PLASMA EFFECTS ON THE LOCATION OF THE OUTERMOST MAGNETIC SURFACE

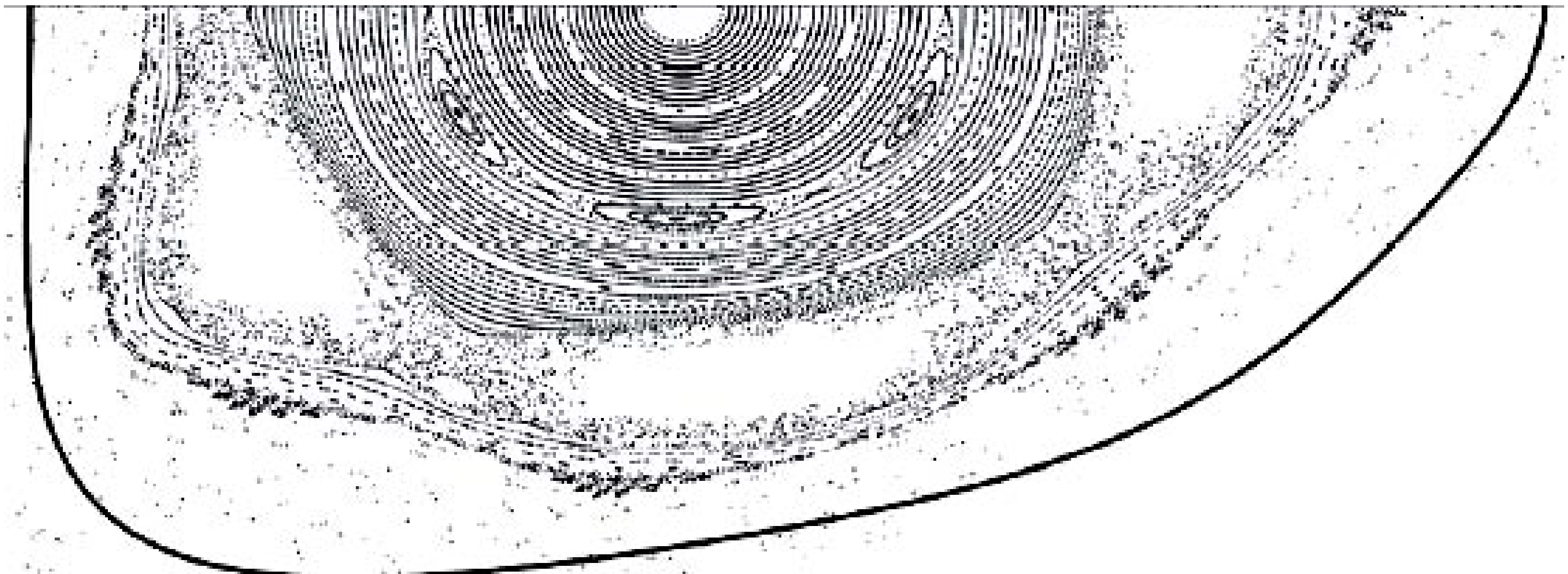
Allen H. Boozer
Columbia University
New York, NY, USA

Physics of Plasmas 12, 092504 (2005)

The Issue

Magnetic field lines can be stochastic and circuit the torus a very large number of times before crossing the region or striking a wall.

Plasma rotation and pressure can modify the width of the stochastic region.



“Unhealed” NCSX from NF 43, 1040 (2003).

Condition for Pressure Shielding

Plasma pressure can shield out stochasiticy causing perturbations if

$$\Delta\beta > \frac{\iota^2}{16}\frac{w}{a}\frac{a}{R_0}\left(\frac{a\iota'}{2\iota}\right) \approx 10^{-4}$$

$\Delta\beta = 2\mu_0\Delta p/B^2$ measures pressure drop across stochastic layer
w is stochastic layer width
$\iota = 1/q$ is rotational transform and $\iota' = d\iota/dr$
a is minor and R_0 the major radius.

Near an axisymmetric separatrix $\iota'/\iota \approx 1/w$, so

$$\Delta\beta > \frac{\iota^2}{32}\frac{a}{R_0} \approx 10^{-3}$$

Calculation of Pressure Effect On Width

Magnetic stochasticity caused by resonances between $\delta A_{||}/B$ and the underlying magnetic structure where

$$\nabla^2 \frac{\delta A_{\|}}{B} \approx -\mu_0 \frac{j_{\|}}{B}.$$

Pressure gradient drives a parallel current, the P.S. current. Let

$$\mathcal{D} \equiv -\frac{\vec{\nabla} \cdot \vec{j}_{\perp}}{B} = \frac{\vec{B}}{B} \cdot \vec{\nabla} \frac{j_{\|}}{B}.$$

Plasma force balance is $\vec{f}_a + \vec{\nabla} p = \vec{j} \times \vec{B}$. Additional force needed for pressure variation in stochastic region since $\vec{B} \cdot \vec{f}_a = -\vec{B} \cdot \vec{\nabla} p$, but $\left|\hat{b} \cdot f_a\right| = \left|\hat{b} \cdot \vec{\nabla} p\right| << \left|\vec{\nabla} p\right|$.

$$\mathcal{D} = \frac{2}{B^2}(\hat{b} \times \vec{\nabla} p) \cdot \vec{\kappa} - \frac{\vec{B}}{B^2} \cdot \vec{\nabla} \times \vec{f}_a$$ where curvature $\vec{\kappa} \equiv \hat{b} \cdot \vec{\nabla} \hat{b}$.

