Visualization Data Analysis

Tobin Munsat

University of Colorado

Center for Integrated Plasma Studies

US-KSTAR Workshop

San Diego, CA

April 15, 2009

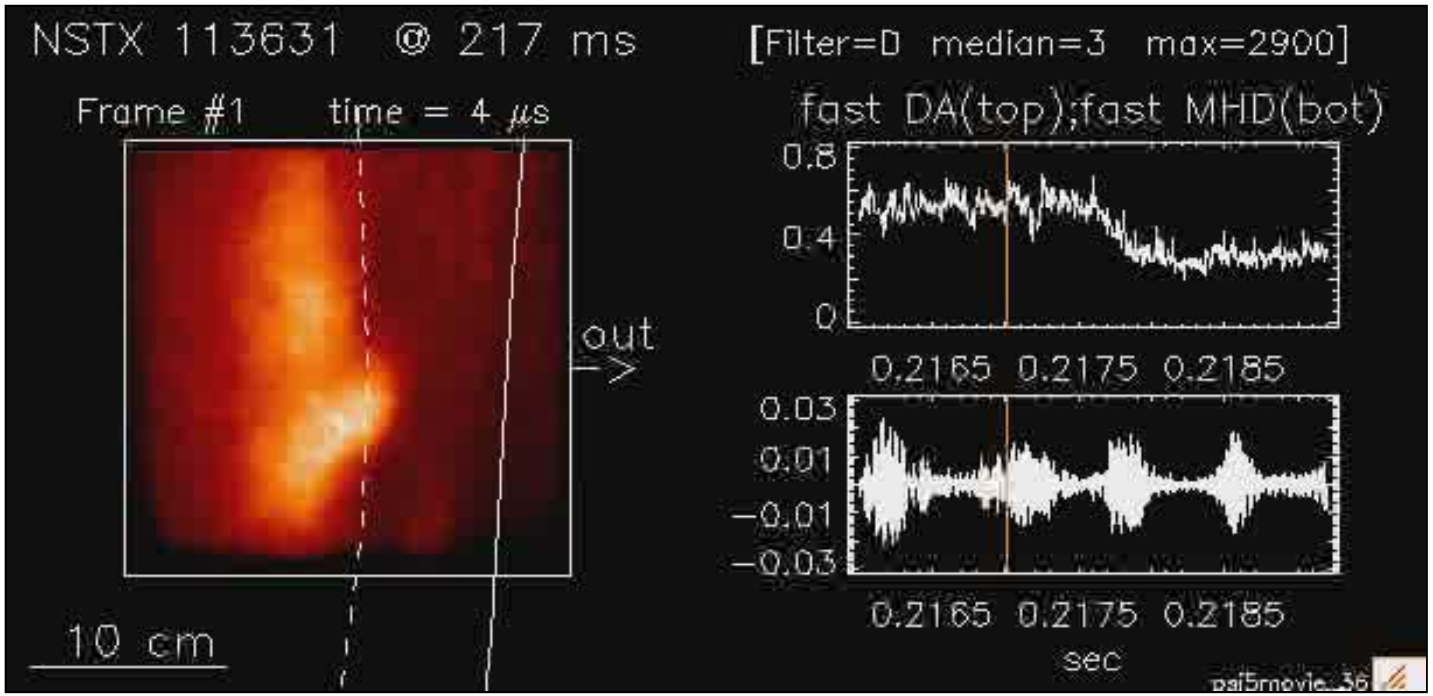
Intro

- **KSTAR will have a suite of imaging diagnostics very early on (SXR, BES, ECEI/MIR, etc.)**
- **Data is very dense and rich!**
- **Analysis methods can be developed and shared between measurements**

- ***Information in basic graphical images***
- ***Correlation and coherency***
- ***Frequency/wavenumber visualization***
- ***Bispectral analysis***
- ***Velocimetry***

Qualitative information from graphical image sets

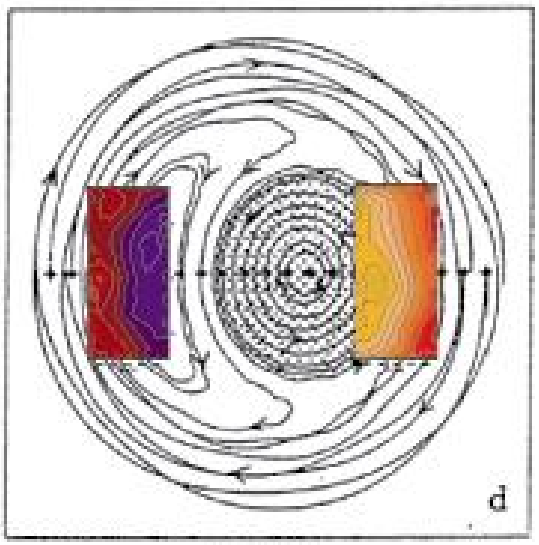
- NSTX shot 113631 (GPI movie)
- Highly intermittent transport
- 2-D view of $\tilde{n}_e$ at H-mode transition

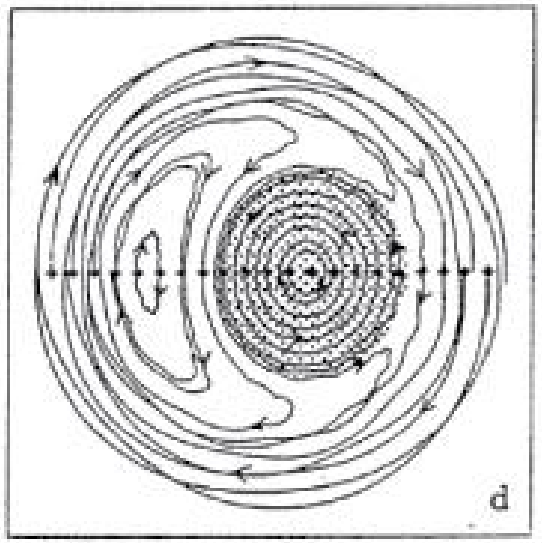


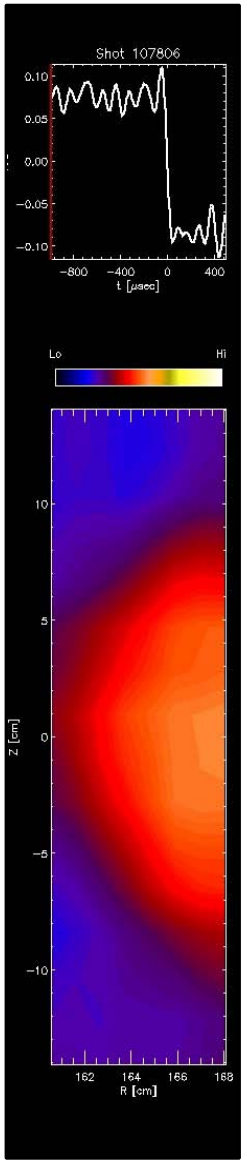
Qualitative information from graphical image sets

