Predicting Optimal 3d Coil Configurations

by

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[2] General Atomics

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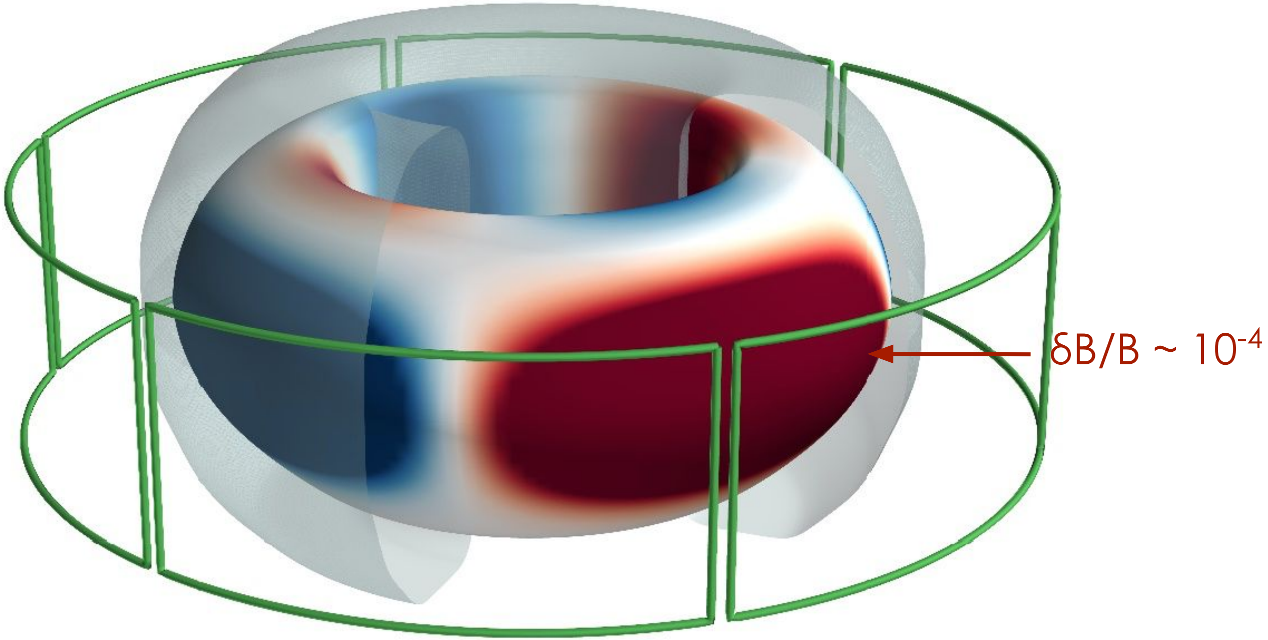
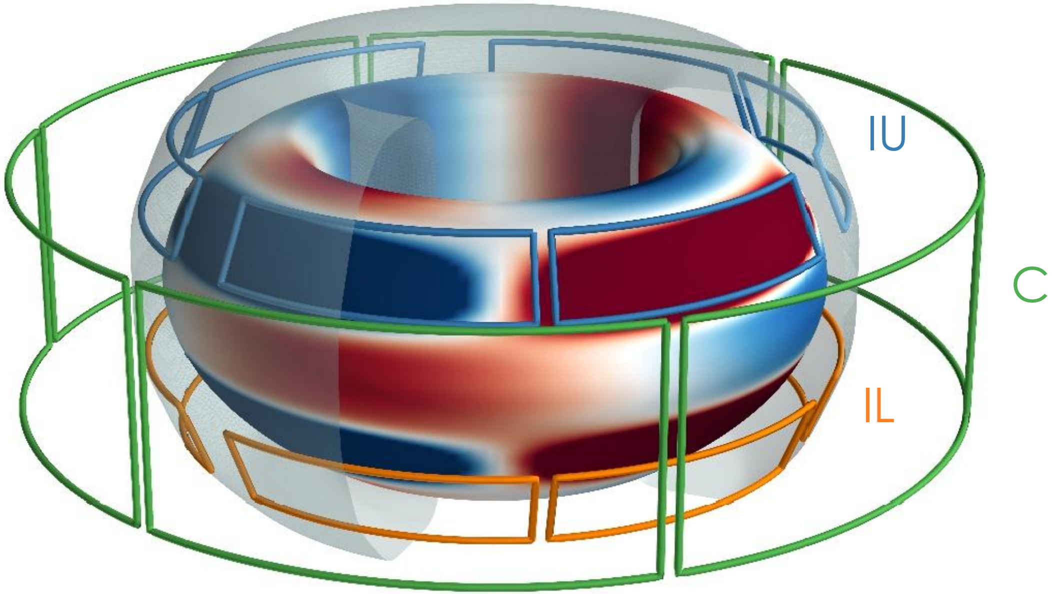
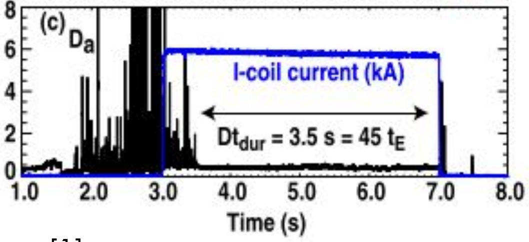
Adjustable Error Field Correction Coils Are a Stable of Today's Tokamaks



