ANALYSIS OF STABLE RESISTIVE WALL MODES IN ROTATING PLASMAS

by
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KEY RESULT

- Measured interaction between plasma and externally applied fields in quantitative agreement with predictions of semi-empirical single-mode MHD model

KEY PHYSICS

- Validation of single rigid mode approach, the basis of several RWM feedback models
- RWM rotation w.r.t. the wall is NOT required for stabilization by plasma rotation

  ☆ Most physics based RWM models DO NOT agree with this result

  Resistive MHD:

  Finn, *Phys. Plasmas* (1995); Boozer, *Phys. Plasmas* (1995);
  Gimblett and Hastie, *Phys. Plasmas* (2000)

  Ideal MHD with dissipation:

  Bondeson and Ward, *Phys. Rev. Lett.* (1994); Betti and Freidberg, *Phys. Rev. Lett.* (1995);
  Fitzpatrick and A. Aydemir, *Nucl. Fusion* (1996); Fitzpatrick, *Phys. Plasmas* (2002)

INSTABILITIES OBSERVED AT □$_N$ ABOVE NO-WALL LIMIT HAVE CHARACTERISTICS OF PREDICTED RESISTIVE WALL MODE

- **Theory**
  - □~ □$_W^{□1}$ for □$_N$ ≥ □$_N^{no\text{-}wall}$
  - □ ~ □$_W^{□1}$ << □$_{plasma}$
  - Mode structure similar to ideal external kink
  - Stable for □ $_{plasma}$ > □ $_{crit}$
  - □>> □$_W^{□1}$ for □$_N$ ≥ □$_N^{ideal\text{-}wall}$

- **Experiment** □$_W$ (n=1) ~ 4 ms
  - □$^{-1}$ ~ 1 to 8 ms in good agreement with □$_W$
  - Mode nearly stationary from its onset while plasma rotates
    - ★ *f* ~ 0 to 60 Hz in good agreement with 1/2□□$_W$ (~ 40 Hz)
  - Radial mode structure agrees with ideal MHD prediction
  - □ $_{crit}$ clearly observed
    - ★ □ $_{crit}$/2□ ~ 5 kHz in good agreement with ~2% (1/2□□$_A$) at q=2
  - □$^{□1}$□ 200 □s and □$_N^{ideal\text{-}wall}$□ 2x□$_N^{no\text{-}wall}$ in good agreement with calculations

DIII-D MEASUREMENTS OF THE ROTATIONALLY STABILIZED RWM HIGHLIGHT NEED FOR NEW PHYSICS MODEL

- Analyzed measurements of the RWM response to external resonant fields vs. Ω, at $\beta > \beta_{nowall}$
- Generalized an ideal MHD model (Garofalo-Jensen-Strait, Phys. Plasmas, 2002) to include the effects of plasma rotation and dissipation
  - Slab-geometry formulation of general-geometry theory by Chu et al. (Nucl. Fusion, 2003)
  - Describe the effects of error fields on a high-β plasma
  - Describe the RWM dispersion relation in the parameter range explored (special case of no external fields)
- Found that the new model can explain quantitatively the experimental observations
  - Need a physics mechanismm for RWM stabilization without (much) dissipation
  - RWM stabilization and plasma rotation braking must be two aspects of the same physics mechanism
- Hu and Betti, "Ion kinetic effects on resistive wall modes", APS '03

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Analysis of stable resistive wall modes in a rotating plasma

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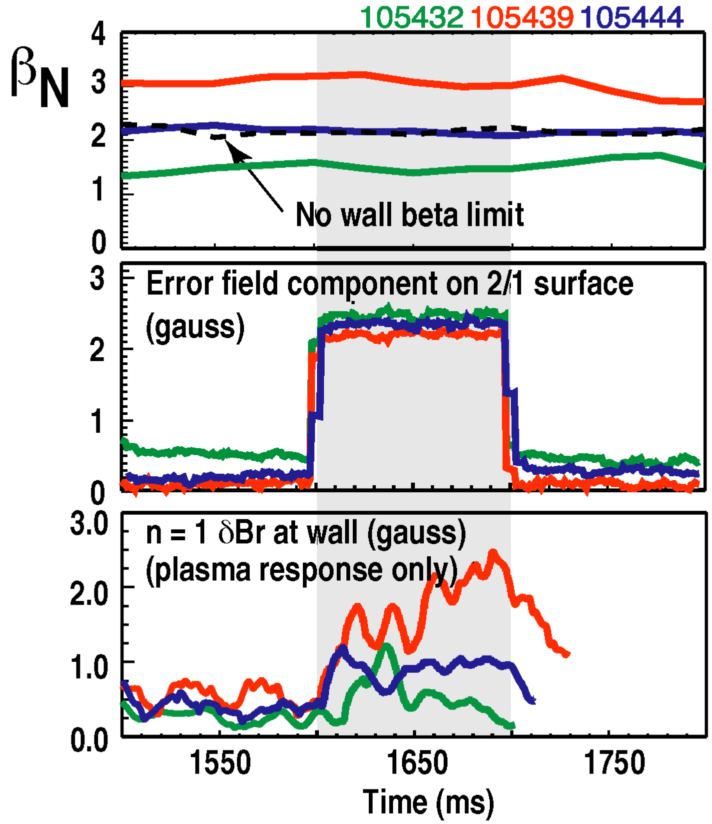
RESISTIVE WALL MODE STABILIZED BY ROTATION IS WEAKLY DAMPED - HAS STRONG RESONANT RESPONSE TO EXTERNAL PERTURBATIONS

- Use external n = 1 field pulses to probe RWM dispersion relation at $\beta_N > \beta_N^{no\text{-}wall}$.
  - Mode is rotationally stabilized

- Resonant field amplification yields three RWM measurements:
  - Growth rate (negative)
  - Toroidal phase relative to external pulse
  - Amplitude (asymptotic)



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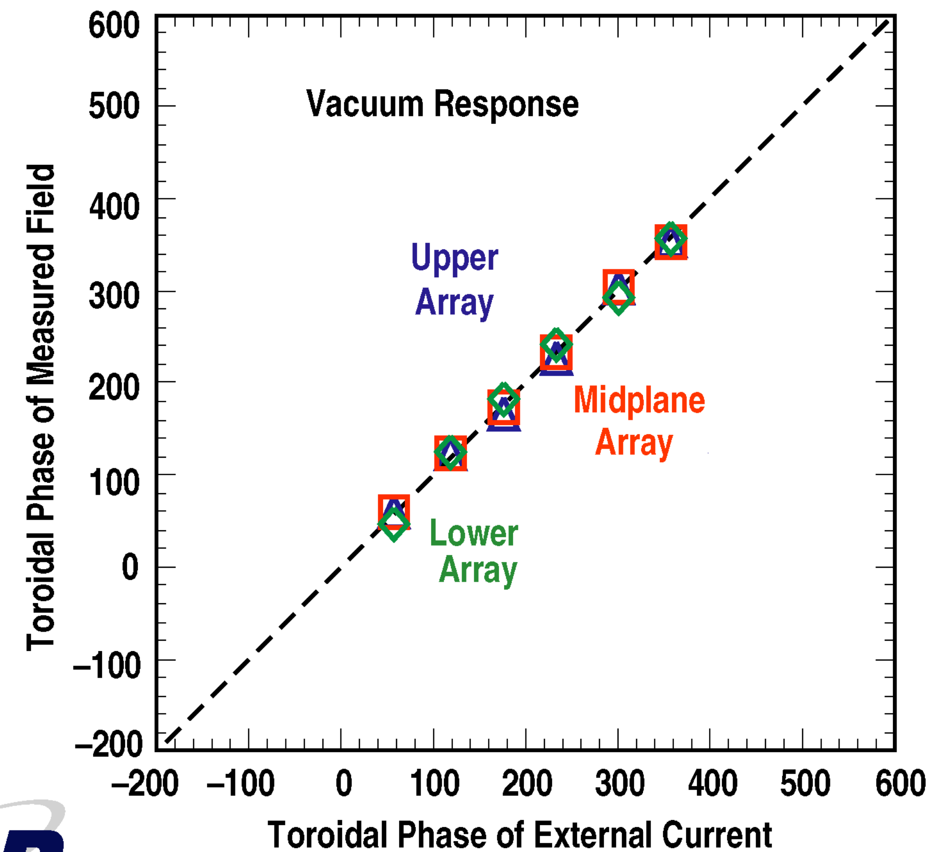
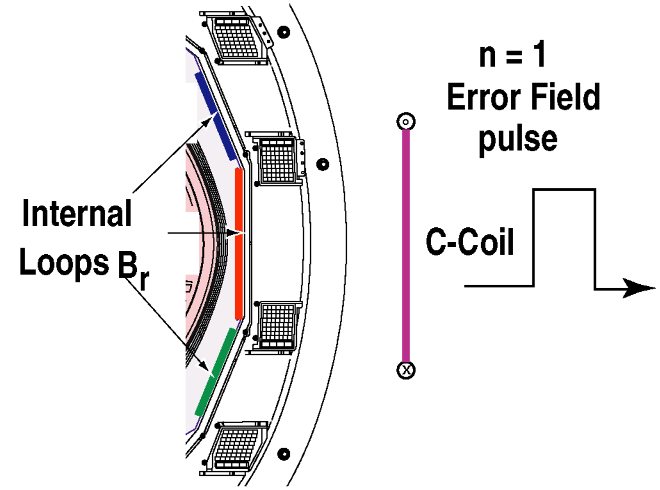
CLEAR EVIDENCE OF RESONANT RWM-ERROR FIELD INTERACTION IS FOUND IN MEASUREMENT OF HELICAL PLASMA RESPONSE