ECEI comparison with topology from MHD models of sawtooth crashes

- Resemblance between 2-D images of the hot spot/island and images from simulation results of the full reconnection model (tearing type) (Sykes et al.)
- High-field side crashed clearly observed
- Partial sawtooth observed/characterized
- Projection of 2-D images determines poloidal and toroidal localization/extent of reconnection zone



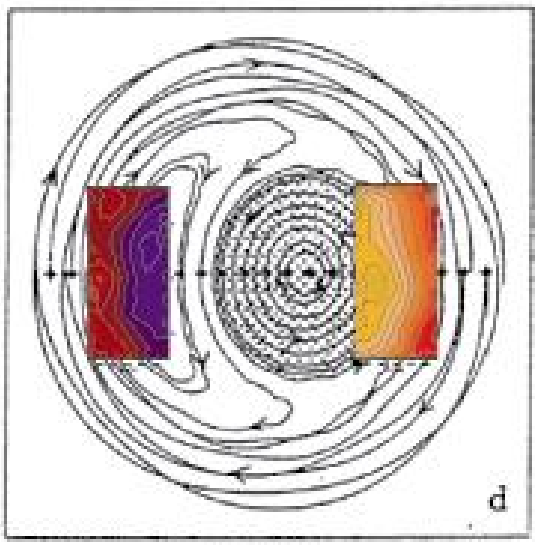


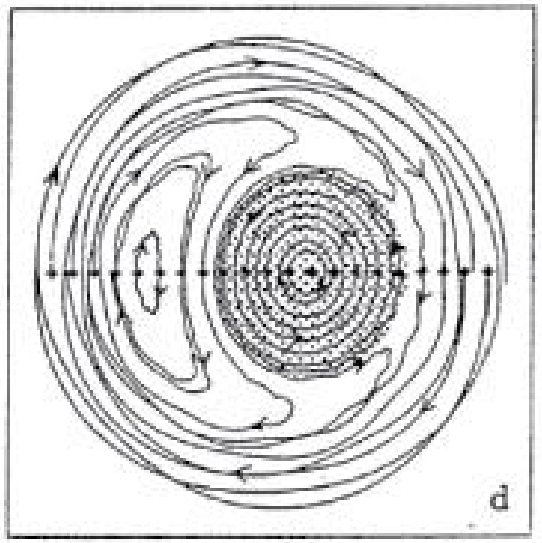


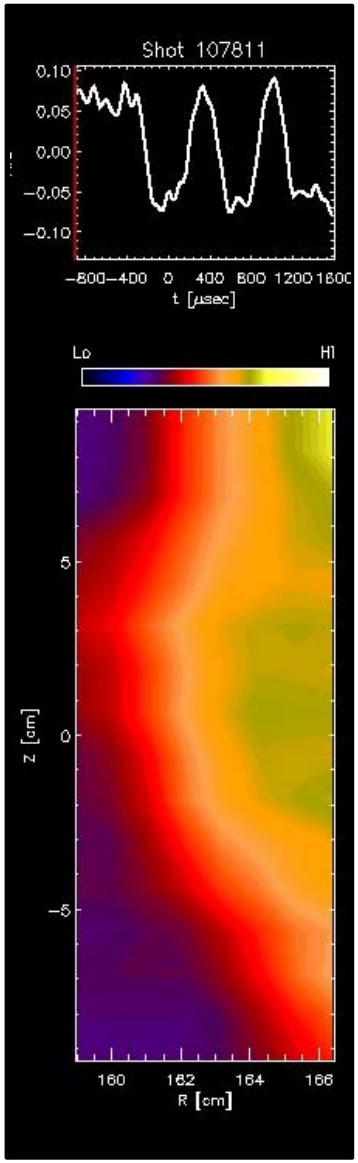
Qualitative information from graphical image sets

ECEI comparison with topology from MHD models of sawtooth crashes

- Resemblance between 2-D images of the hot spot/island and images from simulation results of the full reconnection model (tearing type) (Sykes et al.)
- High-field side crashed clearly observed
- Partial sawtooth observed/characterized
- Projection of 2-D images determines poloidal and toroidal localization/extent of reconnection zone

