An overview of RWM theory

Richard Fitzpatrick

Institute for Fusion Studies

Department of Physics

University of Texas at Austin

Introduction

- Aim of talk is to present simple overview of analytic RWM theory.
- What are leading theories? What assumptions do they make? How are they related to one another?
- To what extent do leading theories account for experimental observations?
- To what extent are leading theories *quantitative*, rather than *qualitative*?

Single mode approximation

- Experimental results from DIII-D, PBX-M, HBT-EP, and NSTX suggest that plasma response to wall dominated by *single toroidal eigenmode* (whose leading poloidal harmonic is resonant just outside plasma).
- Approximate plasma response as due *solely* to this eigenmode.

Cylindrical approximation

- For sake of simplicity, treat plasma as *periodic cylinder* with concentric resistive wall.
- Consider stability of m, n harmonic, which is resonant just outside plasma.
- Hope that single-harmonic cylindrical theory generates RWM dispersion relation which has *same general form* as that which would be obtained from far more complicated single-harmonic toroidal theory (lumped parameter approximation?).

Definitions

- Cylindrical polar coordinates, (r, θ, z). Periodicity length, $2\pi\, R_0$. Simulated major radius, R_0. Simulated toroidal angle, $\phi = z/R_0$. Minor radius, a. Wall radius, r_w.

- Let

$$\mathbf{B} = \nabla\psi \wedge \hat{\mathbf{z}} + B_z\, \hat{\mathbf{z}},$$

where

$$\psi(\mathbf{r}, t) = \psi_0(r) + \tilde{\psi}(r, t)\, \mathrm{e}^{\,\mathrm{i}\,(m\theta - n\phi)}.$$

- Parameterize wall proximity to plasma via

$$d = \frac{1}{m}\, \frac{(r_w/a)^{2m} - 1}{(r_w/a)^{2m} + 1}.$$

Plasma response parameter

- *Plasma response parameter*, λ, defined

$$\left.\frac{d\ln\tilde{\psi}}{d\ln r}\right|_{r=a} = \left.\frac{d\ln\tilde{\psi}}{d\ln r}\right|_{r=a}^{\rm ideal} + \frac{\lambda}{d}.$$

  Here, "ideal" refers to *marginally stable ideal MHD* theory.

- Thus, λ measures that part of plasma response which is *not* accounted for by marginally stable ideal MHD. Expect λ to depend on plasma *rotational inertia* and plasma *dissipation*.

RWM dispersion relation

- RWM dispersion relation written[a]

$$\lambda\,\Psi_a = -(1-\kappa)\,(1-md)\,\Psi_a + \sqrt{1-(md)^2}\,\Psi_w,$$
$$\gamma\,d\tau_w\,\Psi_w = -(1+md)\,\Psi_w + \sqrt{1-(md)^2}\,\Psi_a + 2\,md\,\Psi_c.$$

- Ψ_a is helical magnetic flux at plasma boundary. Ψ_w is wall flux. Ψ_c is vacuum error-field flux at wall.
- Parameter κ determines plasma ideal stability. $\kappa = 0$ corresponds to *no-wall* stability boundary. $\kappa = 1$ corresponds to *perfect-wall* stability boundary.
- τ_w is wall time-constant.

[a] R. Fitzpatrick, Phys. Plasmas **9**, 3459 (2002).

RWM stability boundaries

- RWM stability boundaries determined by solving dispersion relation in absence of error-field. Obtain

$$[\lambda + (1-\kappa)\,(1-md)]\,[\gamma\, d\tau_w + 1 + md] = 1 - (md)^2.$$

Error-field amplification factor

- Error-field amplification factor obtained by solving dispersion relation with $\gamma = 0$. Obtain

$$\mathcal{A} \equiv \frac{\Psi_a}{\Psi_c} = \left(\frac{1-md}{1+md}\right)^{1/2} \frac{2\,md}{\lambda - \kappa\,(1-md)}.$$

Slowing down torque

- Toroidal electromagnetic slowing down torque acting on plasma:

$$T_\phi = \frac{2\,\pi^2\,R_0}{\mu_0}\,\frac{n\,|\Psi_a|^2}{d}\,\mathrm{Im}(\lambda).$$

Garofalo-Jensen model

- Garofalo-Jensen model[a] determines λ *empirically* from measured growth-rate and phase (w.r.t. error-field) of wall flux.
- Once λ is determined, error-field amplification factor $\mathcal{A}$ can be calculated and compared with experimental data. Good agreement is found, which validates general approach.
- G-J model can be used to predict response of RWM to external feedback.
- Main advantage of G-J model is that theoretical expression for λ not required. Main disadvantage is that model cannot predict RWM stability in absence of experimental data.