- Applied n = 1 field pulse from C-coil has no helicity
  - Same toroidal phase at three arrays





- Three toroidal arrays of saddle loops are at different poloidal locations

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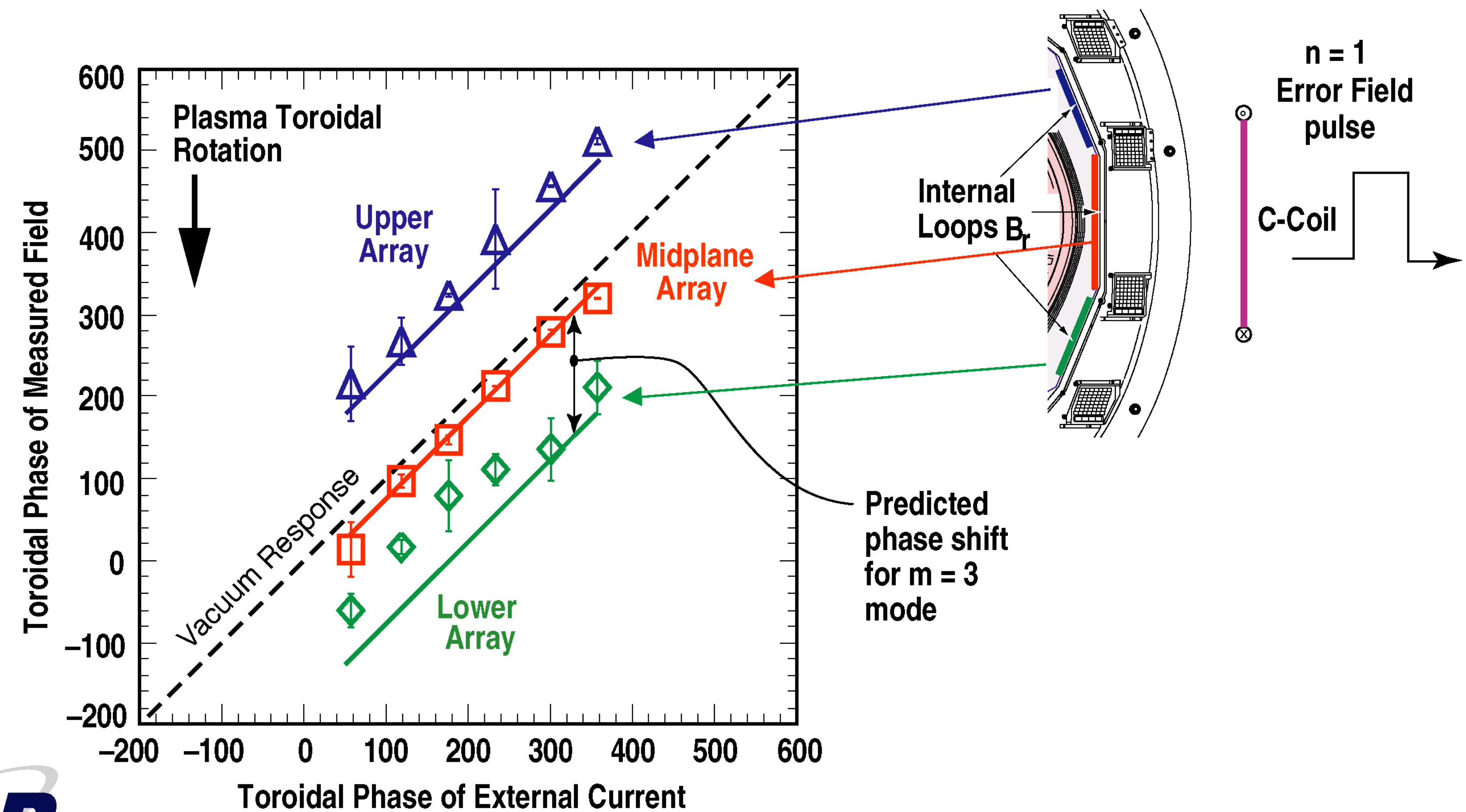
CLEAR EVIDENCE OF RESONANT RWM-ERROR FIELD INTERACTION IS FOUND IN MEASUREMENT OF HELICAL PLASMA RESPONSE

- Plasma response shows a distinct helicity
  - Toroidal phase shift between arrays consistent with m = 3 mode



