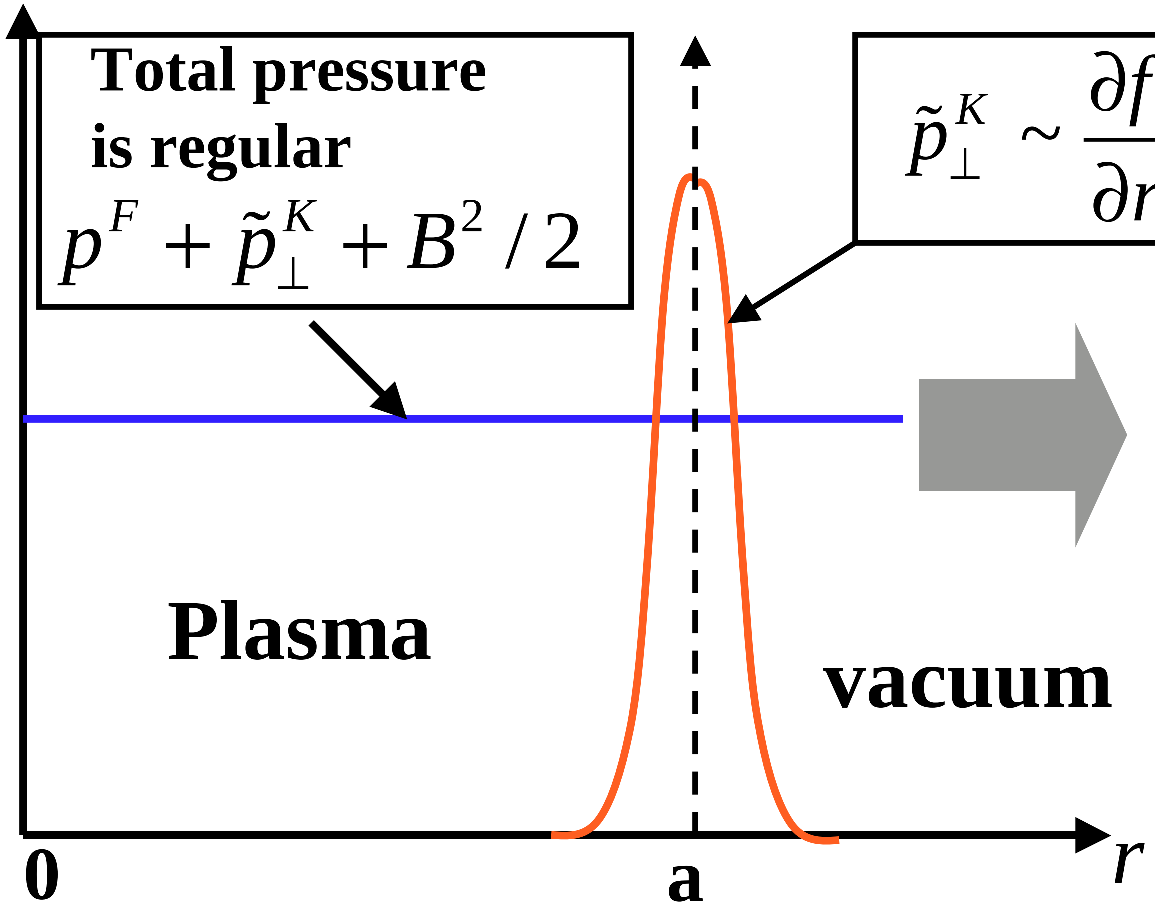
- **The constants are determined through the matching conditions at *r*=*c***

$\tilde{\psi}_m$ and $d\tilde{\xi}_m / dr$ are continuous at $r = c$

$$\tilde{\psi}_m = rBh_m\tilde{\xi}_m / m \qquad h_m = n - m/q$$

The singular kinetic pressure leads to a singular perturbed magnetic field

- **Delta-Function Like Kinetic Pressure at Plasma Edge**



$$\tilde{p}_\perp^K + \frac{B^2}{2} = \text{regular}$$

$$\left[\tilde{p}_\perp^K = -B\tilde{B}_\parallel\right]_a$$

$\tilde{B}_\parallel$ is singular

The kinetic pressure enters through the boundary conditions at the plasma edge

- **Boundary condition at the plasma edge**

$$\left[\, \tilde{p}_\perp^K = -B\tilde{B}_{||} \,\right]_a$$

$$-\hat{n}\bullet\nabla\left[\, p^F + \tilde{p}_\perp^K + B^2/2 \,\right] + \kappa\bullet\hat{n}\left[\, \tilde{p}_\perp^K + 2B\tilde{B}_{||} \,\right] = 0$$