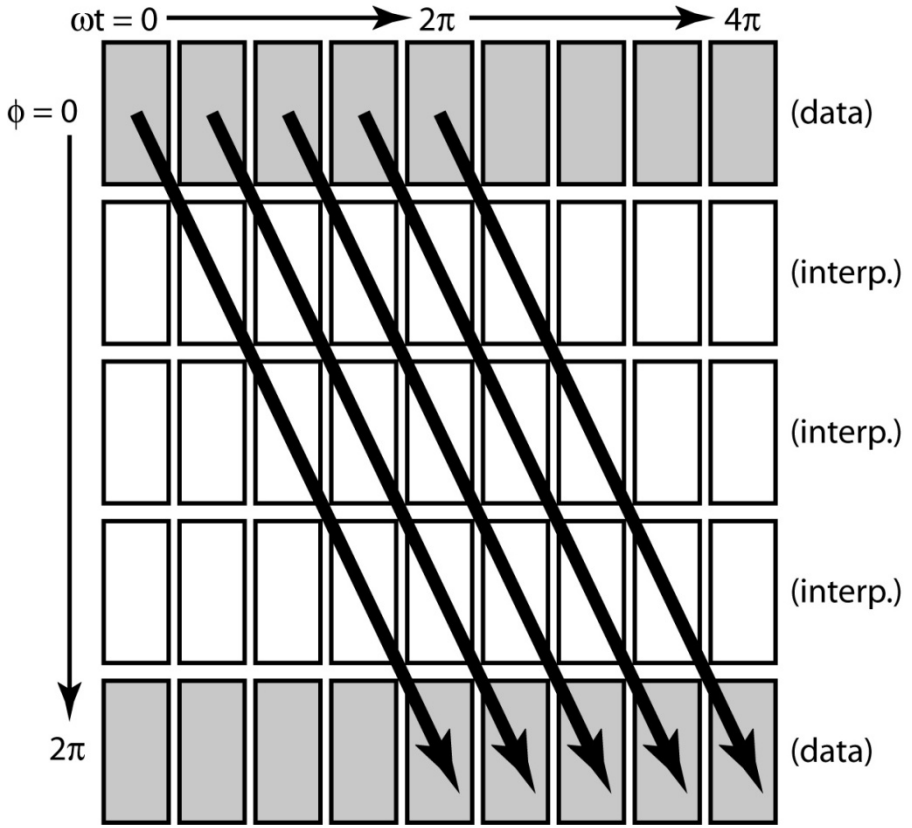






3-d projection from toroidally spaced systems

- Poloidal view of q=1 surface directly observed in *lab frame*
- *Plasma frame* views can be extracted by interpolation (toroidally rotating plasma where $\tau_{evolution} >> \tau_{rotation}$)
- Quantify poloidal extent and time history of image moving through view
- Toroidal extent of particular q=1 helical arc estimated by exclusion from view
- *Toroidal information can be directly quantified with multiple toroidal views*



Vertical: fixed t, resolved in ϕ

Horizontal: fixed ϕ, resolved in t

Diagonal: fixed ϕ *in lab frame*

Cross spectra and coherency

Cross-coherency calculated between poloidally separated channels

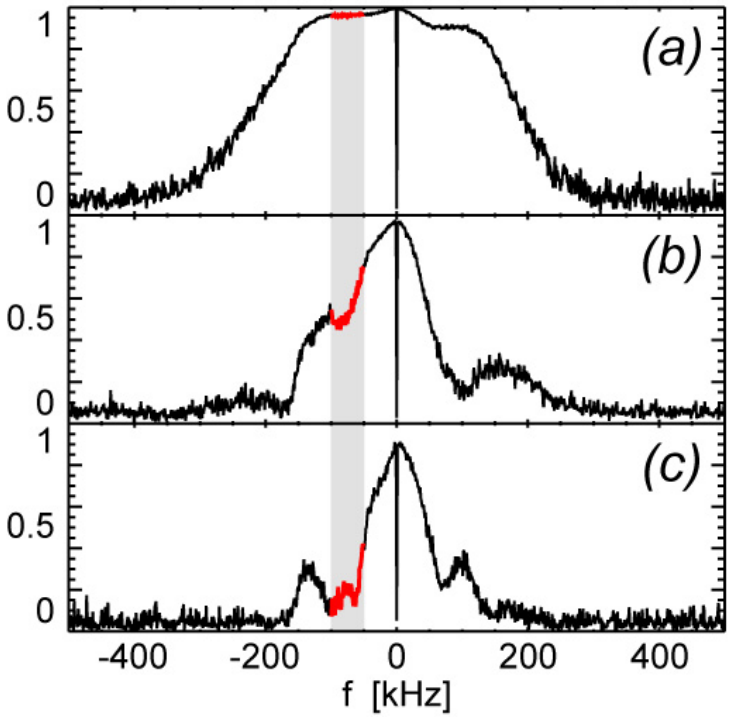
$$\gamma_{xy}^2(f) = \frac{\left|G_{xy}(f)\right|^2}{G_{xx}(f)\cdot G_{yy}(f)}$$

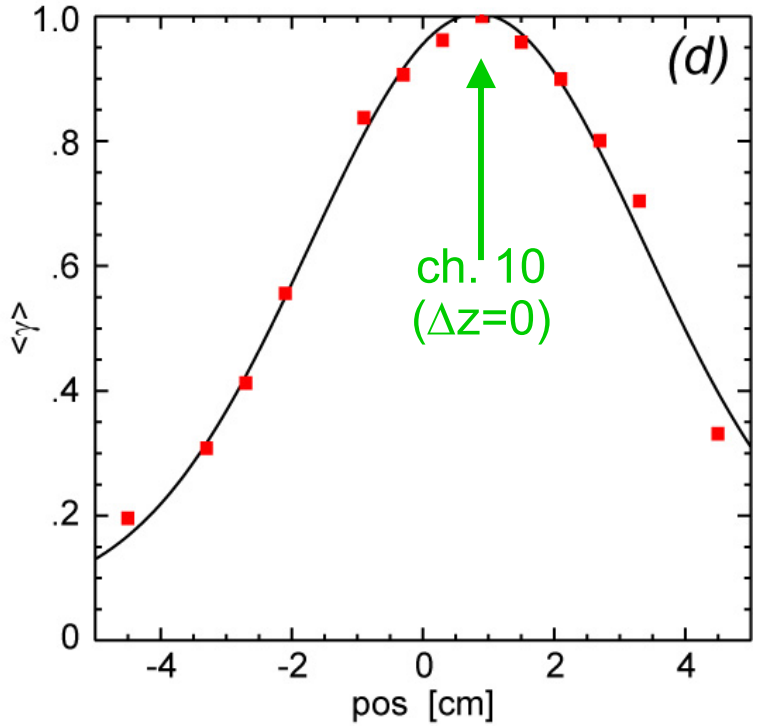
$$G_{xy}(f) = \frac{2}{T}\left(X^*(f)\cdot Y(f)\right)$$

