

# Two-fluid theory of error-field penetration in tokamaks

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## Error-field penetration

- Low mode-number helical error-fields can drive magnetic reconnection in otherwise tearing-stable tokamak plasmas.
- Error-field driven reconnection has highly deleterious effect on plasma confinement.
- Natural mode rotation effectively “shields” error-field from plasma.
- Error-field exerts EM torque on plasma which slows natural mode rotation. Above threshold error-field strength there is sudden collapse in mode rotation, and shielding effect lost. This scenario known as “error-field penetration”.

## Experimental observations

- Error-field penetration has been observed in ohmic target plasmas on COMPASS-C/D, DIII-D, JET, and ALCATOR C-MOD.
- Penetration threshold typically  $(b_r/B_\phi)_{\text{crit}} \sim 10^{-4}$ .
- What is likely penetration threshold in ITER?
- Scaling experiments yield

$$(b_r/B_\phi)_{\text{crit}} \sim n_e^{\alpha_n} B_\phi^{\alpha_B} R_0^{\alpha_R}.$$

Exponents  $\alpha_n$  and  $\alpha_B$  directly measured. Exponent  $\alpha_R$  inferred from dimensionless scaling argument:  $\alpha_R = 2\alpha_n + 1.25\alpha_B$ .

## Results of scaling studies<sup>a</sup>

- All experiments find that  $\alpha_n$  very close to unity. So, approximately *linear* increase of penetration threshold with plasma density.
- JET/DIII-D/C-MOD find  $\alpha_B \sim -1$ . COMPASS-D finds  $\alpha_B \sim -3$ .
- Inferred value of  $\alpha_R$  is *positive* for all experiments. Hence, penetration threshold *increases* with major radius. Suggests that error-fields likely to be no more of problem in ITER than in present-day tokamaks.

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<sup>a</sup>S.M. Wolfe, *et al.*, Phys. Plasmas **12**, 056110 (2005).

## Error-field penetration theory<sup>a</sup>

- Error-field penetration theory based on idea that shielding effect of natural mode rotation sufficiently strong to allow response of inner region to be governed by *linear layer physics*.
- Given layer response, can calculate EM torque acting on inner region.
- EM torque balanced against phenomenological viscous restoring torque.
- Force balance found to break down above critical error-field amplitude. Break down leads to bifurcation to non-shielded solution in which error-field drives locked magnetic island in plasma.

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<sup>a</sup>R. Fitzpatrick, Phys. Plasmas **5**, 3325 (1998).

## Torque balance equation

- Torque balance equation takes form

$$\left(\frac{b_r}{B_\phi}\right)^2 = \frac{2P(Q_0 - Q)}{\kappa S \operatorname{Im}[-\Delta(Q)^{-1}]}.$$