We keep adding more and more of them



All These Coils Do More Than Just Correct Error Fields



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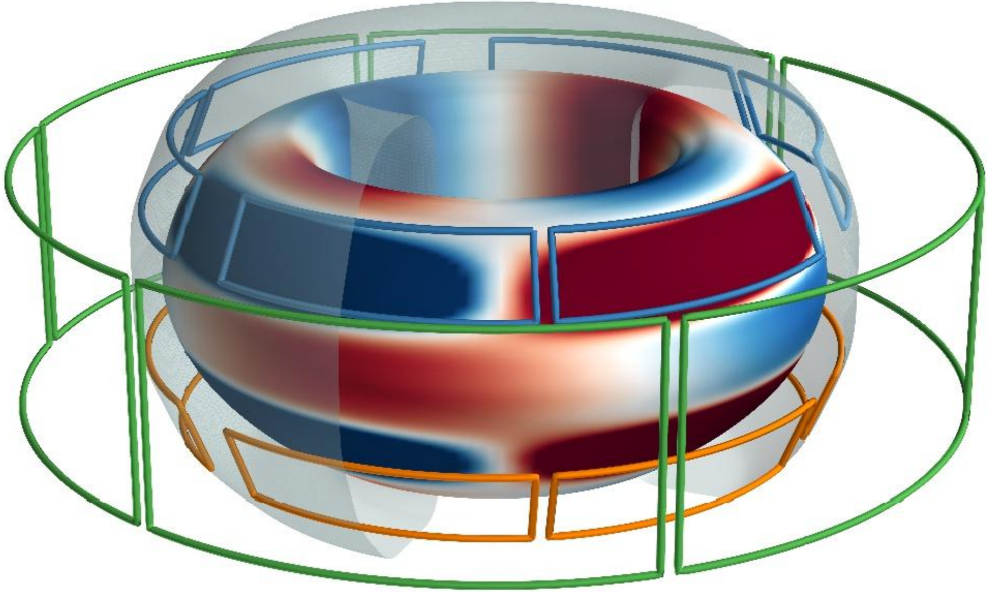
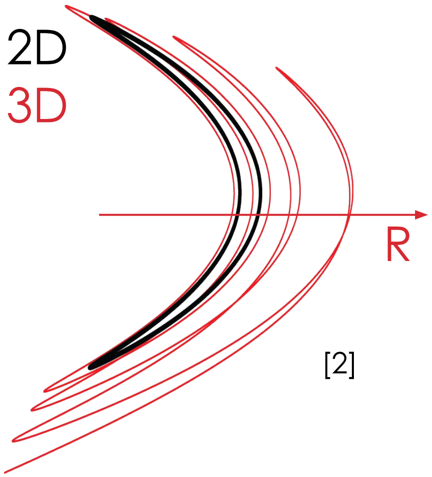