- **B.C. from integration across the plasma edge**

$$[|\, p^F + B^2/2 \,|]_{edge} = -\hat{n}\bullet\kappa\int_{a-}^{a+} dr\,\tilde{p}_\perp^K$$

The trapped particle contribution to the boundary pressure jump condition is only √ϵ lower than the fluid terms

Fluid instability drive

$$-\frac{h_m}{m} a\tilde{\psi}'_m(a) + \frac{3}{2}\frac{\beta}{\in}\left(\frac{m+1}{h^{\mathbf{B}_v}_{m+1}}\tilde{\psi}_{m+1} + \frac{m-1}{h_{m-1}}\tilde{\psi}_{m-1}\right)_a$$

$$-\frac{\beta}{\in}\sqrt{\in}\sum_k\left(kA_{m,k}\frac{\tilde{\psi}_k}{h_k} + B_{m,k}\, r\frac{d}{dr}\frac{\tilde{\psi}_k}{h_k}\right)_a$$

Trapped particle term

$$= \frac{R_0^2}{B}\left(\mathbf{B}_{Vacuum} \bullet \tilde{\mathbf{B}}_{Vacuum}\right)_a$$

Vacuum contribution

The coefficients of the kinetic terms are integrals over the pitch angle

$$A_{m,k} = \frac{3}{8\sqrt{2\pi}} \int_0^1 \frac{K(u)(\sigma_{m-1} + \sigma_{m+1})(\sigma_{k-1} + \sigma_{k+1})}{(4s_m + 2)\dfrac{E(u)}{K(u)} + 4s_m(u-1) - 1} du$$

$$B_{m,k} = \frac{3}{8\sqrt{2\pi}} \int_0^1 \frac{K(u)(\sigma_{m-1} + \sigma_{m+1})(\sigma_{k-1} - \sigma_{k+1})}{(4s_m + 2)\dfrac{E(u)}{K(u)} + 4s_m(u-1) - 1} du$$

$$\sigma_m = \frac{1}{K(u)} \int_0^{\pi/2} \frac{\cos\left[2(m - nq_a)\arcsin(\sqrt{u}\sin x)\right]}{\sqrt{1 - u\sin^2 x}}$$

$$s_m = \left(\frac{r}{q}\frac{dq}{dr}\right)_a$$

The full dispersion relation is derived by matching the vacuum to the plasma solution

$$\frac{R_0^2}{B}\boldsymbol{B}_{Vac}\bullet\tilde{\boldsymbol{B}}_{Vac}=\sum_{j=-\infty}^{\infty}\delta_{m,j}\tilde{\psi}_j(a)$$

Resistive wall terms

$$\delta_{m,j}=\frac{j}{h_j}\sum_{l=-\infty,\neq 0}^{\infty}|\,l\,|^{-1}\mu_l G_{l-j}^l G_{m-j}^l$$

$$\mu_l=\frac{2|\,l\,|-i\omega\tau_w[1+(a/b)^{2|l|}]}{2|\,l\,|-i\omega\tau_w[1-(a/b)^{2|l|}]}$$

$$G_l^m=\frac{1}{\pi}\int_0^{\pi}d\theta\cos(l\theta)\left[\frac{m}{q(a,\theta)}-n\right]$$

τ_w = resistive wall magnetic diffusion time

The RWM growth rate is found by setting to zero the determinant of the linear system

- **The only parameters affecting the stability are:**

  β/ϵ, edge $q=q(a)$, central $q=q\,(0)$ and $\epsilon=a/R_0$

- **Here $q(a)$ is set to 2.5, $q\,(0)=1.1$ and the RWM growth rate is calculated for varying β/ϵ**

The relevant parameter is the fraction of the maximum beta-limit improvement

- Maximum beta limit improvement = β_b^{MHD} - β_∞^{MHD}
- **Define the degree of β limit improvement χ**