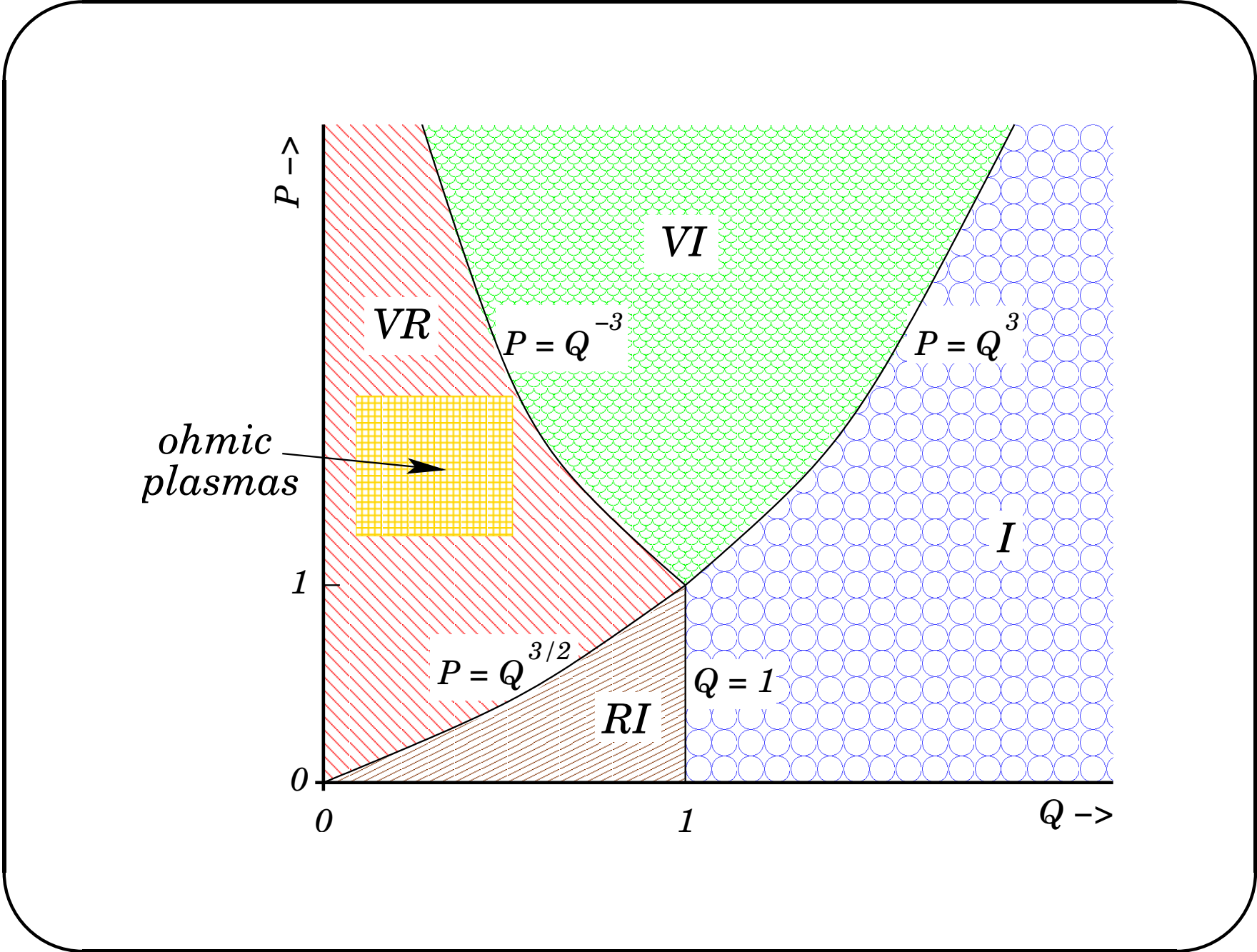
- Here,  $S$  is Lundquist no.,  $P$  magnetic Prandtl no.,  $Q_0$  normalized (to  $S^{-1/3}$ ) natural mode frequency,  $Q$  normalized slowed natural mode frequency,  $\Delta$  normalized layer response function,  $\kappa$  geometric factor.
- Once we know layer response function,  $\Delta(Q)$ , can find critical value of  $b_r/B_\phi$  above which torque balance lost. Corresponds to *maximum* value of r.h.s versus  $Q$ .
- To understand error-field penetration, simply need to classify all possible layer response regimes.

## Single-fluid response regimes <sup>a</sup>

- In single-fluid theory there are *four* possible layer response regimes:
  - **Visco-resistive (VR)** - viscosity and resistivity dominant.
  - **Resistive-inertial (RI)** - resistivity and inertia dominant.
  - **Visco-inertial (VI)** - viscosity and inertia dominant.
  - **Inertial (I)** - inertia dominant.
- Can plot out extents of these regimes in Q-P space.

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<sup>a</sup>R. Fitzpatrick, Phys. Plasmas **5**, 3325 (1998).



## Predictions of single-fluid theory<sup>a</sup>

- Ohmic tokamak plasmas in *visco-resistive regime*.
- Scaling with physics parameters:

$$(\mathbf{b}_r/B_\phi)_{\text{crit}} \sim Q_0 S^{-1/2} P^{7/12}.$$

- Scaling with standard dimensionless parameters:

$$(\mathbf{b}_r/B_\phi)_{\text{crit}} \sim \rho_*^{4/3} \nu_*^{1/6} \beta^{-1/6}.$$

- Scaling with engineering parameters (see later):

$$(\mathbf{b}_r/B_\phi)_{\text{crit}} \sim n_e^0 T_e^{1/6} B_\phi^{-1} R_0^{-7/6}.$$

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<sup>a</sup>A. Cole, R. Fitzpatrick, submitted to Phys. Plasmas (2005).

## Problems with single-fluid theory

- Single-fluid theory predicts about right magnitude for penetration threshold [*i.e.*,  $(b_r/B_\phi)_{\text{crit}} \sim 10^{-4}$ ].
- Scaling of penetration threshold with engineering parameters predicted by single-fluid theory *does not* agree very well with experimental data.
- Scaling with  $B_\phi$  about right. Scaling with  $n_e$  much too weak. Given dimensionless scaling constraint,  $\alpha_R = 2 \alpha_n + 1.25 \alpha_B$ , scaling with  $R_0$  inevitably also wrong—exponent *negative*.
- Single-fluid theory predicts that penetration threshold *decreases markedly* with increasing machine size, signaling trouble for ITER.