[a] A.M. Garofalo, *et al.*, 2003 Sherwood meeting.

Boozer model: Basic assumptions

- Boozer model[a] assumes λ is *purely imaginary*: $\lambda = -\mathrm{i}\,\alpha$.
- Simplest physical implementation:

$$\lambda = -\mathrm{i}\,\Omega_\phi\,\tau_D,$$

where Ω_ϕ is plasma toroidal angular velocity, and τ_D is some plasma dissipation time-scale.

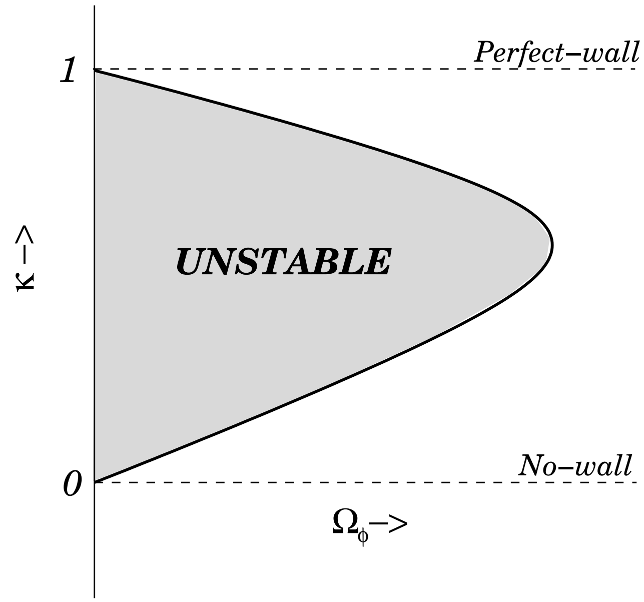
- Error-field amplification factor:

$$|\mathcal{A}| = \left(\frac{1-md}{1+md}\right)^{1/2}\frac{2\,md}{[\kappa^2\,(1-md)^2+(\Omega_\phi\,\tau_D)^2]^{1/2}}.$$

Factor *strongly peaked* at no-wall stability boundary $(\kappa = 0)$ when $|\Omega_\phi\,\tau_D| \ll 1$.

[a] A.H. Boozer, Phys. Plasmas **2**, 4521 (1995),

Boozer model: RWM stability boundaries



- Critical rotation rate needed to completely stabilize RWM:

$$(\Omega_\phi)_c = \frac{1 - md}{2\,\tau_D}.$$

Boozer model: Summary

- Critical rotation rate varies inversely with strength of plasma dissipation.
- Critical rotation rate decreases as plasma-wall distance increases.
- Error-field drag torque peaks strongly at no-wall stability boundary in low dissipation limit.
- Boozer model does not take plasma inertia into account. Hence, model fails completely as ideal stability boundary approached.

Fitzpatrick model: Basic assumptions

- In Fitzpatrick model[a]

$$\lambda = \tau_C^2\,(\gamma - \mathrm{i}\,\Omega_\phi)^2 + \tau_D\,(\gamma - \mathrm{i}\,\Omega_\phi),$$