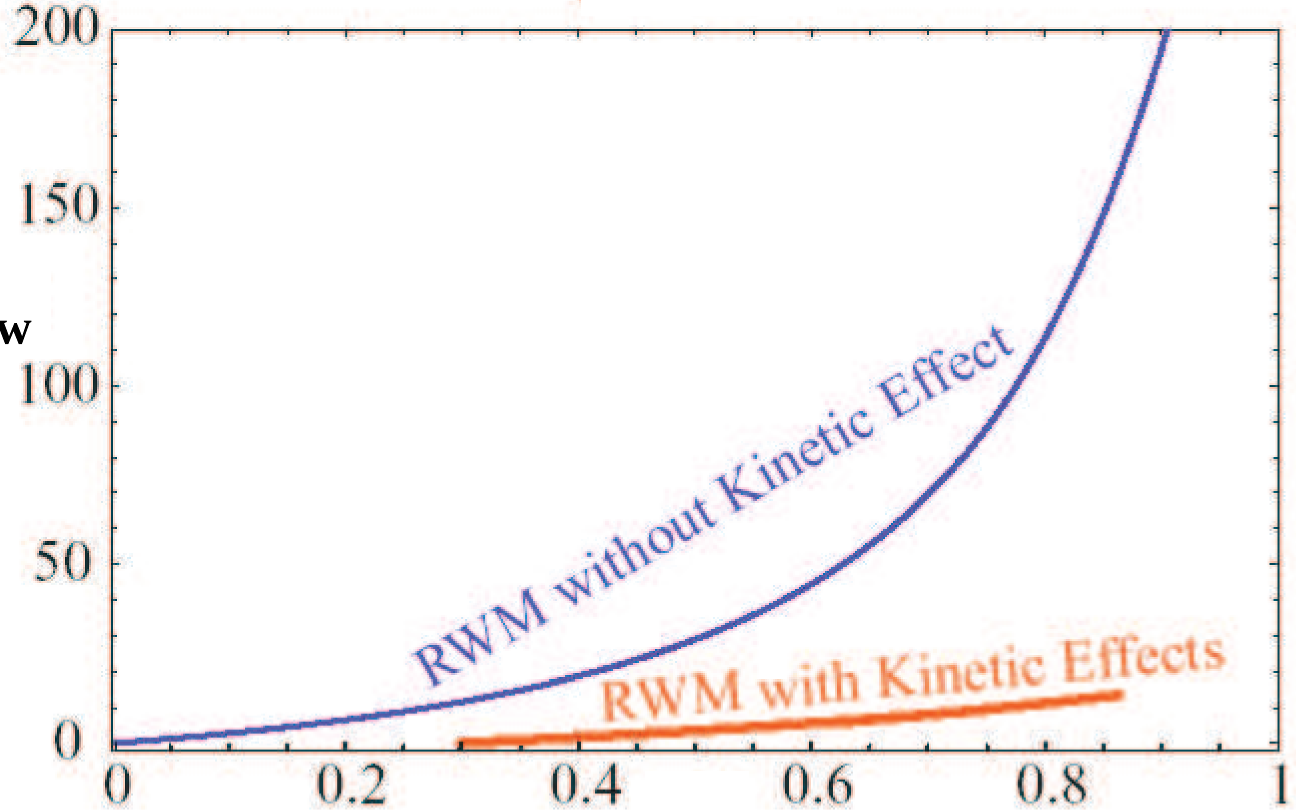
$$\chi = \frac{\beta - \beta_\infty^{MHD}}{\beta_b^{MHD} - \beta_\infty^{MHD}}$$

- β_∞^{MHD} beta limits without wall
- β_b^{MHD} beta limits with ideal wall at r=b

- If Marginal Stability at χ=1 → full ideal wall β limits are recovered
- If Marginal Stability at χ=0 → beta limits are same as no wall
- If Marginal Stability at $\chi = \chi_0$ → fraction χ_0 of the ideal wall limits is recovered.