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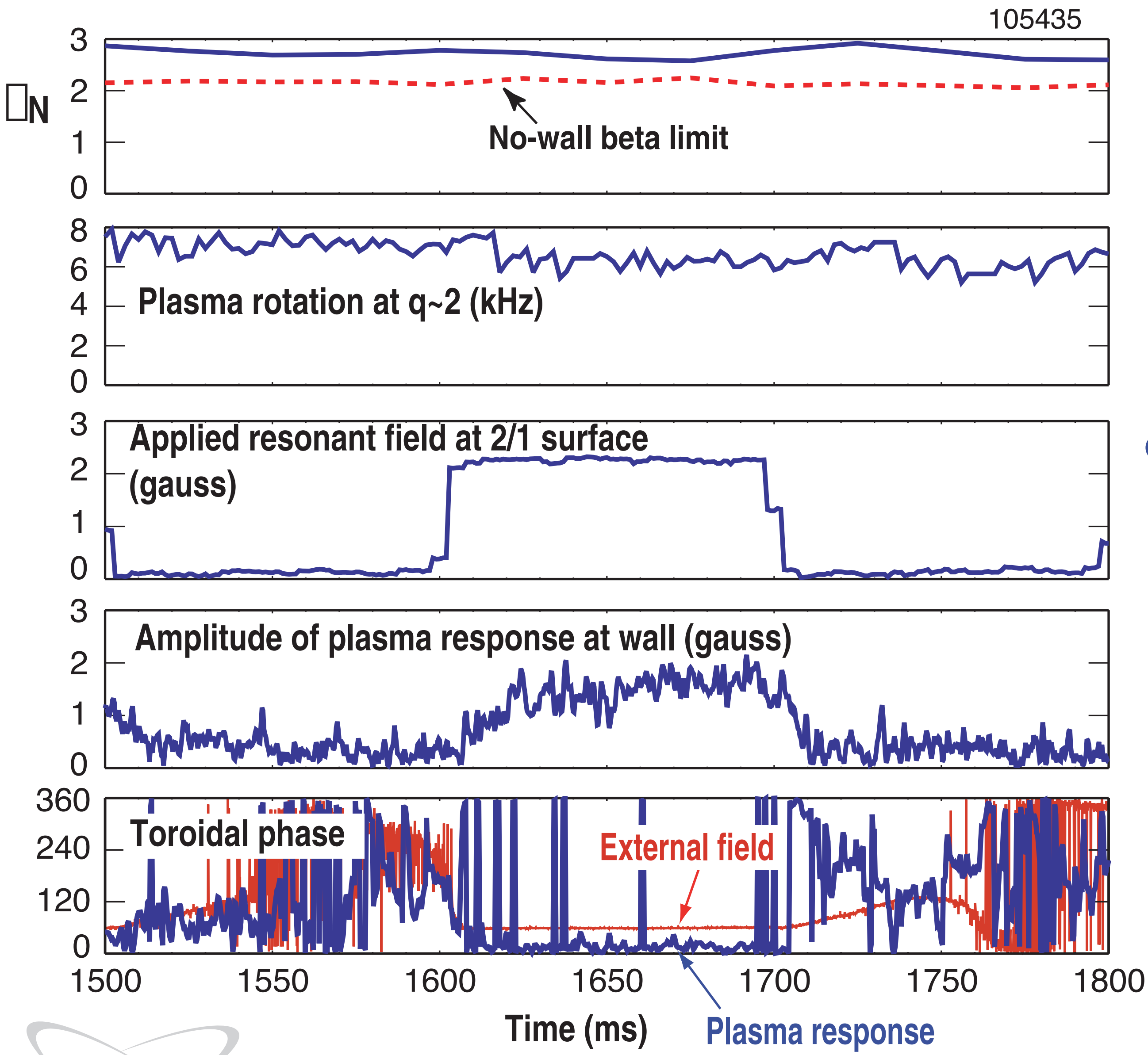
RESISTIVE WALL MODE DRIVEN BY EXTERNAL FIELD DOES NOT ROTATE (CONSTANT TOROIDAL PHASE) BUT REMAINS STABLE



- Mode rotation w.r.t. wall NOT required for stabilization by plasma rotation

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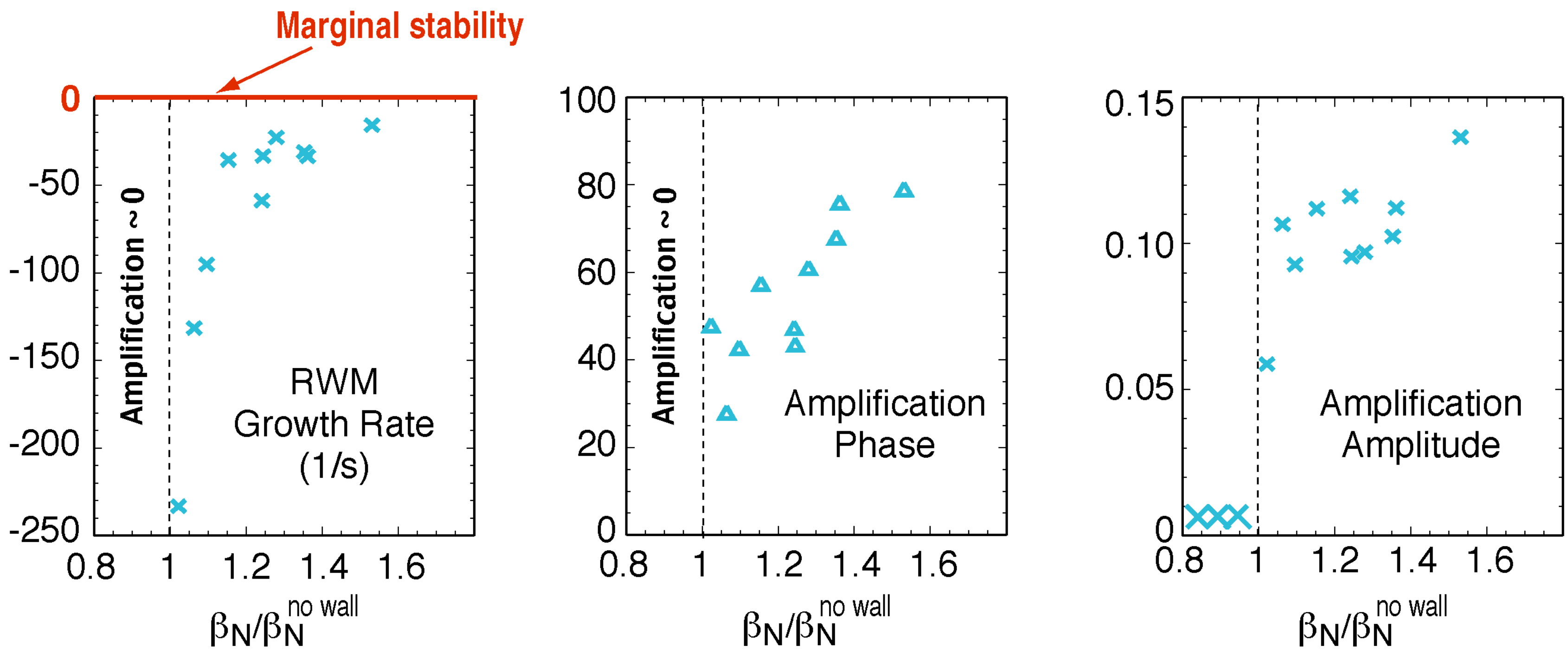
PLASMA BECOMES LESS STABLE AS β_N INCREASES

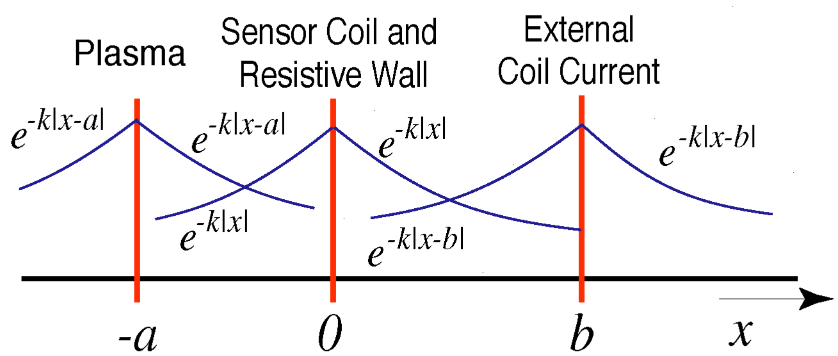
- Plasma response to pulsed n=1 field increases as β_N approaches the RWM stability boundary (γ=0)



- In contrast with predictions of some RWM models [6, 7], which calculate a maximum resonance at the no-wall limit, not at γ=0

[6] A.H. Boozer, Phys. Rev. Lett., (2002).
[7] R. Fitzpatrick, Phys. Plasmas, (2002).