## Possible source of problems

- Single-fluid theory models natural mode rotation as due entirely to  $\mathbf{E} \times \mathbf{B}$  flow. In reality, mode rotation has large *diamagnetic* component. Is shielding effect due to diamagnetic mode rotation substantially different from that due to  $\mathbf{E} \times \mathbf{B}$  rotation?
- In large, hot tokamaks even ohmic plasmas are in low collisionality regime. Do low collisionality effects substantially modify layer response functions?
- In order to investigate both of these questions, need to adopt *two-fluid* model of plasma.

## Two-fluid model <sup>a</sup>

- We employ reduced two-fluid model valid for arbitrary  $\beta$ .
- Model contains electron and ion diamagnetic effects, including ion gyroviscous correction.
- Model contains semi-collisional and Hall effects appropriate to low-collisionality plasmas.
- Model does not contain electron inertia, anisotropic heat conduction, neoclassical flow damping, or trapped particle effects.

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<sup>a</sup>R. Fitzpatrick, F.L. Waelbroeck, Phys. Plasmas **12**, 022307, (2005).

## Diamagnetic effects<sup>a</sup>

- Using two-fluid model, we conclude that diamagnetic mode rotation just as effective as  $\mathbf{E} \times \mathbf{B}$  mode rotation at shielding error-fields.
- Penetration threshold largely indifferent to whether natural mode rotation due to diamagnetic or  $\mathbf{E} \times \mathbf{B}$  flows, or to some combination of two. Penetration threshold essentially just scales with total mode rotation frequency.

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<sup>a</sup>A. Cole, R. Fitzpatrick, submitted to Phys. Plasmas (2005).

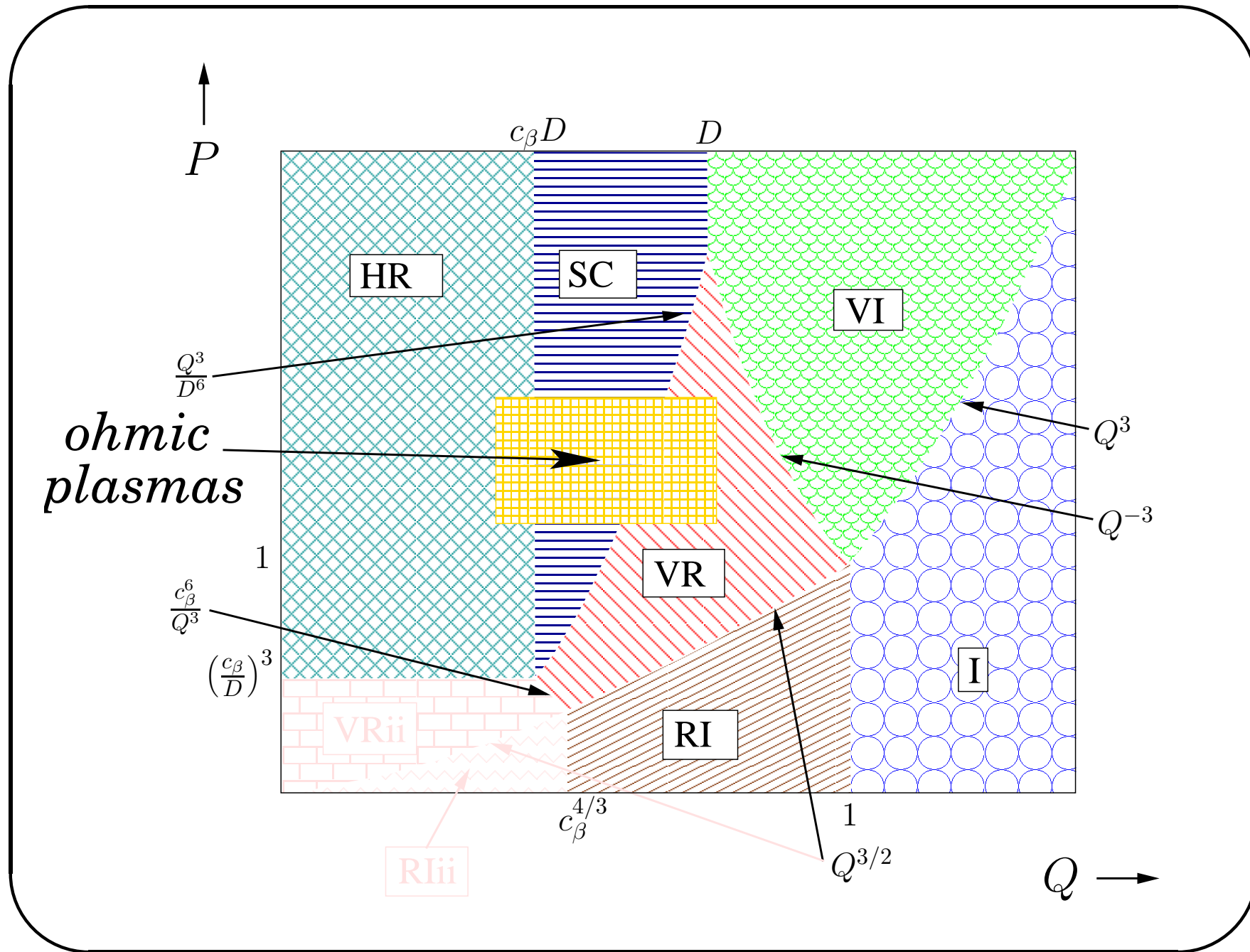
## Low collisionality effects<sup>a</sup>