Results for standard aspect ratios: R/a=3

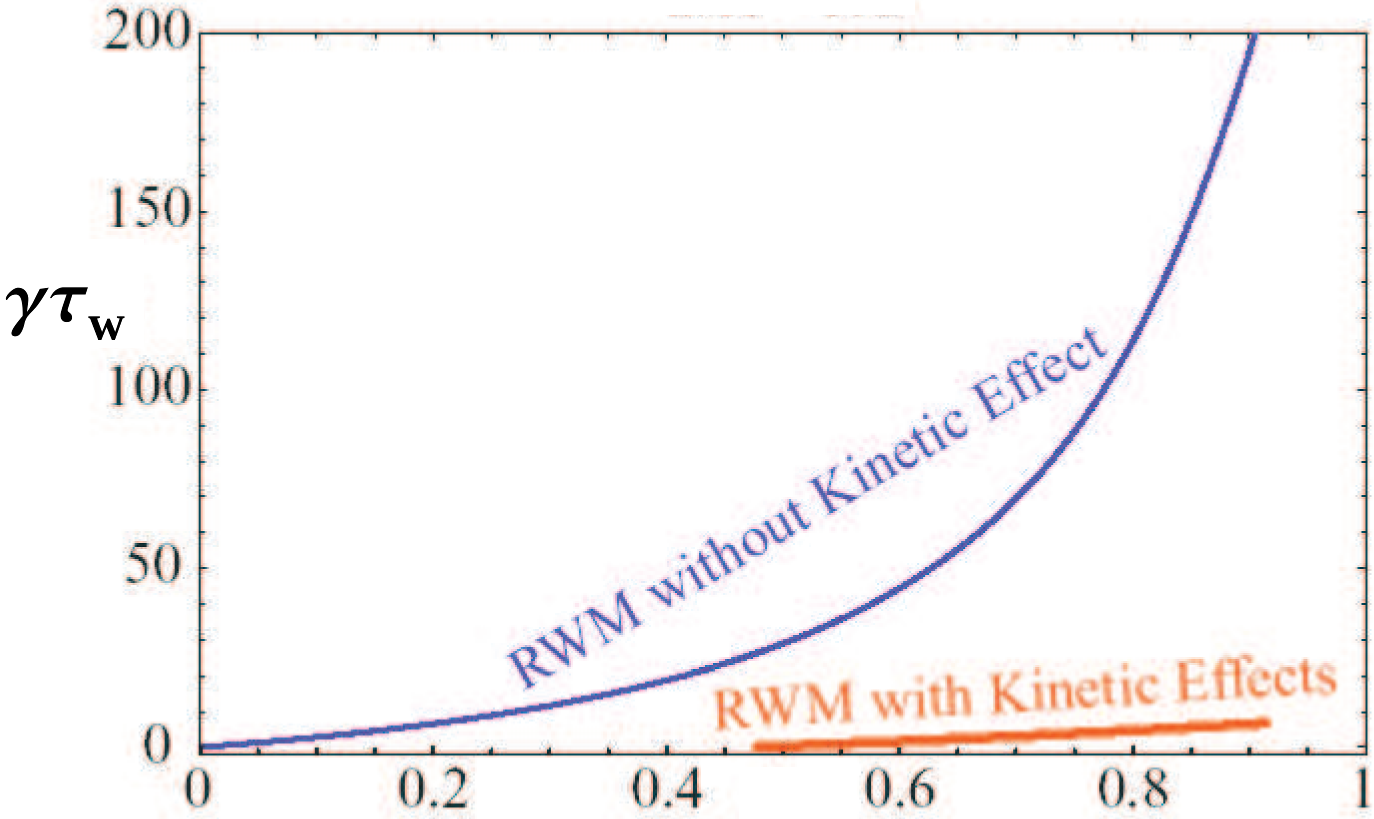
$\gamma\tau_w$



χ = β-limits improvement

30% of β_b- β_∞ is recovered by kinetic effects

Results for tight aspect ratios: *R*/*a*=1



χ = β-limits improvement

47% of β_b- β_∞ is recovered by kinetic effects

Fast rotation $\Omega_{rot} > \omega_*$ requires a large electric field that gives a dissipative character to the kinetic terms

The kinetic terms are resonant:

$$\delta W_K(\omega = 0) \sim \in^{3/2} \delta W_f \frac{\omega_{E\times B} + \omega_*}{\omega_{E\times B} + \omega_D}$$

- **The sign of ω_{EXB} depends on the rotation direction**

$$\omega_{E\times B}\,\mathrm{sgn}(I_\phi) = \Omega_{rot} - \omega_{*i}\,\mathrm{sgn}(I_\phi)$$