PLASMA MODEL WITH ONLY ONE MODE => PLASMA RESPONSE IS ENTIRELY GIVEN BY ONLY TWO PARAMETERS (e.g. MODE AMPLITUDE AND PHASE)



Assume: $\partial/\partial t = i\omega t$, **and the symmetries:**

$\partial/\partial y = ik_t$ **and** $\partial/\partial z = ik_p$,

where $k_t = n/R$ **is the toroidal wavenumber,**

$k_p = m/a$ **is the poloidal wavenumber,**

and $k = \sqrt{k_t^2 + k_p^2}$

- **The perturbed magnetic field is:** $\bar{b} = \bar{\nabla} \times \bar{A}$, **where:** $\bar{A} = (\hat{z} - \frac{k_p}{k_t}\hat{y})\,\varphi(x)\,e^{i(k_t y + k_p z)}$
- **The value of $\bar{A}$ at x can be calculated using the Green's functions for a current sheet J_i at x_i:**

$$\bar{A}(x, y, z) = \sum_i \frac{\mu_0}{2\sqrt{k_t^2 + k_p^2}} J_i(x)\,e^{i(k_t y + k_p z)}\,(\hat{z} - \frac{k_p}{k_t}\hat{y})\,e^{-\sqrt{k_t^2 + k_p^2}\,|x - x_i|}$$

- **External currents:** $\bar{J}_E =$ **known**
- **Wall currents:** $\bar{J}_W = -\frac{1}{\mu_0} i\omega\bar{A}(0)2k\tau_W$, **where:** $\tau_W \equiv \frac{\delta\mu_0}{2k\eta}$
- **Plasma currents:** $\bar{J}_P = \frac{1}{\mu_0}\mathcal{A}\bar{A}(0)$ **, equivalent to the boundary condition:** $\frac{1}{\varphi(x)}\frac{\partial\varphi(x)}{\partial x}\Big|_{x=0^-} \equiv \Lambda$

where $\Lambda = \lambda_{re} + i\lambda_{im}$ **, and** $\Lambda = k - \mathcal{A}e^{-ka}$

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THE RESISIVE WALL MODE DISPERSION RELATION IS A SPECIAL CASE (NO EXTERNAL CURRENTS) OF THE PLASMA RESPONSE EQUATION

- We can now evaluate $\bar{A}(0)$ due to the plasma current, the resistive wall current, and a time dependent external current:

$$\bar{A}(0) = \frac{\mu_0 \bar{J}_E e^{-kb}}{k\left(1+\frac{\Lambda}{k}+2i\omega\tau_W\right)}$$

In vacuum is $\Lambda = k$

- The dispersion relation describes the mode oscillations that happen in the absence of external currents, therefore it is obtained from the plasma response equation by setting $J_E = 0$:

$$1+\frac{\Lambda}{k}+2i\omega\tau_W = 0$$

  - ❒ From here, the growth rate as a function of the plasma parameter Λ is: $\gamma = -\frac{1+\frac{\lambda_{re}}{k}}{2\tau_W}$

- The resonant field amplification, as defined in the experiment, is (after transient behavior):

$$\overline{RFA} = \frac{\bar{A}(0) - \bar{A}_V(0)}{\bar{A}_V(0)} = \frac{2}{1+\frac{\Lambda}{k}} - 1$$

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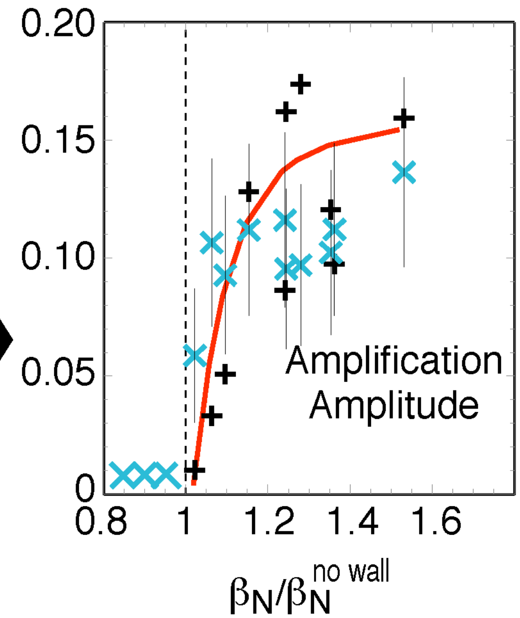
RFA AMPLITUDES ARE PREDICTED (IN PRINCIPLE) WITH NO FREE PARAMETERS. THE PROFILE vs. β IS SCALED WITH A CONSTANT TO ACCOUNT FOR GEOMETRY

- Resonant field amplification yields three RWM measurements:
  - Growth rate (negative)
  - Toroidal phase relative to external pulse
  - Amplitude (asymptotic)

- RWM growth rate and phase are used as input to the model
- Amplitude is predicted

× Measurements
+ Model predictions
— Fit to model



- The one-mode model overestimates the amplification by assuming that 100% of the externally applied field is resonant with the plasma mode.
  - External field has n=-1 and n=+1 components and m=0,1,2,3 components (mostly)
  - Only the n=+1 and virtually only the m=2 and m=3 are resonant with the plasma mode
- The slab geometry of the model gives the fields a slower spatial variation than in a torus. This also leads the model to overestimate the amplification.

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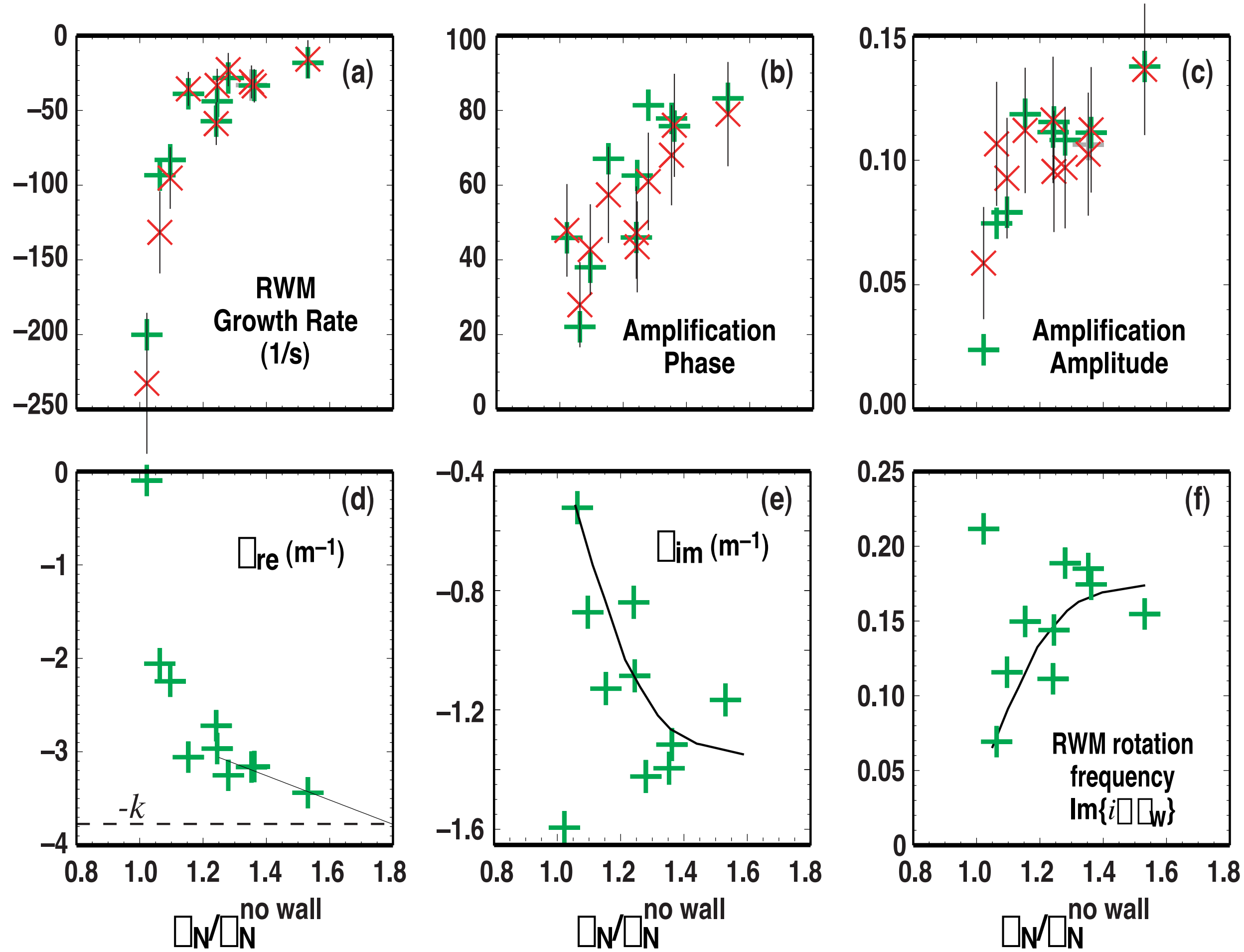
KNOWING THE PLASMA RESPONSE FUNCTION □(□N) -> CAN CALCULATE □N-DEPENDENCE OF RWM CHARACTERISTICS NOT DIRECTLY MEASURABLE