  where

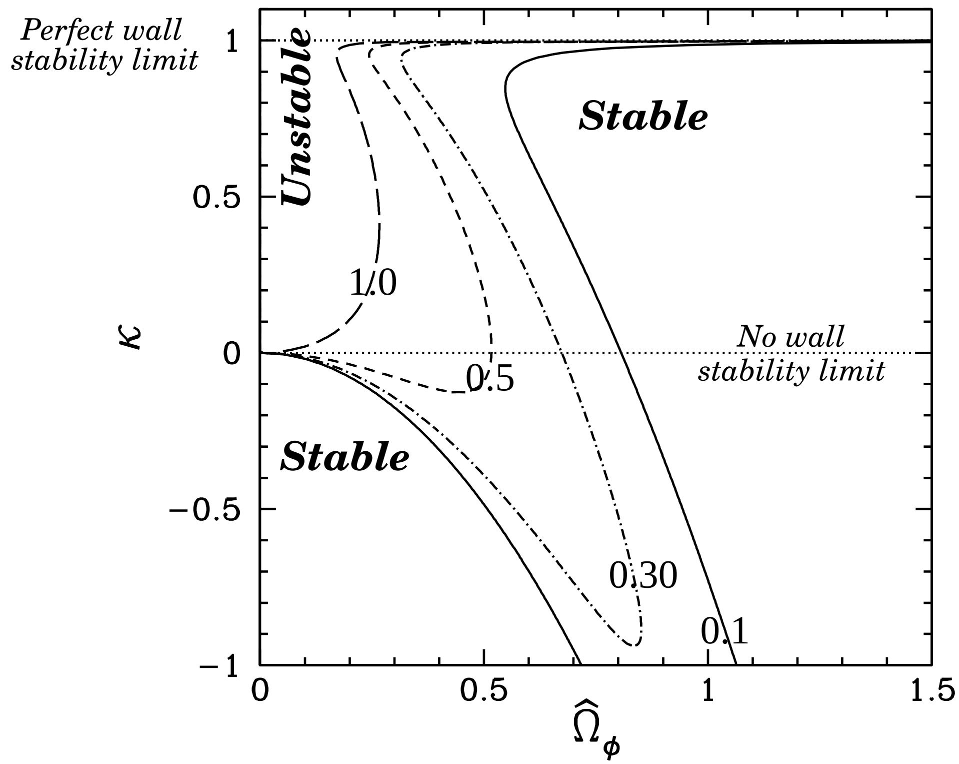
$$\tau_C = (k_\parallel\,r)_{r=a}\,\tau_A.$$

  Here, τ_A is Alfvén time. Note that $(k_\parallel\,r)_{r=a} \ll 1$.

- First term corresponds to plasma *inertia*. Second term corresponds to plasma *dissipation*.

[a] R. Fitzpatrick, Phys. Plasmas, **9**, 3459 (2002).

Fitzpatrick model: RWM stability boundaries



- $\hat{\Omega}_\phi = \Omega_\phi \, \tau_C$. Curves correspond to $\tau_D/\tau_C = 0.1$, 0.3, 0.5, 1.0.

Fitzpatrick model: Summary

- Critical rotation rate:

$$(\Omega_\phi)_c \simeq \tau_C^{-1} \ll \tau_A^{-1}.$$

- In high dissipation limit, $\tau_D \gg \tau_C$, Fitzpatrick model closely resembles Boozer model.

- In low dissipation limit, Fitzpatrick model predicts unstable window in Ω_ϕ, below no-wall stability boundary. Error-field resonance lies on *lower boundary* of this window.

Dynamical RWM model

- Fitzpatrick model can be converted into system of (normalized) o.d.e.s:

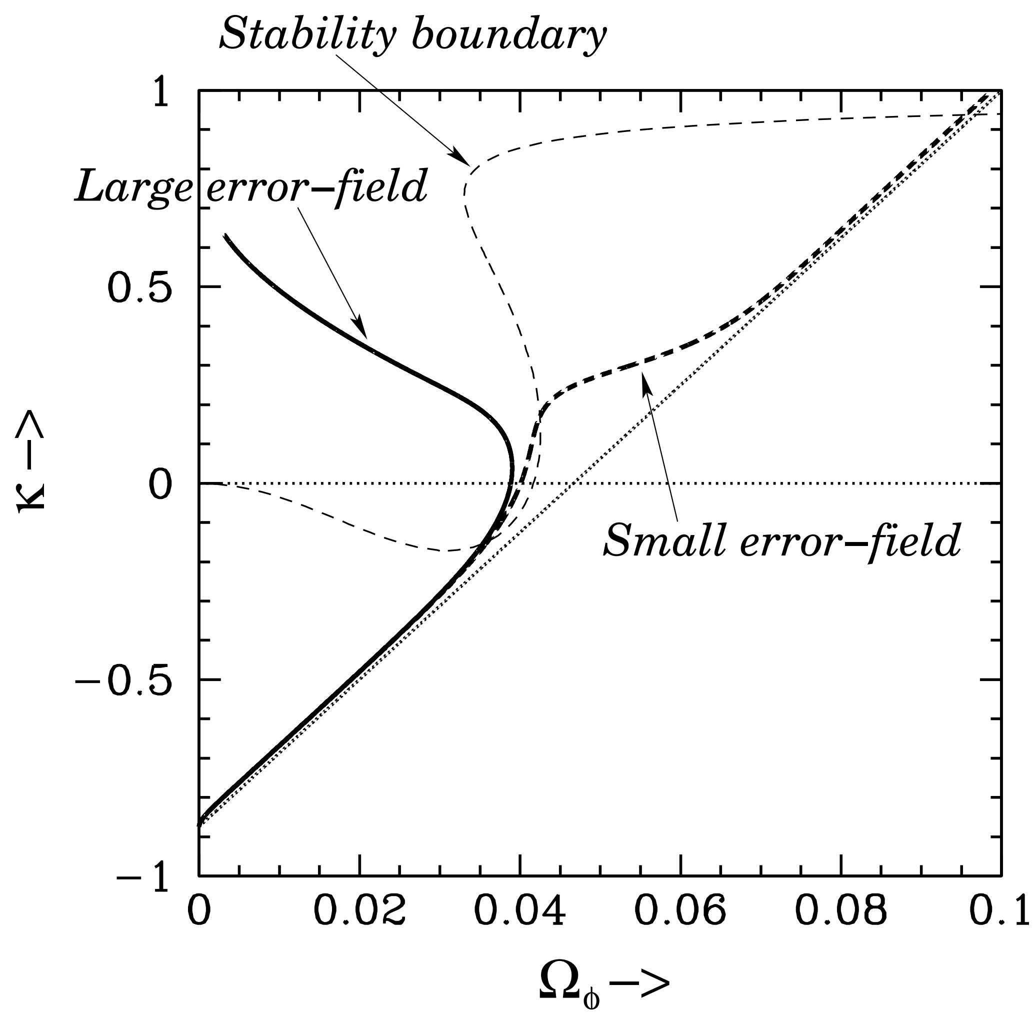
$$\frac{d^2\Psi_a}{dt^2}+(\tau_D-2\,\mathrm{i}\,\Omega_\phi)\,\frac{d\Psi_a}{dt}+\left[(1-\kappa)\,(1-md)-\Omega_\phi^2-\mathrm{i}\,\tau_D\,\Omega_\phi\right]\Psi_a = \sqrt{1-(md)^2}\,\Psi_w,$$

$$d\tau_w\,\frac{d\Psi_w}{dt}+(1+md)\,\Psi_w=\sqrt{1-(md)^2}\;\Psi_a+2\,md\;\Psi_c,$$