- **ω_{EXB} >0 for fast co-injection**

$$\omega_{E\times B} = \Omega_{rot} - \omega_{*i} > 0$$

- **ω_{EXB} <0 for counter-injection**

$$\omega_{E\times B} = -\Omega_{rot} - \omega_{*i} < 0$$

Co-injection leads to resonance with the electrons while counter-injection leads to resonance with ions

Resonance condition $\Rightarrow \omega_{E\times B} + \omega_D = \omega_{E\times B} + \langle \omega_D \rangle Z_{i,e} \dfrac{\varepsilon}{T_{i,e}} = 0$

- **Here $Z_{i,e}$ = ±1 for ions (+) and electrons (-1)**

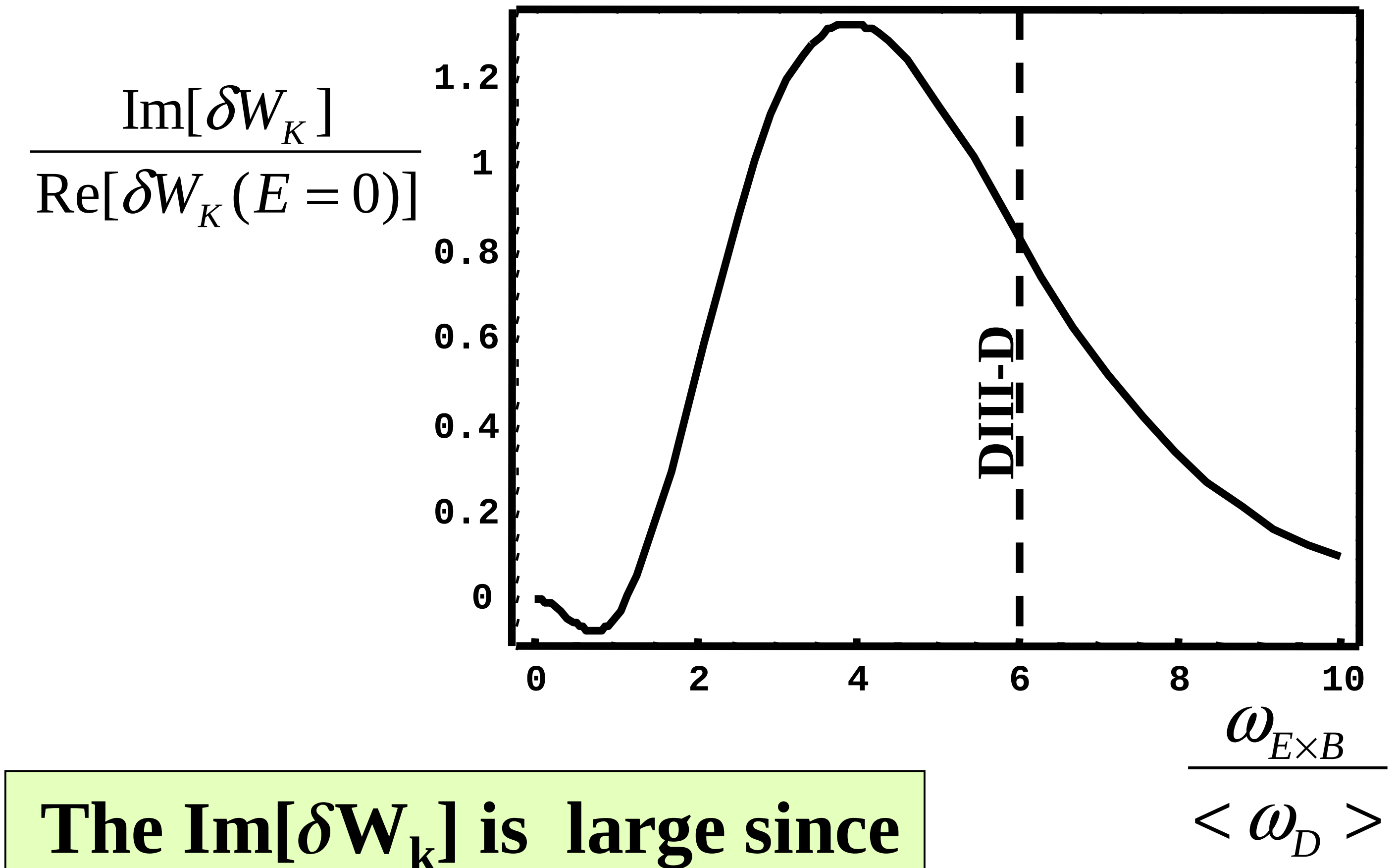
- **Consider DIII-D values for very high beta discharges**