- Better overall agreement is obtained by allowing the input parameters (growth rates and phase shifts) to have some deviation from the measured values

Experimental measurements (x) and model predictions (+)



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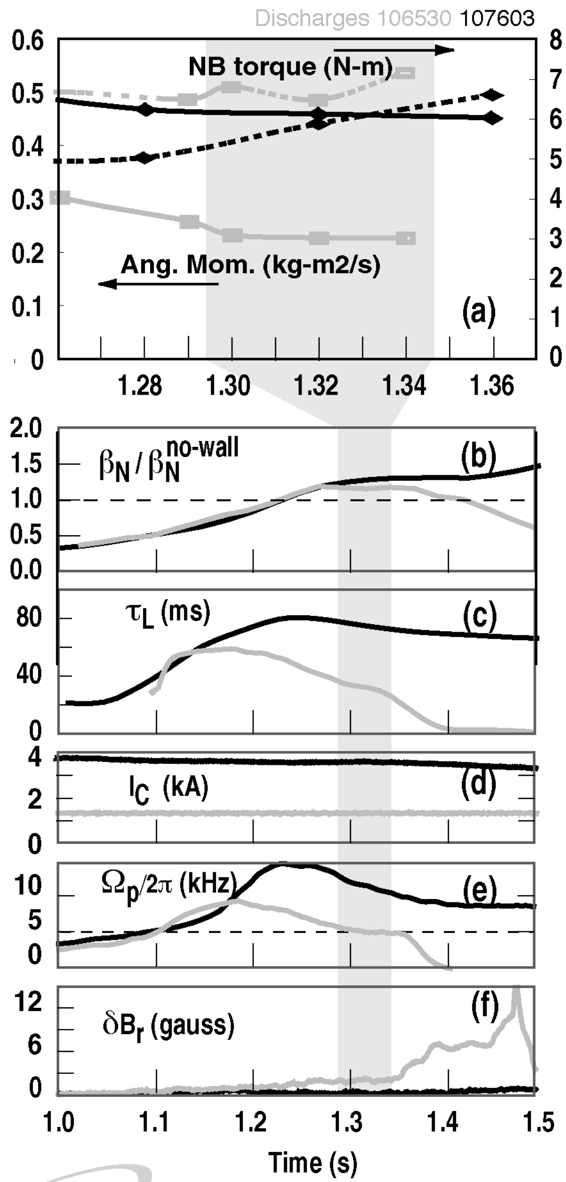
IN PLASMAS WITH DIFFERENT ERROR FIELD CORRECTION THE EVOLUTION OF τ_L STARTS TO DIFFER WHEN $\beta_N \sim \beta_N^{NO\text{-}WALL}$



- Torque exerted on plasma by resonant field response to uncorrected magnetic error is estimated assuming it is solely responsible for decay of τ_L

$$\frac{dL^{7603}}{dt} = T_{NB}^{7603} - \frac{L^{7603}}{\tau_L^{7603}}$$

$$\frac{dL^{6530}}{dt} = T_{NB}^{6530} - \frac{L^{6530}}{\tau_L^{7603}} - T_{RF}^{6530}$$

- A resonant field response δB_r ~ 2 G (at the wall) gives a torque on the plasma T_{RF} ~ 3.5±1 N-m
  - ❑ The force exerted on the flowing plasma is:

$$F_{RF} = T_{RF}/R \sim 1.4 \pm 0.5 \text{ N}$$

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THE RESONANT FIELD AMPLITUDE OBTAINED FOR GIVEN FORCE FROM MODEL'S $\Box_{im}$ IS CONSISTENT WITH EXPERIMENTAL MEASUREMENTS AND TRANSPORT ANALYSIS

- The resonant RWM exerts a braking force F_y on the plasma flowing in the y-direction. The time average force per unit area in the y-direction is given by:

$$dF_y \big/ ds = \frac{1}{2\Box_0} \mathrm{Re}\left\{ \left(\overline{\Box} \, \Box \overline{A} \, \Box \hat{y} \right) \left(\overline{\Box} \, \Box \overline{A} \, \Box \hat{x} \right) \right\}$$

- The force exerted on the flowing plasma by a finite radial field at the wall, $b_x(0) = \left[\overline{\Box} \, \Box \overline{A} \right]_x (0)$ is given by:

$$F_y = \frac{1}{2\Box_0} \frac{\left| b_x(0) \right|^2}{k_t + k_p^2/k_t} \left| \Box_{im} \right| 2 \Box R \; 2 \Box r$$

- Since the radial field measured in the experiment is a plasma response only:

$$\left| b_x(0) \right| = \left| b_{P,x}(0) \right| \frac{\left| \mathrm{RFA} \right| + 1}{\left| \mathrm{RFA} \right|}$$

- Therefore the magnitude of the RWM that would exert a force F_y ~1.4 N is:

$$\left| b_{P,x}(0) \right| = \frac{\left| \mathrm{RFA} \right|}{\left| \mathrm{RFA} \right| + 1} \sqrt{\frac{2\Box_0 \Box k_t + \frac{k_p^2}{k_t} \Box F_y}{\left| \Box_{im} \right| 2 \Box R 2 \Box r}}$$

~ 1.3±0.4 Gauss

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DIII-D MEASUREMENTS INCONSISTENT WITH FIZPATRICK'S LOW-DISSIPATION RWM REGIME