ch. 10 & 9
Δz= 0.6 cm

ch. 10 & 5
Δz= 3.0 cm

ch. 10 & 1
Δz= 5.4 cm



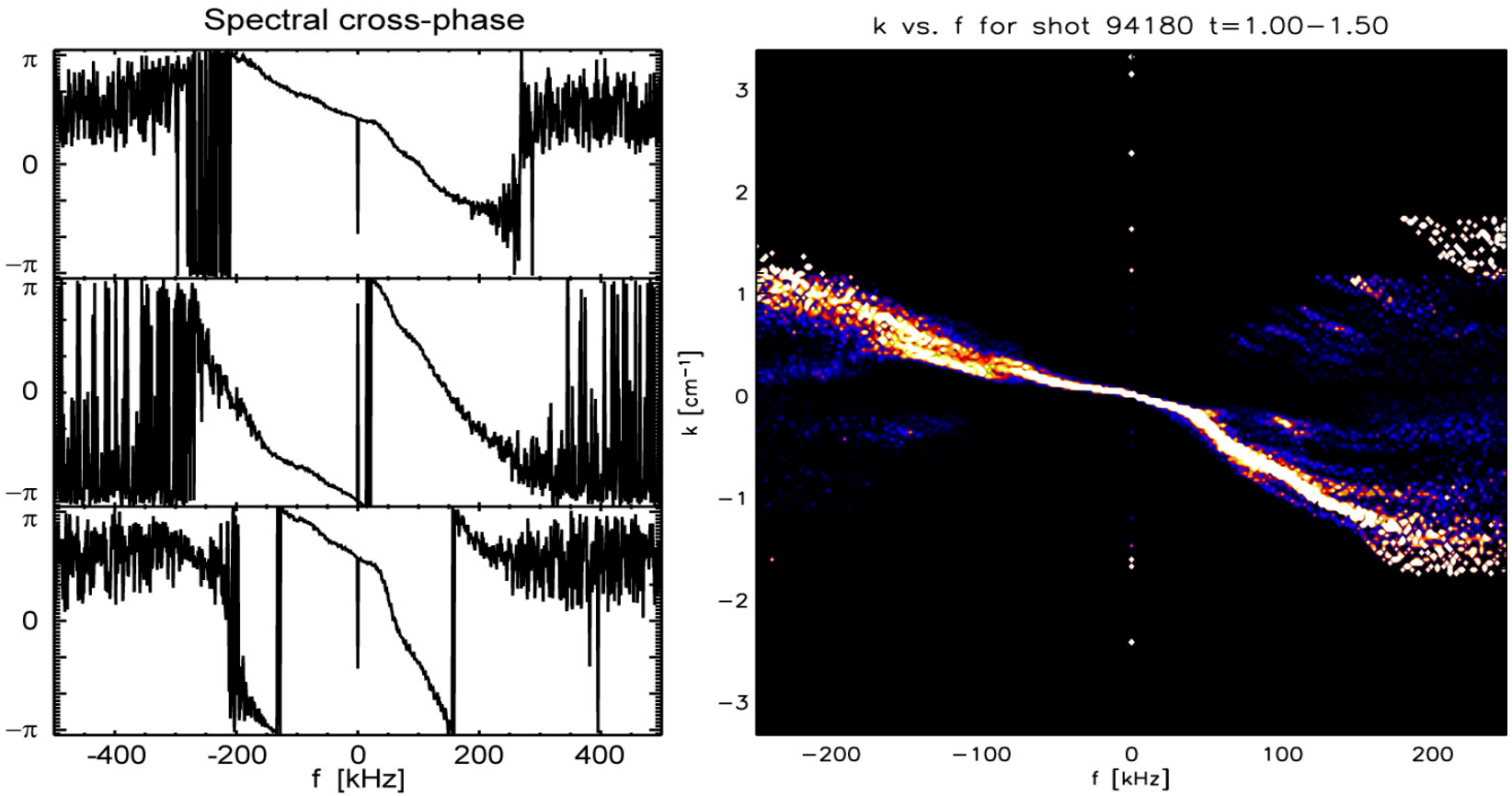


Cross spectra and coherency

Cross-phase for all channel pairs (weighted cross-spectral power) combined into map of cross-power vs. f,k

$$\phi(f) = \tan^{-1}\left[im(G_{xy}) \Big/ re(G_{xy}) \right]$$

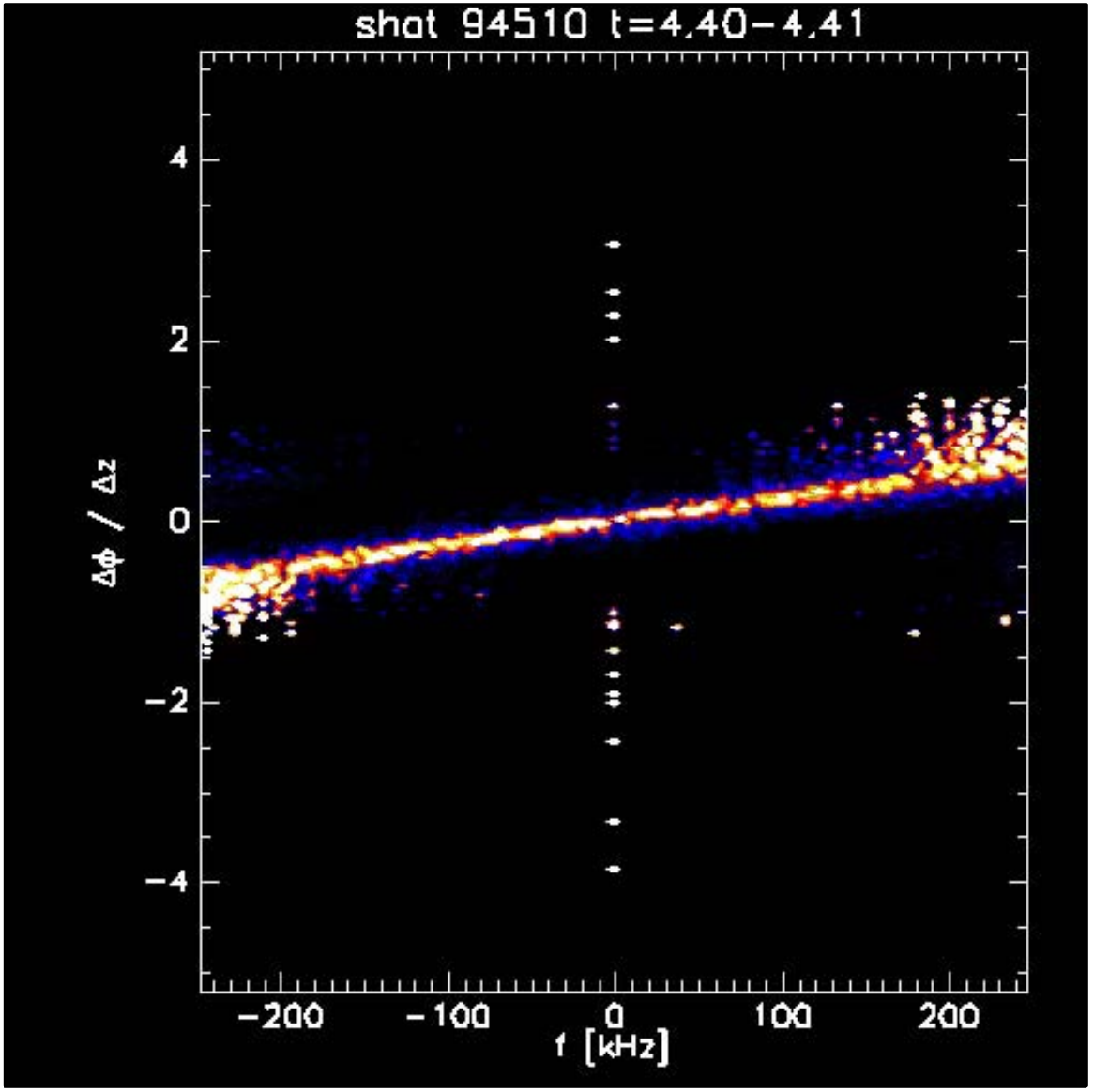
$$k(f) = \frac{\phi(f)}{\Delta z}$$



Cross spectra and coherency

Co-injected beam spins up plasma, then rotation reverts to electron-diamagnetic direction after NBI turn-off.