- **Assume co-injection:**
  Ω_{rot}~ 36 kHz, ω_{*i}~ 12kHz → ω_{EXB}~24kHz

- **Resonance requires high energy electrons (use ω_D~4kHz)**

$$\frac{\omega_{E\times B}}{\langle \omega_D \rangle} \approx 6 \approx \frac{\varepsilon}{T_e}$$

The trapped particle dissipation is large

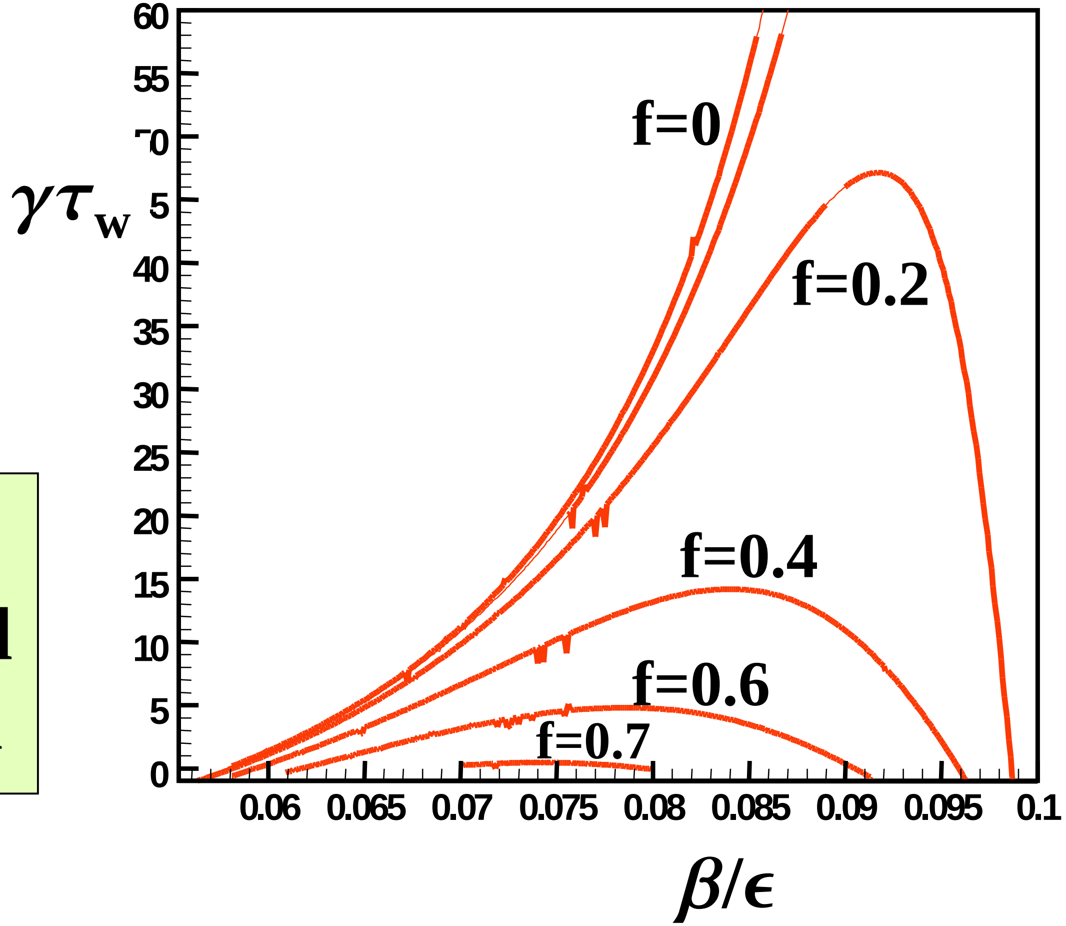


The Im[δW$_k$] is large since Re[δW$_k$(E=0)]~$\sqrt{\epsilon}\,\delta$W$_F$

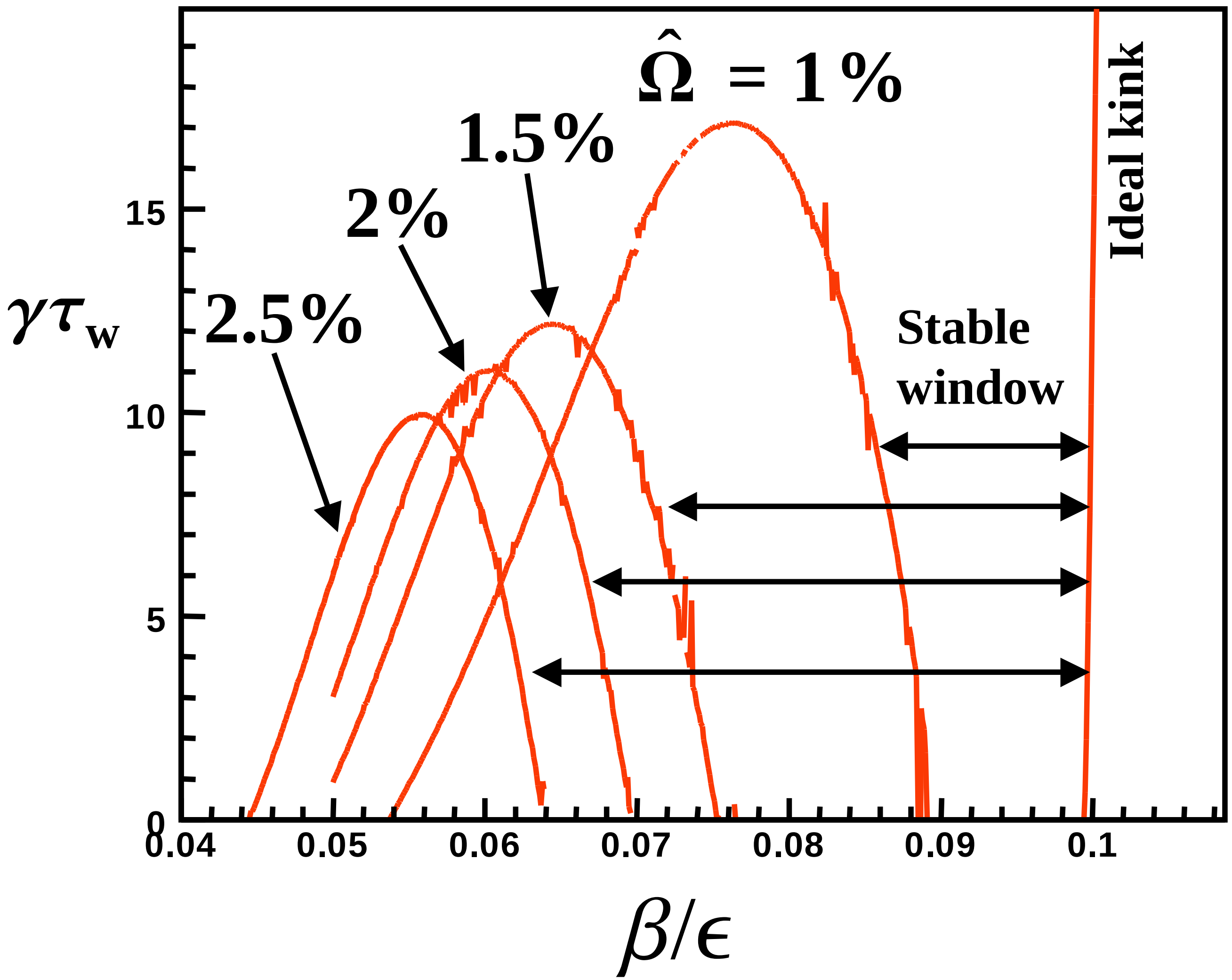
The large trapped particle dissipation stabilizes the RWM without plasma inertia

Plasma inertia is not included

f = fraction of Im[δW_k] retained In the calculation



20% of the dissipation is sufficient to open a stability window for rotations of 1% of ω_A



f=0.2

Plasma inertia is included

$$\hat{\Omega} = \frac{\Omega_{rot}}{\omega_A}$$

Conclusions

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- **For large equilibrium electric fields, the trapped particle contribution is still large but dissipative. This causes the RWM to rotate leading to AC wall stabilization.**

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References

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