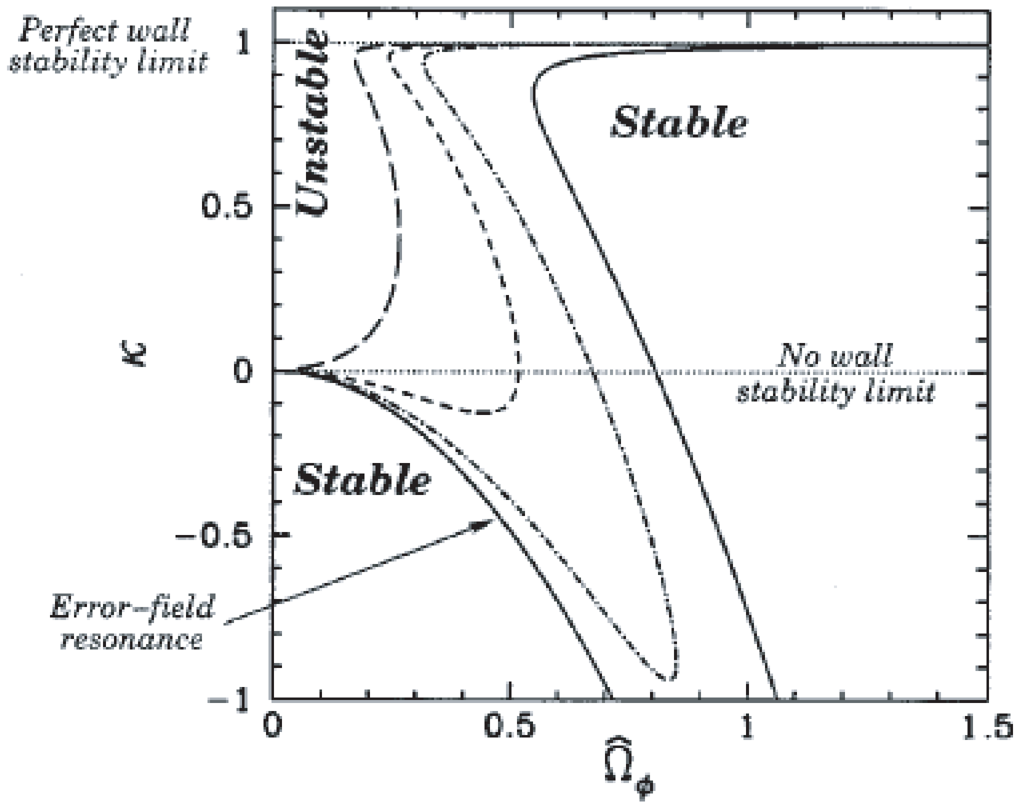

FIG. 3. Stability boundaries for the Fitzpatrick–Aydemir RWM dispersion relation, evaluated numerically for $\nu_* = 0.10$ (solid curve), $\nu_* = 0.30$ (dotted–dashed curve), $\nu_* = 0.50$ (short-dashed curve), and $\nu_* = 1.00$ (long-dashed curve), as well as $S_* = 100$, $m = 3$, and $r_w = 1.2a$.

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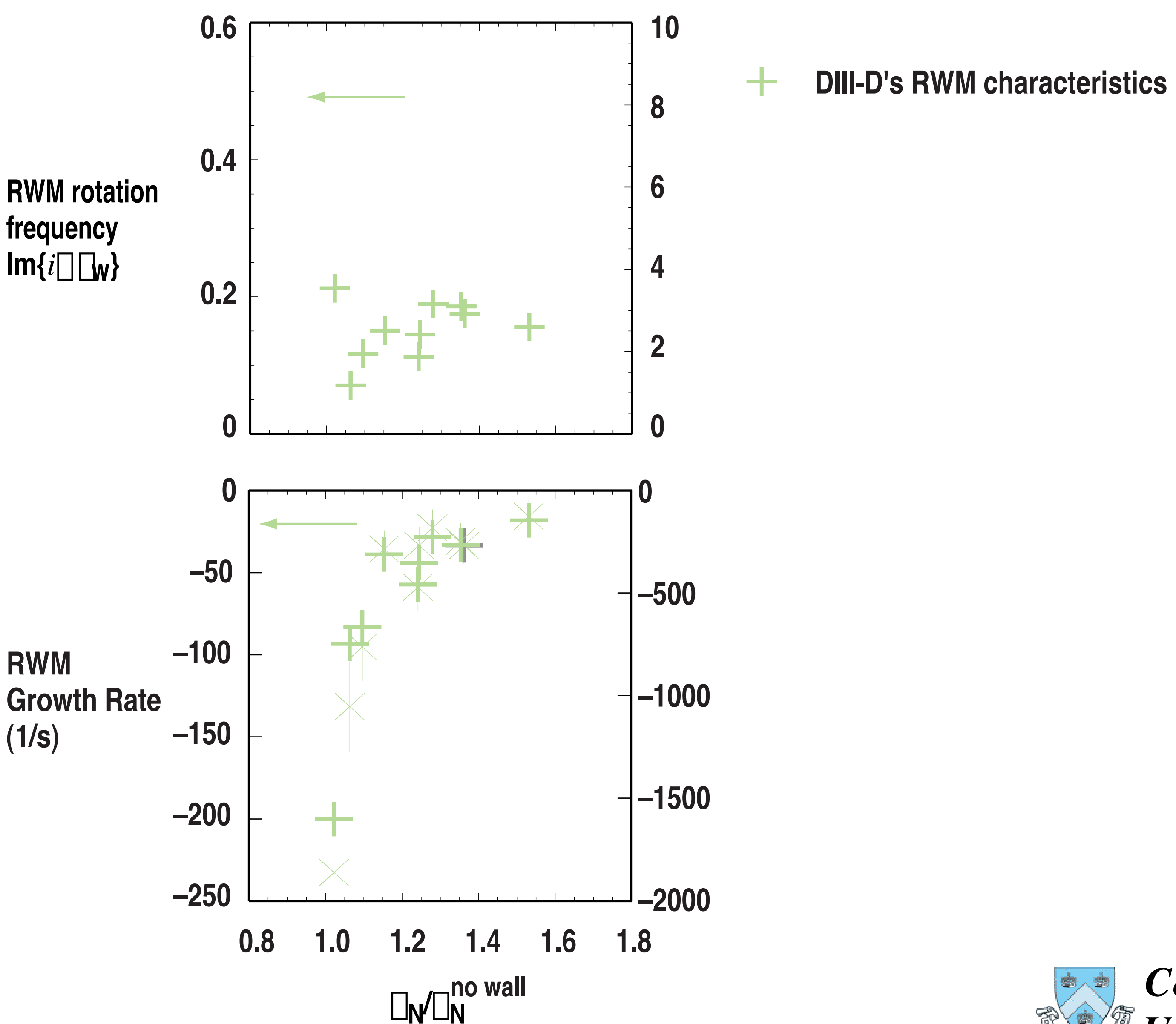
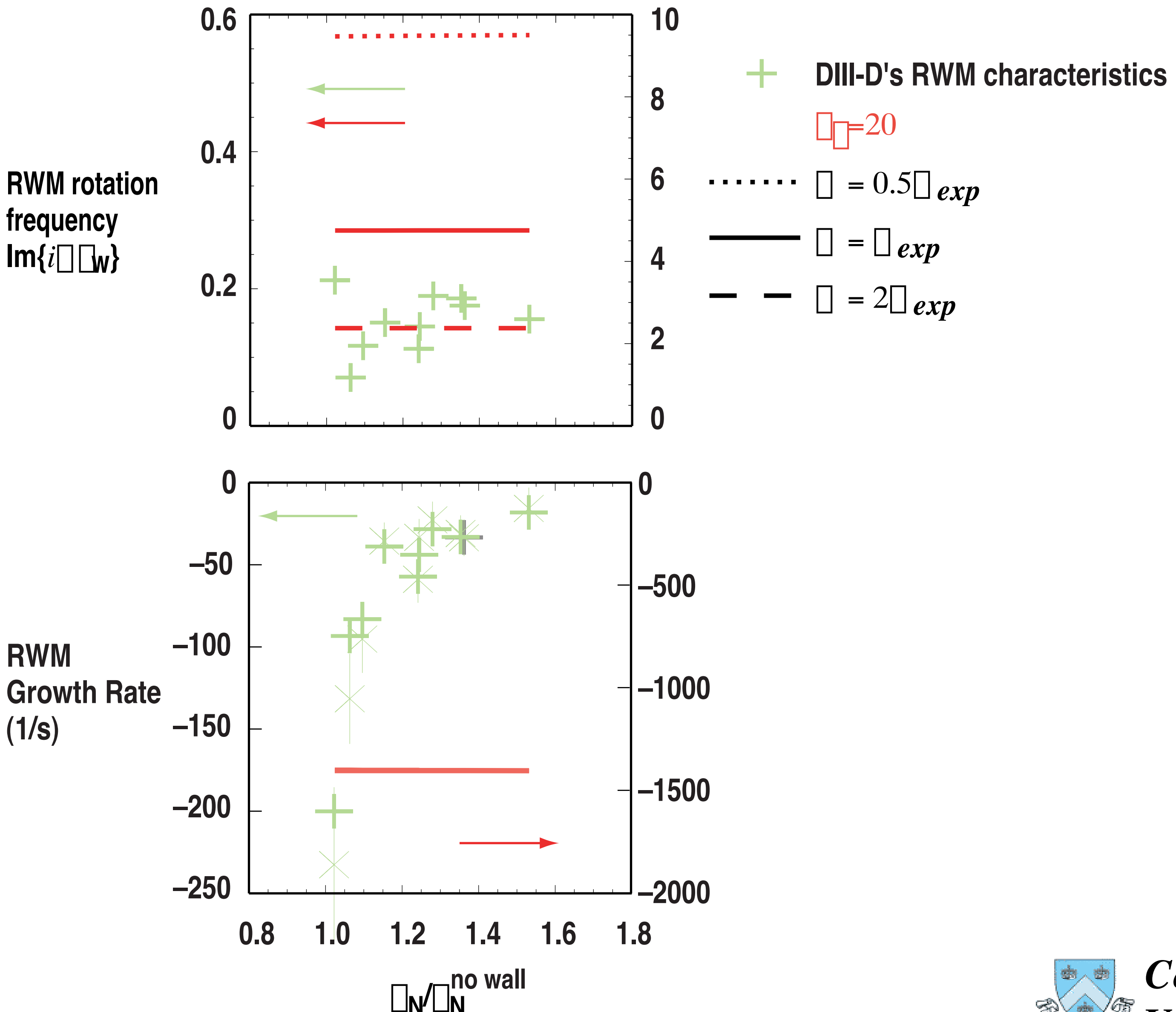
DIII-D MEASUREMENTS INCONSISTENT WITH FIZPATRICK'S HIGH-DISSIPATION RWM REGIME