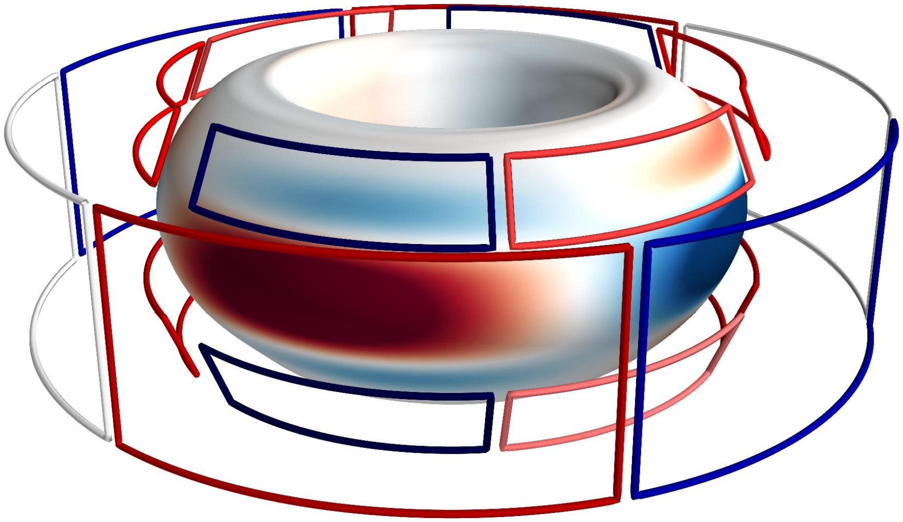


[2]

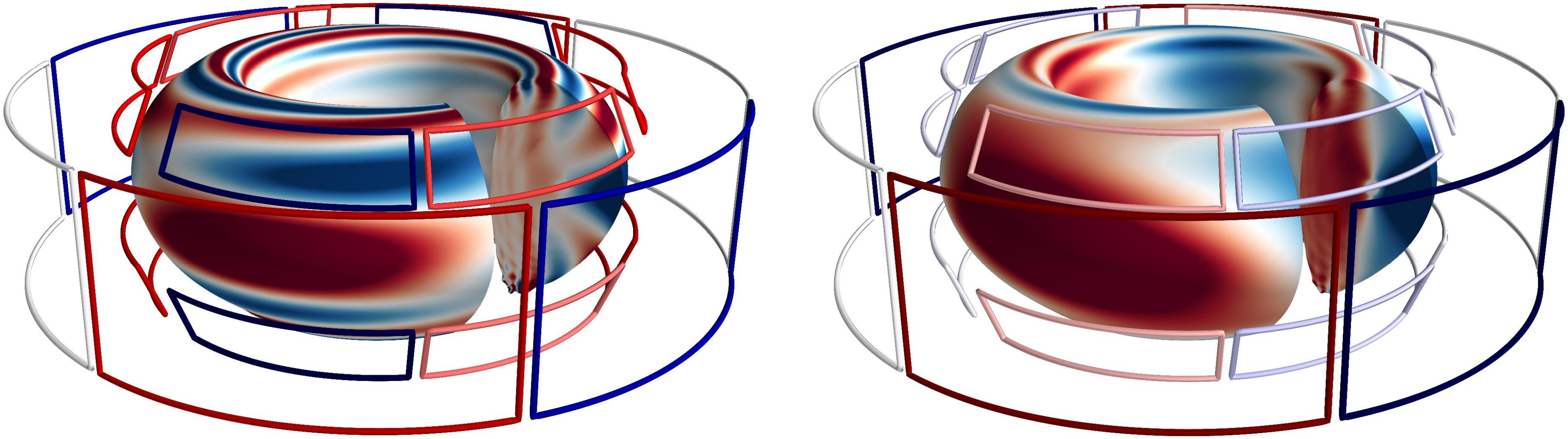
[1] Wade Et al., Nuclear Fusion 55 (2015).

[2] Kim, K., et al., *Phys. Plasmas* 19,(2012).