More complicated ‘dispersion relations’ can emerge from coherency analysis of imaged data



Bispectral Analysis

Measure of *coherence* between waves satisfying coupling/resonance conditions: $\omega_1 + \omega_2 = \omega_3 \quad k_1 + k_2 = k_3$

(To discern *coupled* waves from independent modes)

Bispectrum

$$B = \left\langle Z_1 Z_2 Z_3^{\,*} \right\rangle$$

Bicoherence

$$b = \sqrt{\frac{|B|^2}{\left\langle |Z_1 Z_2|^2 \right\rangle \left\langle |Z_3|^2 \right\rangle}}$$

Biphase

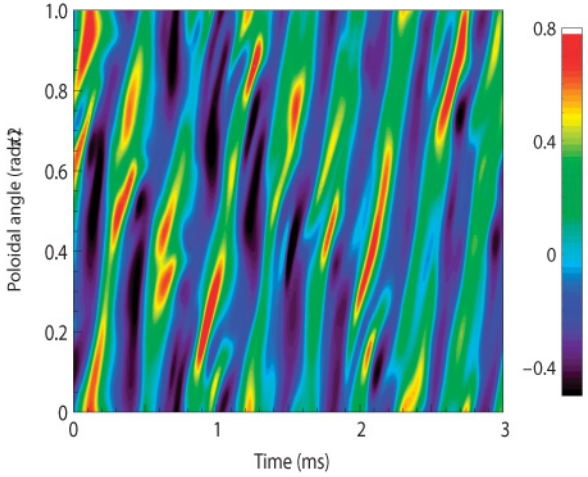
$$\varphi_b = \tan^{-1} \frac{Im(b)}{Re(b)}$$

- ***Spectral power transfer in plasma turbulence***
- ***Turbulent energy cascading***
- ***Nonlinear waves/structures from self-excited fluctuations***

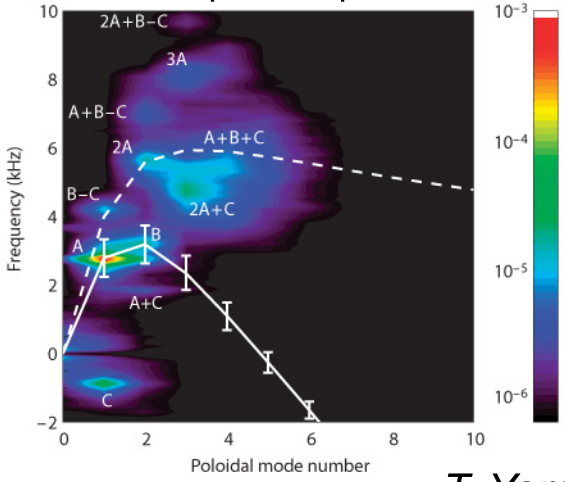
Bispectral Analysis

Coupling of drift waves & flute mode → cascade to smaller-scale fluctuations:

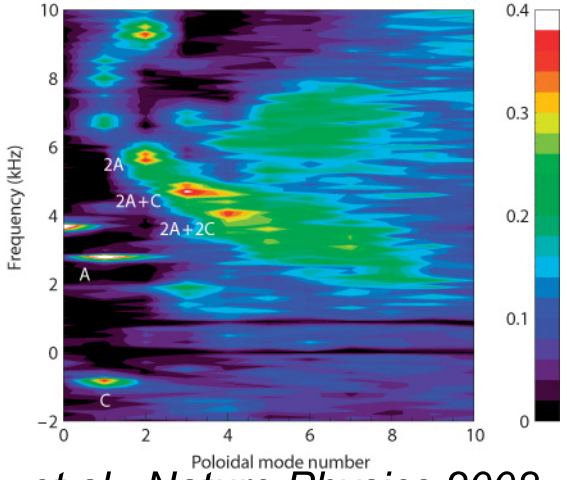
Poloidal mapping of $\tilde{n}_i$



Fourier power spectrum

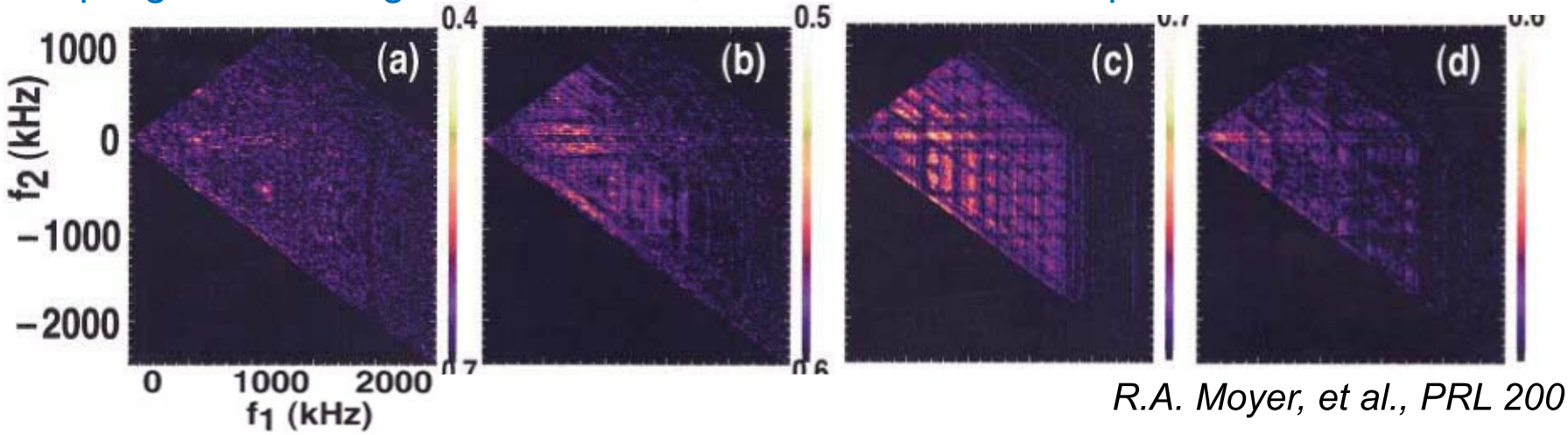


Bicoherence



T. Yamada, et al., Nature Physics 2008

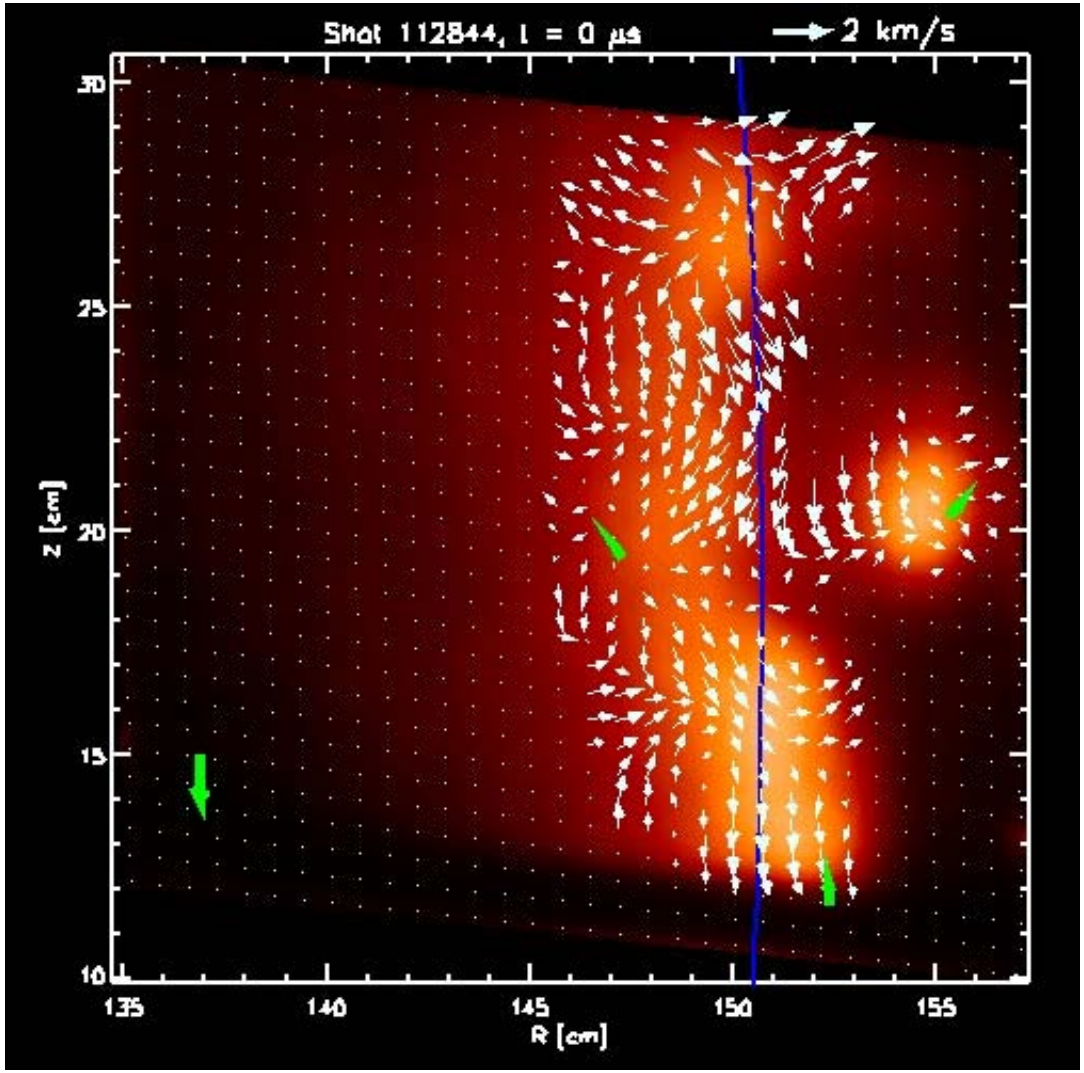
Coupling between high k,ω and low k,ω fluctuations near separatrix in L-H transition:



R.A. Moyer, et al., PRL 2001

Velocimetry

- Combination of Optical Flow and Pattern Matching techniques used to derive velocity fields from image frames
- Match structure evolution, trajectories, etc. to evolving models
- Characterize turbulent plasmas and assess intermittent transport
- Derive higher-order statistics from dense flow fields



Optical Flow : The Basics

- Optical flow condition = local constant brightness
  (Solve continuity equation for intensity):

$$\frac{\partial I}{\partial t} + u\frac{\partial I}{\partial x} + v\frac{\partial I}{\partial y} = \frac{\partial I}{\partial t} + \vec{v}\cdot\vec{\nabla} I = 0 \qquad \text{where} \qquad \vec{v} \equiv (u, v)$$

Now, can define spatial basis functions $\phi(x, y)$ so that

$$u = \sum_k u_k \phi_k \qquad \text{and} \qquad v = \sum_k v_k \phi_k$$

$$\frac{\partial I}{\partial t} + \sum_k u_k \phi_k \frac{\partial I}{\partial x} + \sum_k v_k \phi_k \frac{\partial I}{\partial y} = 0$$

(solved for each pixel)

Optical Flow : Implemented

$$\frac{\partial I}{\partial t} + \sum_k u_k \phi_k \frac{\partial I}{\partial x} + \sum_k v_k \phi_k \frac{\partial I}{\partial y} = 0$$

$$[x_0, x_1, x_2, x_3, x_4, x_5, \ldots] \equiv [u_0, v_0, u_1, v_1, u_2, v_2, \ldots]$$