DIII-D MEASUREMENTS INCONSISTENT WITH FIZPATRICK'S HIGH-DISSIPATION RWM REGIME



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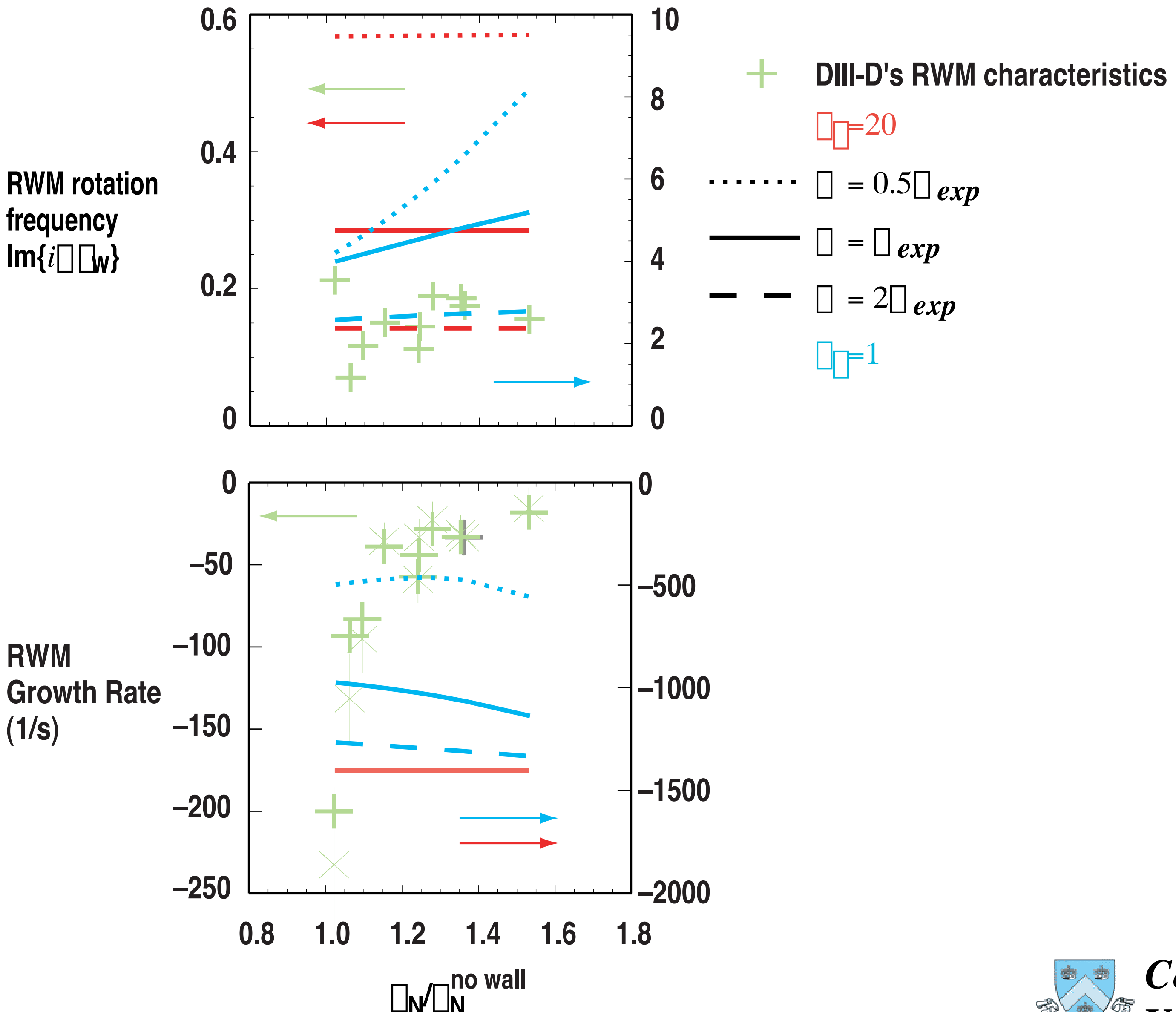

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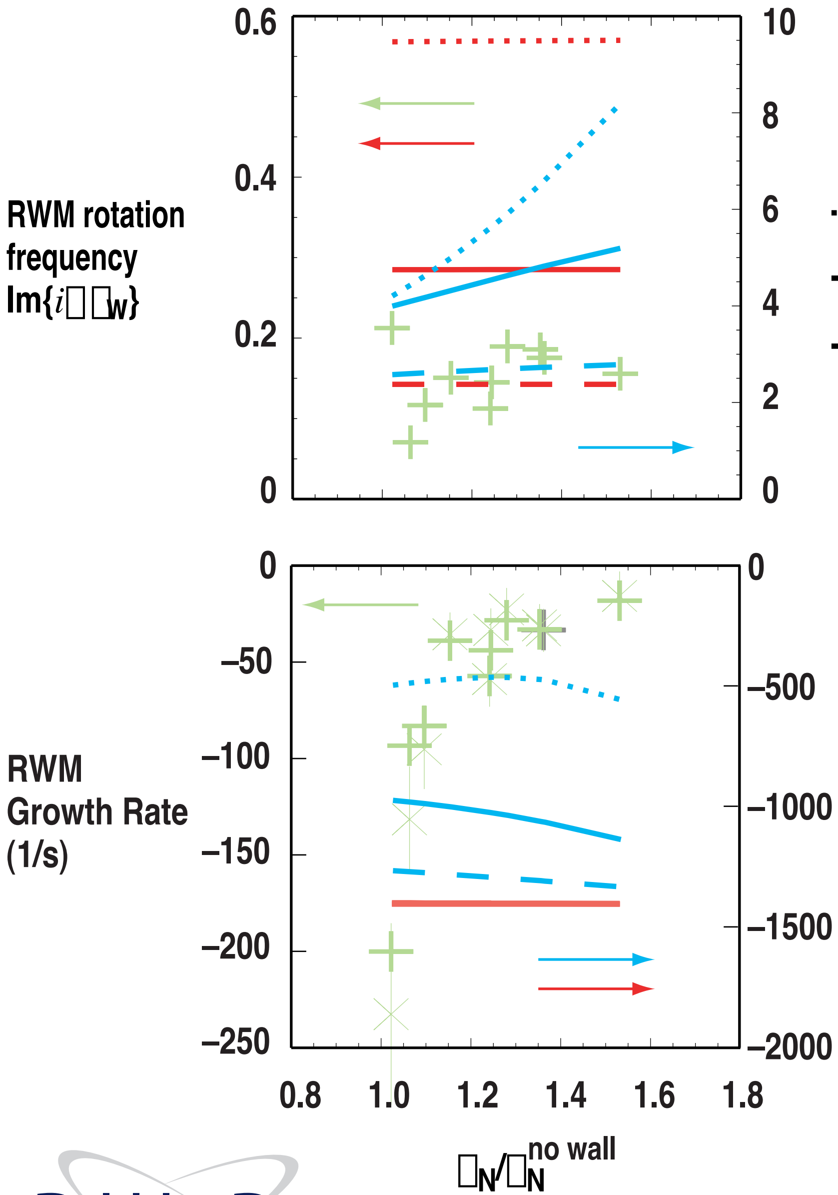
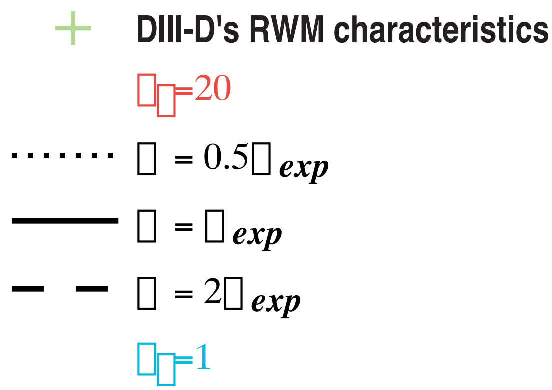
DIII-D MEASUREMENTS INCONSISTENT WITH FIZPATRICK'S HIGH-DISSIPATION RWM REGIME