Many Combinations of Coils Are Available



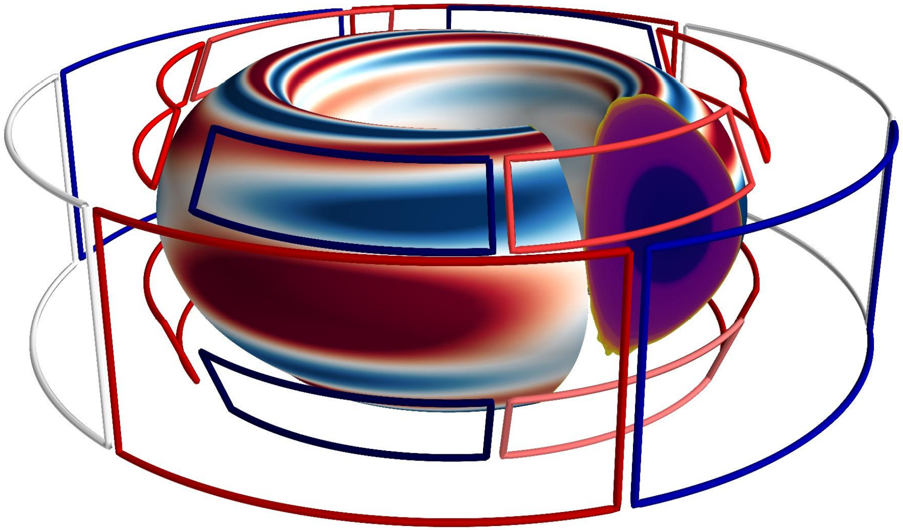
Even the Plasma Response is Multi-modal



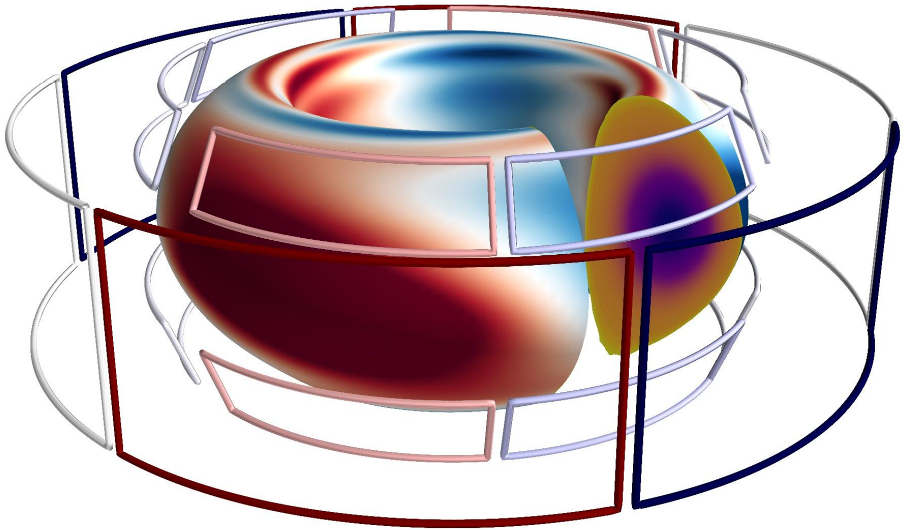
Paz-Soldan, *et al.*, *Phys. Rev. Lett.* 114, (2015).

This Talk: How to Optimize a *5D* Coil Phase Space for Desired Physics

NTV in the Edge



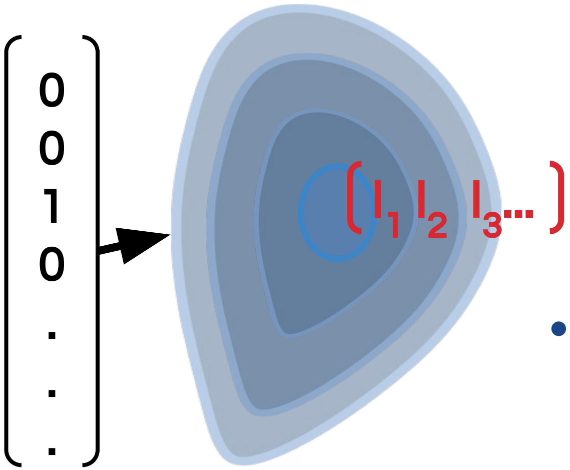
NTV Throughout



Review: Resonant Coupling

The ideal MHD resonant coupling "dominant mode" is the leading error field correction metric for ITER