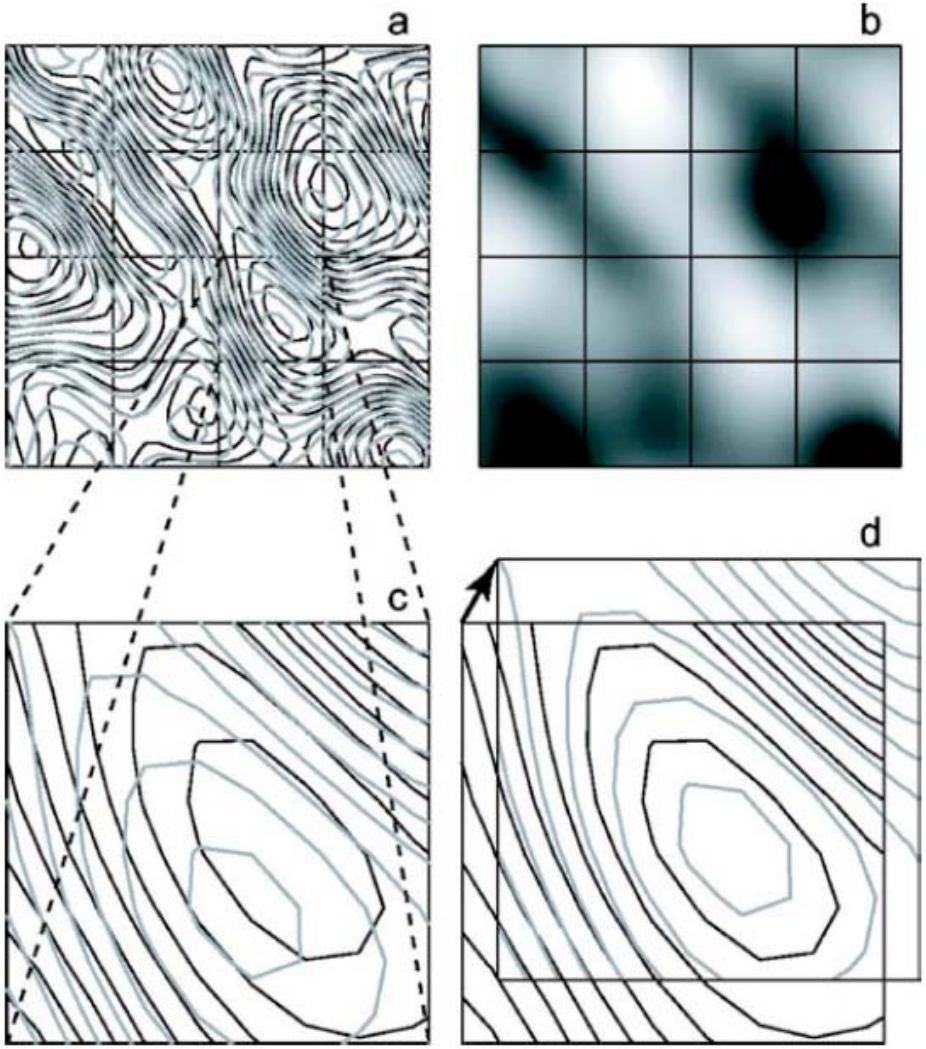
$$A_{p,0} \equiv \phi_0 \left[\frac{\partial I}{\partial x} \right]_p, A_{p,1} \equiv \phi_0 \left[\frac{\partial I}{\partial y} \right]_p, A_{p,2} \equiv \phi_1 \left[\frac{\partial I}{\partial x} \right]_p, A_{p,3} \equiv \phi_1 \left[\frac{\partial I}{\partial y} \right]_p, \ldots$$

$$B_p \equiv -\left[\frac{\partial I}{\partial t} \right]_p \quad \rightarrow \quad Ax = B$$

Combined Optical Flow / Pattern Matching

- Decompose each image into grid of tiled sub-images.
- Best matching position is found using error function:
  - Reward smaller integrated absolute difference over each sub-image.
  - Reward (weakly) “smoother” velocity fields. Assumes minimal shearing in true field.

- Velocity field is not constrained to lie along the gradient of the intensity.
- Does not exhibit a maximum velocity limit due to time resolution.
- Output highly dependent on initial guess for velocity field on each frame (use of optical flow output solves this).



Divergence-Controlled Optical Flow

- **Decompose the velocity field into the gradient of *two* scalar potentials:**

$$\vec{v} = \vec{\nabla}\psi + \vec{\nabla}\phi$$

- **Laminar and curl-free flows are captured by the gradient of *ψ*, solenoidal flows are captured by the gradient of *φ***

$$\vec{\nabla}\psi = \vec{v}_{laminar} + \vec{v}_{curl-free},\ \vec{\nabla}\times\left(\vec{\nabla}\psi\right) = 0$$

$$\vec{\nabla}\phi = \vec{v}_{solenoidal},\ \vec{\nabla}\cdot\left(\vec{\nabla}\phi\right) = 0$$

- **Design minimization functions for each velocity potential that are minimized by the "best" optical flow field estimate with vanishing divergence:**

-> Optical flow objective functional

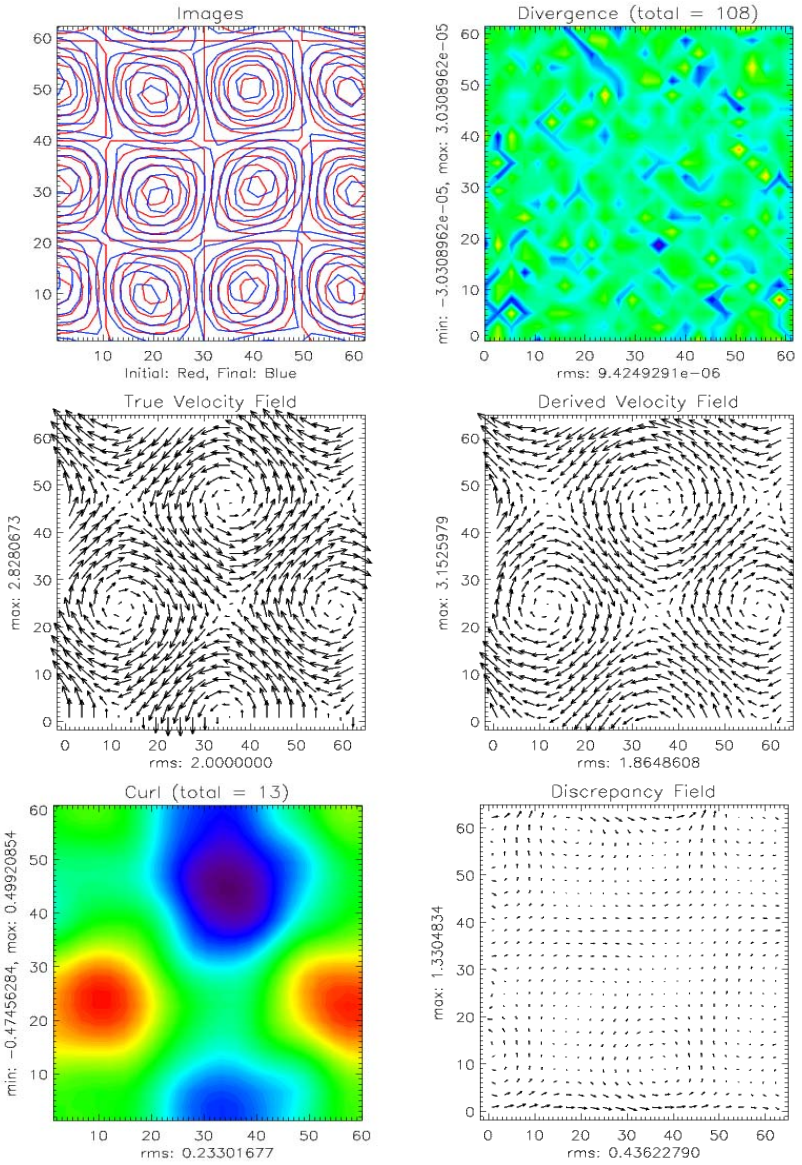
$$J_0(\psi,\phi) = \left| D_t I + \vec{\nabla} I \cdot \left(\vec{\nabla}\psi + \vec{\nabla}\phi\right)\right|^2$$