- Using two-fluid model, we find two new layer response regimes which may be appropriate to ohmic plasmas:
  - **Semi-collisional (SC)**<sup>b</sup> - resistivity, and collisionless ion skin-depth effects dominant.
  - **Hall-resistive (HR)** - resistivity, and Hall effects dominant.
- Two new normalized parameters introduced into theory:
  - **D** - normalized (to  $S^{-1/3}$ )  $\rho_s$ .
  - **$c_\beta$**   $\sim \sqrt{\beta}$ .
- Can plot out extents of these regimes in Q-P space.

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<sup>a</sup>A. Cole, R. Fitzpatrick, submitted to Phys. Plasmas (2005).

<sup>b</sup>F.L. Waelbreock, Phys. Plasmas **10**, 4040 (2003).



## Scaling of threshold with physics parameters

- Visco-resistive:

$$(b_r/B_\phi)_{\text{crit}} \sim Q_0 S^{-1/2} p^{7/12}.$$

- Semi-collisional:

$$(b_r/B_\phi)_{\text{crit}} \sim Q_0^{5/4} S^{-1/2} p^{1/2} D^{-1/2}.$$

- Hall-resistive:

$$(B_r/B_\phi)_{\text{crit}} \sim Q_0 S^{-1/2} p^{1/2} D^{-1/4} c_\beta^{1/4}.$$

## Scaling with standard dimensionless parameters

- Visco-resistive:

$$(\mathbf{b}_r/B_\phi)_{\text{crit}} \sim \rho_*^{4/3} \nu_*^{1/6} \beta^{-1/6}.$$

- Semi-collisional:

$$(\mathbf{b}_r/B_\phi)_{\text{crit}} \sim \rho_*^{5/4} \nu_*^{1/4}.$$

- Hall-resistive:

$$(B_r/B_\phi)_{\text{crit}} \sim \rho_*^{5/4} \nu_*^{1/4}.$$

## Transformation to engineering parameters

- Wish to express penetration threshold in form

$$(b_r/B_\phi)_{\text{crit}} \sim n_e^{\alpha_n} B_\phi^{\alpha_B} R_0^{\alpha_R}.$$

- Assume natural frequency scales as diamagnetic frequency.
- Assume momentum confinement time-scale has *exactly same scaling* as energy confinement time-scale.
- Eliminate energy confinement time-scale using ohmic power balance. Introduces additional engineering parameter  $T_e$ . Have

$$(b_r/B_\phi)_{\text{crit}} \sim n_e^{\alpha_n} T_e^{\alpha_T} B_\phi^{\alpha_B} R_0^{\alpha_R},$$

where  $\alpha_R = 2\alpha_n + 0.5\alpha_T + 1.25\alpha_B$ .

## Scaling with engineering parameters

- Visco-resistive:

$$(b_r/B_\phi)_{\text{crit}} \sim n_e^0 T_e^{1/6} B_\phi^{-1} R_0^{-7/6}.$$

- Semi-collisional:

$$(b_r/B_\phi)_{\text{crit}} \sim n_e^{1/4} T_e^{1/8} B_\phi^{-5/4} R_0^{-1}.$$

- Hall-resistive:

$$(b_r/B_\phi)_{\text{crit}} \sim n_e^{1/4} T_e^{1/8} B_\phi^{-5/4} R_0^{-1}.$$

## Discussion

- New low-collisionality scalings *very similar* to old high-collisionality scaling.
- Scaling with  $n_e$  still *too weak* (unless  $T_e$  increases very rapidly with  $n_e$ —seems rather unlikely).
- Scaling with  $R_0$  still exhibits large *negative exponent*.
- Assumption that momentum confinement time-scale scales in exactly same manner as energy confinement time-scale *highly questionable*. Need experimental data to make better assumption. Maybe this will help close gap between theory and experiment.

## Summary

- Have generalized single-fluid error-field penetration theory to take two-fluid effects into account.
- Two-fluid effects have surprisingly little influence on scaling of penetration threshold. Neither diamagnetic nor low-collisionality effects seem to make much difference.
- Discrepancy between theory and experiment still remains.
- Any suggestions welcome!