$\vec{B} \cdot \vec{\nabla} \times \vec{f}_a$ gives viscosity (rotation) effect. Neglected here.

In a stochastic field neighboring field lines separate as $\exp(\ell / L_L)$. Lyapunov length L_L provides a decorrelation distance

$$\frac{j_{\parallel}(\vec{x})}{B} = \int_{-\infty}^{\infty} e^{-(\ell'/L_0)^2} \mathcal{D}[\vec{x}(\ell - \ell')] d\ell'$$ with L_0 a few times L_L.

If resonant $\delta A_{||}/B$ Fourier components caused by currents due local plasma pressure gradient are comparable to those caused by external currents, stochastic layer is either shielded out, or plasma is unstable to setting up a stochastic layer.

Pressure Distribution in Stochastic Layer

Plasma pressure varies slowly along $\vec{B}$ but rapidly across $\vec{B}$. This feature occurs in solutions to

$$\frac{\partial p}{\partial t} + \vec{v}_{\|} \cdot \vec{\nabla} p = \vec{\nabla} \cdot D \vec{\nabla} p, \text{ where } \vec{v}_{\|} = \pm \frac{3C_s}{B} \vec{B}, \text{ so } \vec{\nabla} \cdot \vec{v}_{\|} = 0.$$

Solve using Monte Carlo method: $\mu_n = (1 - \nu \Delta t)\mu_o \pm \sqrt{(1 - \mu_o^2)\nu \Delta t}$

For each of the 3 coordinates: $x_n = x_o \pm \sqrt{D \Delta t}$.

$$\text{Let } p(\vec{x}) = \sum_i p_i F_i(\vec{x}), \text{ where } p_i = \frac{c_0}{K} \sum_{k=1}^{K} F_i(\vec{x}_k) \text{ and } \vec{x}_k \equiv \vec{x}(t = k\Delta t).$$

$$\text{Follows from } \lim_{K \to \infty} \frac{1}{K} \sum_{k=1}^{K} F_i(\vec{x}_k) F_j(\vec{x}_k) \propto \delta_{ij}.$$

Estimation of Effect

Critical island half-width for stochasticity is $\square \square \frac{a}{4m}\left(\frac{2\square}{a}\right)$.

Given by $\frac{\square A_{\parallel}}{aB} \square \frac{\square}{32m^2}\frac{a}{R_0}\left(\frac{2\square}{a}\right)$. Mode # of primary island is m.

Near island $p \square p_0 + p_I\{1+\cos(n\square \square m\square)\}\{1+\cos(n\square \square (m+1)\square)\}$
$p_I \square (2\square/w)\square p$ with $\square p$ is pressure drop across stochastic width w.

Gives resonant term $\mathcal{D} \square \frac{p_I}{B^2 \square R_0}$ varying on scale $L_I = \frac{4R_0}{\square m \square}$.

Pressure-driven current gives $\left(\frac{\delta A_{\parallel}}{aB}\right)_p \approx \frac{4\Delta\beta}{m\iota' a}\frac{\iota}{w}$, where $\Delta\beta \equiv \frac{2\mu_0 \Delta p}{B^2}$.

Pressue-driven $\delta A_{||}/B$ is smaller than that externally produced only if

$$\Delta\beta < \frac{\iota^2}{16}\frac{w}{a}\frac{a}{R_0}\left(\frac{a\iota'}{2\iota}\right).$$

If $w/a=0.1$, $\iota=1/3$, and $a/R_0=1/3$ then there are strong pressure corrections unless $\Delta\beta<2x10^{-4}$.

□ Dependence on Mode Number *m*

Critical □ for important plasma effects is independent of *m*.

However, expected □ µ m^4 if parallel transport is diffusive.

Typical distance along a field line to cross stochastic layer is $L_T = (w/2□)^2 \, L_I$ µ m^2.

Distance to go around an island is $L_I = 8R_0/□$.

Island width for overlap is □ µ $1/m$.

Expect strong m dependence in stochastic layer balance.

Summary of Plasma Effects

If pressure drop across a stochastic layer is sufficiently large, □□$>10^{-3}$ *to* 10^{-4}, plasma pressure has a major effect on stochastic layer.

In principle, plasma can shield or amplify stochasticity effects. If it amplifies, plasma is unstable to establishing a stochastic layer.

Though critical □□ is independent of mode number m of the primary islands, the value that can arise increases rapidly with m.

Method was given for calculating the importance of plasma effects.