Fitzpatrick, Phys. Plasmas, 2002: [...] unless the dissipation is fairly weak (i.e., $\nu_* \lesssim 0.1$), the error-field resonance *does not* correspond to a RWM stability boundary. The reason for this is that, unless ν_* is small, the RWM possesses a non-negligible *real frequency* at its marginal stability point. It is easily demonstrated that a real frequency of order the inverse wall time is sufficient to shift the error-field resonance (which is corresponds to the response of the plasma to a *zero frequency* perturbation) away from the RWM stability boundary.

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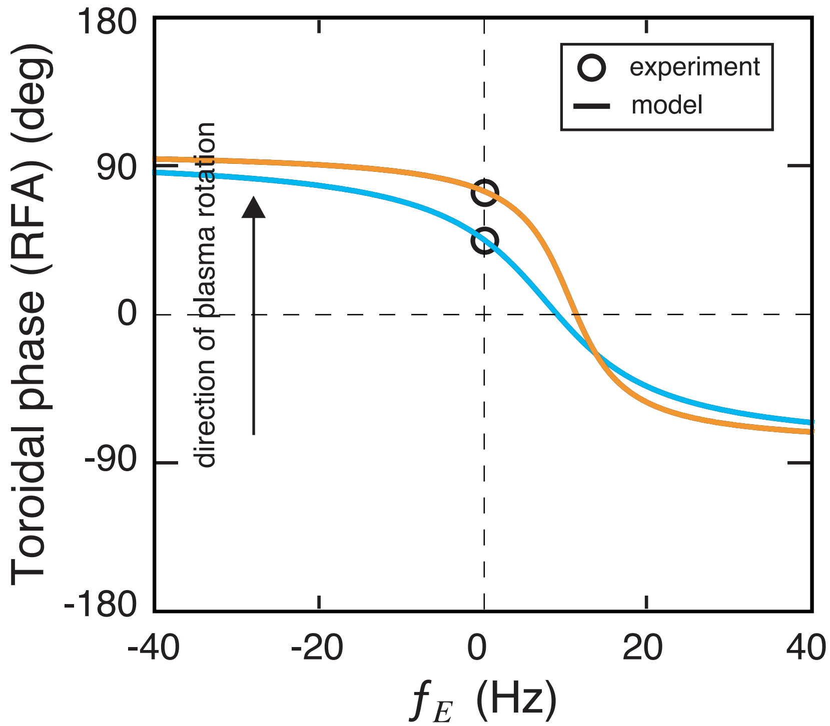
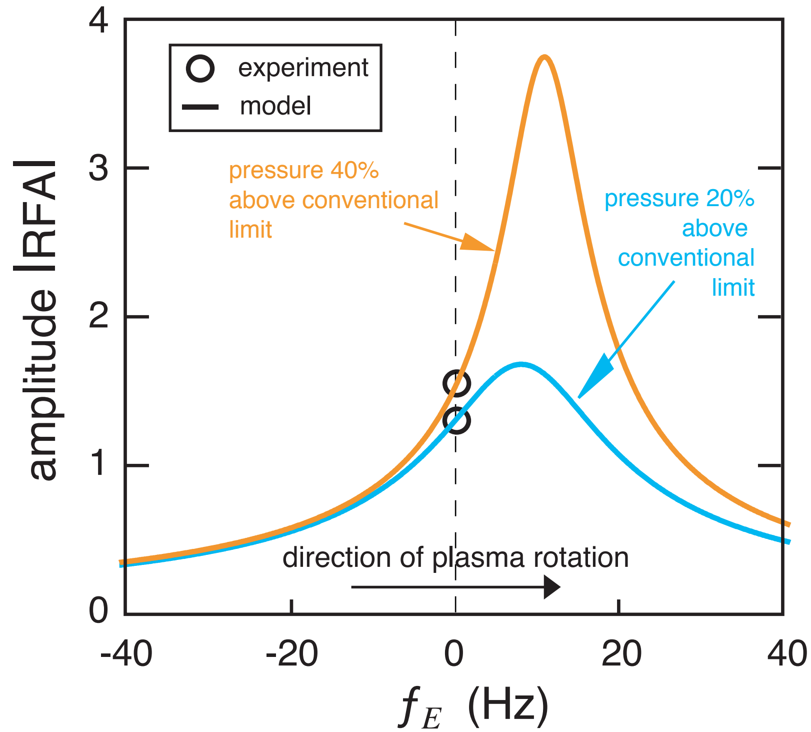
FREQUENCY DEPENDENCE OF RESONANT FIELD AMPLIFICATION PREDICTED BY SIMPLE RWM MODEL

- **External current:** $J_E = J_{E0}\, e^{i\square_E t}$
- **Plasma response (resonant field amplification):** $\overline{RFA} = \dfrac{\overline{A}(0) \square \overline{A}_V(0)}{\overline{A}_V(0)} = \dfrac{k \square \square}{k + \square + 2ki\square_E\square_W}$
- **Stability parameters (□) from square pulse analysis -> Resonance at f 10 Hz**
- **Recent experimental results in excellent agreement with model predictions**

(Reimerdes, et al., EPS '03, PRL to be submitted)





SUMMARY

- Previous common understanding of the rotational stabilization of the RWM appears inconsistent with the recent experimental evidence from DIII-D

- Measurements of plasma response to external field pulses at $\beta_N > \beta_N^{no\ wall}$ yield complete characterization of the RWM dispersion relation

- Beta dependence and frequency dependence of plasma response to external fields correctly predicted by simple, semi-empirical, one-mode RWM model
  - Model estimate of the RWM interaction with plasma rotation consistent with transport calculations

- The RWM rotation with respect to the wall is not needed for mode stabilization by plasma rotation
  - ☆ Most physics based RWM models DO NOT agree with this result
  - ☆ Model by Hu and Betti shows an example of an effective mechanism for RWM stabilization by plasma rotation, without dissipation