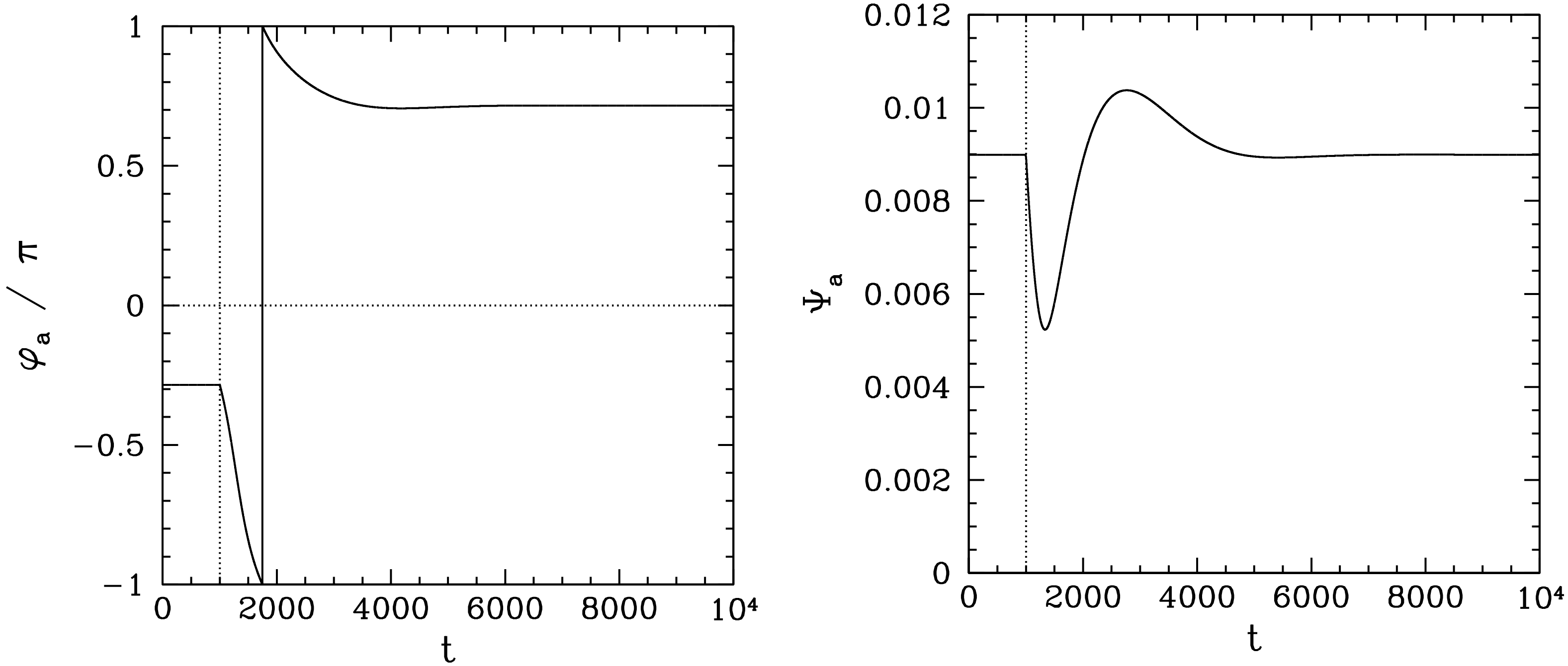
$$\frac{d\Omega_\phi}{dt}+\tau_D\,(\Omega_\phi-\Omega_\phi^{(0)})=-\tau_D\,\Omega_\phi\,|\Psi_a|^2.$$

- Here, $\Omega_\phi^{(0)}$ is unperturbed plasma rotation rate.