- **Linearity and speed of ideal MHD perturbed equilibria provides resonant coupling matrix**



$$\begin{bmatrix} I_r \end{bmatrix} = \left(\begin{bmatrix} \; \end{bmatrix}\begin{bmatrix} \; \end{bmatrix}\begin{bmatrix} \; \end{bmatrix} C \right) \begin{bmatrix} \Phi_x \end{bmatrix}$$

- **SVD provides an ortho-normal basis describing the entire space of Φ_x that drives resonant current**
  - When 1st singular value >> 2nd, we say V^T_1 is a "dominant mode"
- **The "overlap"[1] or "coupling" is a measure of how efficiently a given external field drives this dominant mode**

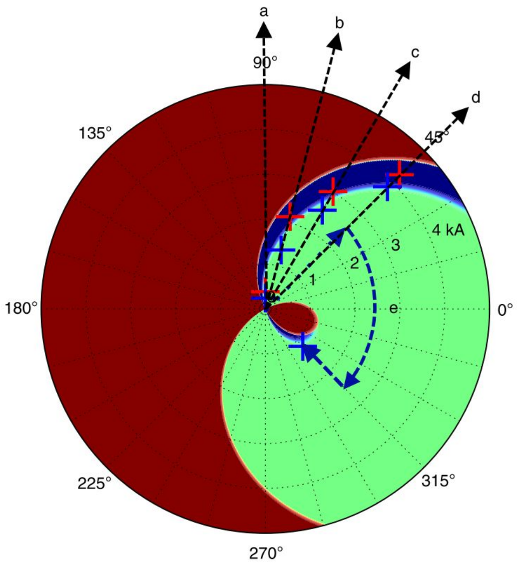
$$\textit{“overlap”} = \Phi_x \cdot \Phi_d \,/\, |\Phi_x|\,|\Phi_d|$$

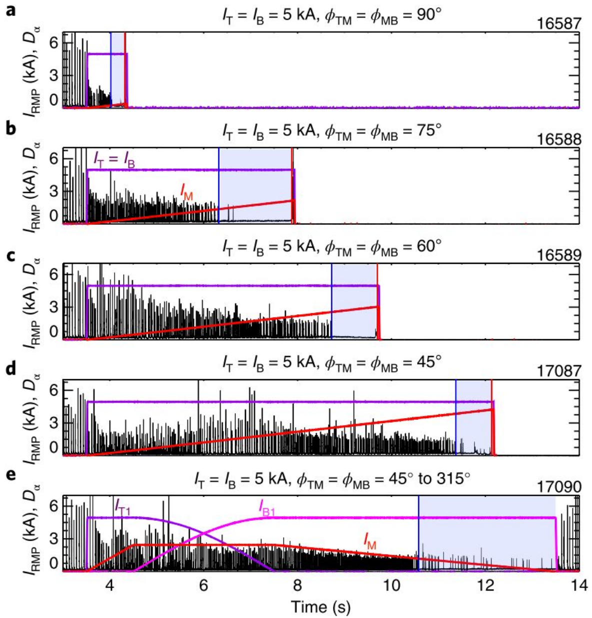
Recent multi-modal work has shown it is possible to separate the edge and core resonances

- **LM disruptions & ELM suppression both governed by resonant field penetration**
- **Use same metric, just separate core & edge**

I_r

- **To date, focus on brute-force mapping**





[1] Park, et al., *Nature Physics*, (2018)

Latest Addition: NTV Torque

Generalized Perturbed Equilibrium Code (GPEC) Provides a Matrix Formulation for the NTV Torque Similar to MHD Stability

- **Including an anisotropic pressure tensor in DCON-like perturbed force balance produces self consistent 3D equilibria and torques**

$$\delta W = -\tfrac{1}{2}\int dx^3\, \boldsymbol{\xi} \cdot (\delta \mathbf{J} \times \mathbf{B} + \mathbf{J} \times \delta\mathbf{B} + \nabla \delta P + \nabla \cdot \boldsymbol{\Pi})$$

$$F = 0 = \delta \mathbf{J} \times \mathbf{B} + \mathbf{J} \times \delta\mathbf{B} + \nabla \delta P + \nabla \cdot \boldsymbol{\Pi}$$

Eigenvectors describe space of force free 3D states

Contains NTV physics, collisionality regimes, etc.

