Magnetic Field Errors: Interactions and Control

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- *Resistive wall modes and error field amplification*, A. H. Boozer, Physics of Plasmas **10**, 1458-1467 (2003).

- *Magnetic islands and perturbed plasma equilibria*, C. Nührenberg and A. H. Boozer, Physics of Plasmas **10**, 2840-2851 (2003).

- *Physics of Magnetically Confined Plasmas*, A. H. Boozer, requested by Reviews of Modern Physics, submitted 1 September 2003, 70 pages.

Major Points

- Why required plasma information for error field analysis is very limited.

  1. Permeability matrix $\overleftrightarrow{P}$ gives plasma information. Eigenvalues have form $\frac{1}{-s+i\alpha}$.

  2. Follows from $\delta\vec{B} = \vec{\nabla}\phi$ with $\nabla^2\phi = 0$ between wall and plasma.

- Upper limit on error field torque.

  1. Follows from Maxwell stress tensor $\frac{\delta B_\varphi \delta B_r}{\mu_0}$.

  2. Implies small islands are shielded.

- Use of ideal δW code to determine island causing perturbations.

- Design of error correction coils.

Plasma Response to a Field Error

An external field error gives a perturbed equilibrium.

For an ideal plasma, perturbed equilibrium can be found by minimizing standard δW specifying

$$\delta\vec{B} \cdot \vec{\nabla}\psi = \vec{B}_0 \cdot \vec{\nabla}(\vec{\xi} \cdot \vec{\nabla}\psi)$$

at location of unperturbed plasma surface.

Plasma equilibrium equations determine relation between:

- perturbed normal field $\delta\vec{B} \cdot \hat{n}$
- perturbed tangential field, $\hat{n} \times \delta\vec{B}$

on plasma surface.

Plasma Modification of Error Field

Between external currents and plasma, change in field is curl and divergence free, $\delta\vec{B} = \vec{\nabla}\phi$ with $\nabla^2\phi = 0$.

Perturbed plasma equilibrium determines relation between:

- Perturbed normal field on plasma surface $\delta\vec{B}\cdot\hat{n}$ or $\hat{n}\cdot\vec{\nabla}\phi$.

- perturbed tangential field on plasma surface, $\hat{n}\times\delta\vec{B} = \hat{n}\times\vec{\nabla}\phi$ or ϕ.

Given ϕ and $\hat{n}\cdot\vec{\nabla}\phi$

Laplace theory determines relation between $\delta\vec{B}_x\cdot\hat{n}$ and $\delta\vec{B}\cdot\hat{n}$ on plasma surface.

External Effects of Field Error

Relation between $\delta\vec{B}_x \cdot \hat{n}$ and $\delta\vec{B} \cdot \hat{n}$ on plasma surface determines all effects of plasmas that are externally measurable (torques, resistive wall modes, etc.).

$f_i(\theta, \varphi)$ are a set of dimensionless functions $\oint f_i f_j^* w da = \delta_{ij}$.

$$\hat{n} \cdot \delta\vec{B} = w \sum_i \Phi_i(t) f_i^*(\theta, \varphi)$$

$$\hat{n} \cdot \delta\vec{B}_x = w \sum_i \Phi_i^x(t) f_i^*(\theta, \varphi)$$

For linear perturbations $\Phi_i = \sum_j P_{ij} \Phi_j^x$. Defines permeability matrix $\overleftrightarrow{P}$.

Permeability Matrix $\overleftrightarrow{P}$

$\overleftrightarrow{P}$ is purely a plasma quantity and determines all effects of plasmas that are externally measurable.

Eigenvalues of $\overleftrightarrow{P}$ have form $\frac{1}{-s+i\alpha}$

$$s \equiv -\frac{\textit{energy with plasma}}{\textit{energy without plasma}}$$

required to produce given $\delta\vec{B}\cdot\hat{n}$ on plasma surface.

$$\alpha \equiv \frac{\textit{torque on plasma}}{\textit{energy without plasma}}$$

associated with a given $\delta\vec{B}\cdot\hat{n}$ on plasma surface.

Error Field Amplification

$\Phi = \frac{1}{-s+i\alpha}\Phi_x$ implies $|\Phi|^2 = \frac{|\Phi_x|^2}{s^2+\alpha^2}$

As plasma approaches marginal stability, $|s| \to 0$, error field on plasma $\delta\vec{B}\cdot\hat{n}$ is enormous for $|\alpha| << 1$.

Torque on plasma, $\tau_\varphi = \alpha\frac{|\Phi|^2}{L_p} = \frac{\alpha}{s^2+\alpha^2}\frac{|\Phi_x|^2}{L_p}$

Externally applied torque has upper limit

$$|\tau_\varphi| \leq \frac{1}{2|s|}\frac{|\Phi_x|^2}{L_p}$$

which is very large near marginal stability.

Creation of an island causes strong damping (large effective α), if $|\alpha_{eff}| > s$ island is shielded out of the plasma.

Parallel currents that shield out island are driven by the viscosity and have a radial spread,

$$\vec{B}_0\cdot\vec{\nabla}(j_{||}/B)) = -\vec{\nabla}\cdot(\vec{B}_0\times\vec{f}_{vis}/B^2)$$

Island Calculations using Ideal MHD δW Codes

Islands associated with each distinct rational number n/m are driven by a single distribution of external normal magnetic field on the plasma surface, $\vec{B}^x_{n/m} \cdot \hat{n} = w\Phi^x_{n/m} f_{n/m}(\theta, \varphi)$.

The externally produced normal field distributions $f_i(\theta, \varphi)$ that are orthogonal, $\oint f_i f^*_{n/m} w da = 0$, to the $f_{n/m}$'s drive no islands.

The singular current that arises in ideal MHD to preserve surfaces is proportional to the jump $[\vec{\xi} \cdot \vec{\nabla}\psi]$ at rational surfaces.

An SVD analysis of the matrix that relates the distributions of $\vec{B}_x \cdot \hat{n}$ on the plasma surface and the $[\vec{\xi} \cdot \vec{\nabla}\psi]$'s on the n/m rational surfaces determines the $f_{n/m}$'s.

Need enough correction coils to control $f_{n/m}$'s.

Design of error correction coils

The important magnetic field errors drive:

- marginally stable, $|s| \to 0$ MHD modes.

  $\vec{B}_x \cdot \hat{n}/w$ has definite dependence $f_s(\theta, \varphi)$

- magnetic islands

  $\vec{B}_x \cdot \hat{n}/w$ has definite dependence $f_{m/n}(\theta, \varphi)$

Correction coils must produce as many (say I) independent normal field distributions on the plasma surface $\vec{B}_x \cdot \hat{n}/w = \Phi_i f_i(\theta, \varphi)$.

Let $\vec{\Phi} = \overleftrightarrow{M} \cdot \vec{J}$ with $\vec{J}$ a vector of any set of J independent coil currents.

If the $I \times J$ matrix $\overleftrightarrow{M}$ has I non-singular eigenvalues, with none too small, then the coils form an adequate correction coil system.

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