-> ψ Augmented Lagrangian (with φ held constant and Lagrange multipliers q)

$$L_\psi(\psi,\bar{\phi},q) = J_0(\psi,\bar{\phi}) + \left\langle q, \vec{\nabla}\cdot\left(\vec{\nabla}\psi\right)\right\rangle + \frac{r}{2}\left|\vec{\nabla}\cdot\left(\vec{\nabla}\psi\right)\right|^2 + \frac{\gamma}{2}\left|B_n\cdot\vec{\nabla}\psi\right|^2$$

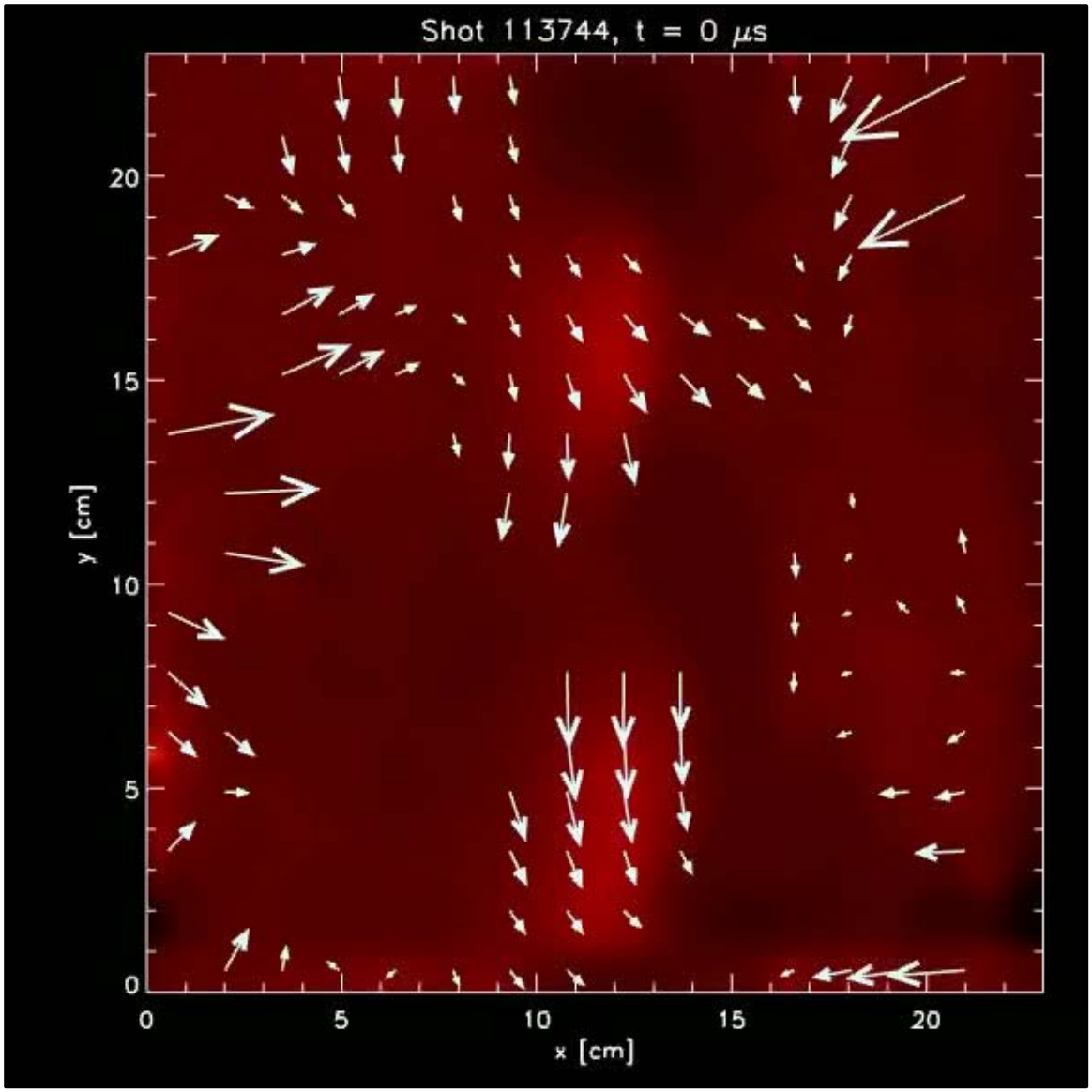
-> φ objective functional (with ψ held constant)

$$J_\phi(\bar{\psi},\phi) = J_0(\bar{\psi},\phi) + \frac{\lambda_c}{2}\left|\vec{\nabla}\left(\vec{\nabla}\times\left(\vec{\nabla}\phi\right)\right)\right|^2$$



Velocimetry on GPI (NSTX)

- "Dense" flow fields
- No a-priori knowledge of velocity behavior required
- No a-priori knowledge of structures required
- Analysis of turbulent flowfields possible
- Techniques applicable to many diagnostic systems



Concluding remarks

- Each technique here can be adapted to many multichannel / imaging instruments
- Techniques have different strengths and weaknesses depending on spatial / time resolution and coverage
- Many crossover studies will be possible!