Simulation of NBI ramp

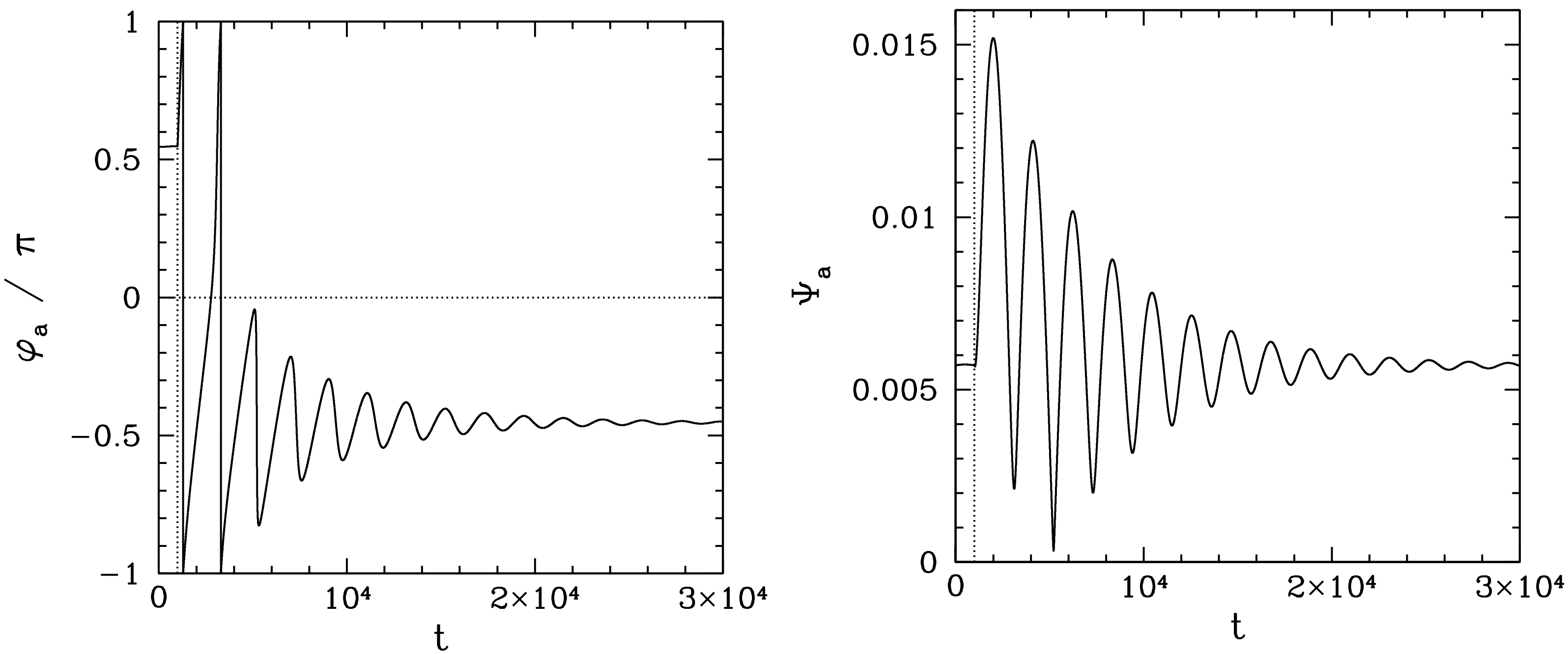


Response to error-field phase flip



Far from RWM stability boundary

Response to error-field phase flip



Close to RWM stability boundary

Nature of plasma dissipation

- Strength of plasma dissipation has profound affect on RWM stability boundaries. What determines dissipation strength?
- One source of dissipation is that due to Alfvén resonances associated with toroidally coupled side-bands within plasma. Strength of this dissipation fairly straightforward to calculate. However, calculated strength seems too small to account for observations.
- Another source of dissipation is Landau damping at sound-wave resonances within plasma. This dissipation is toroidally enhanced,[a] but may be substantially diminished by trapped-particle effects.[b] Strength of sound-wave dissipation seems large enough to account for observations, but actual value remains somewhat uncertain.

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[a] R. Betti, and J.P. Freidberg, Phys. Rev. Letts. **74**, 2949 (1995).
[b] A. Bondeson, and M.S. Chu, Phys. Plasmas **3**, 3013 (1996).

Successes and failures of RWM model

- Dynamical RWM model fairly successful at accounting for RWM stability boundaries and error-field phase-flip experiments on HBT-EP.[a]

- Dynamical RWM model offers good *qualitative* explanation for DIII-D observation that too large a static error-field prevents access to wall stabilized regime.

- Dynamical model *does not* correctly predict behaviour of amplitude and phase of error-field driven flux, in wall-stabilized regime on DIII-D, as β increased. Flux amplitude *does not* peak at no-wall stability boundary. Instead, amplitude seems to increase continuously as perfect-wall boundary approached.

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[a] M. Shilov, invited talk at 2003 APS.

Summary

- Although much progress has been made in RWM theory, there are still some outstanding issues.
- Need accurate method of calculating plasma dissipation which avoids use of ad-hoc assumptions. Toroidal enhancements? Trapped particle effects?
- Need to reconcile DIII-D error-field driven flux observations with theory. Why is there no resonance at no-wall stability boundary?