Park & Logan, *Phys. Plasmas* 24, 032505 (2017).

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- **Torque, like energy, is reduced to boundary term**

$$\delta W = \boldsymbol{\Phi}^\dagger \cdot \mathrm{Re}(\mathbf{W}) \cdot \boldsymbol{\Phi}$$

$$T = \boldsymbol{\Phi}^\dagger \cdot \mathrm{Im}(2n\mathbf{W}) \cdot \boldsymbol{\Phi}$$

- **We use plasma response physics to connect this to any external field**

$$T = \boldsymbol{\Phi}_{\mathbf{x}} \cdot \mathbf{T} \cdot \boldsymbol{\Phi}_{\mathbf{x}}$$

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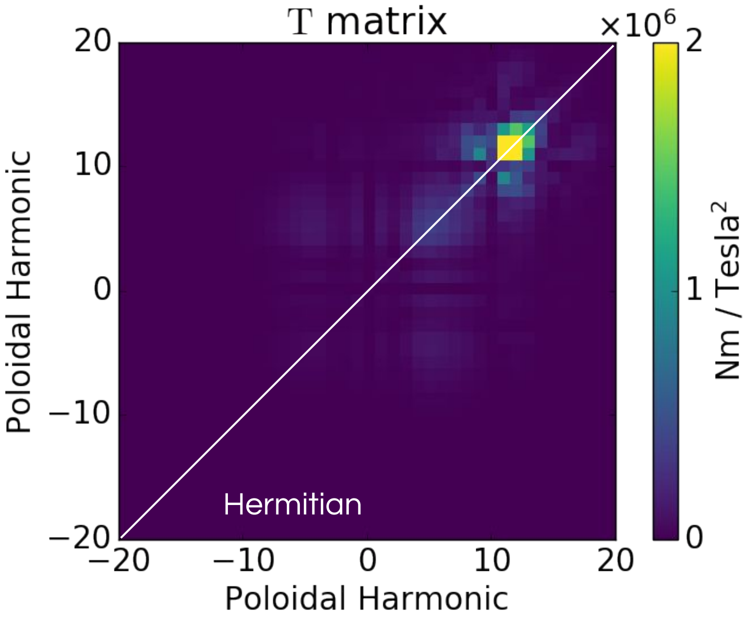
Contains both plasma response & kinetic physics

Park & Logan, *Physics of Plasmas* **24,** 032505 (2017).

Matrix Formulation Insights & Multi-modal Optimization



- **Matrix decomposition quickly provides "optimal" spectrum**
  - Ideal stability: $\delta W = \boldsymbol{\Phi}^\dagger \cdot W \cdot \boldsymbol{\Phi}$
    - 1st eigenmode = "least stable mode"
  - Ideal resonant coupling: $\boldsymbol{\Phi}_r = \boldsymbol{\mathcal{C}} \cdot \boldsymbol{\Phi}_x$
    - 1st SVD is EFC "dominant mode"
  - Kinetic torque: $\tau_{NTV} = \boldsymbol{\Phi}^\dagger \cdot \mathbf{T} \cdot \boldsymbol{\Phi}$
    - 1st eigenmode is **maximum NTV**
- **Eigendecompose:** $\mathbf{T} = \mathbf{U} \cdot \boldsymbol{\Lambda} \cdot \mathbf{U}^{-1}$
  - u_1 spectrum produces maximum λ_1 Nm/G^2
  - Multiple $\lambda_i \sim \lambda_1$ enables profile manipulation

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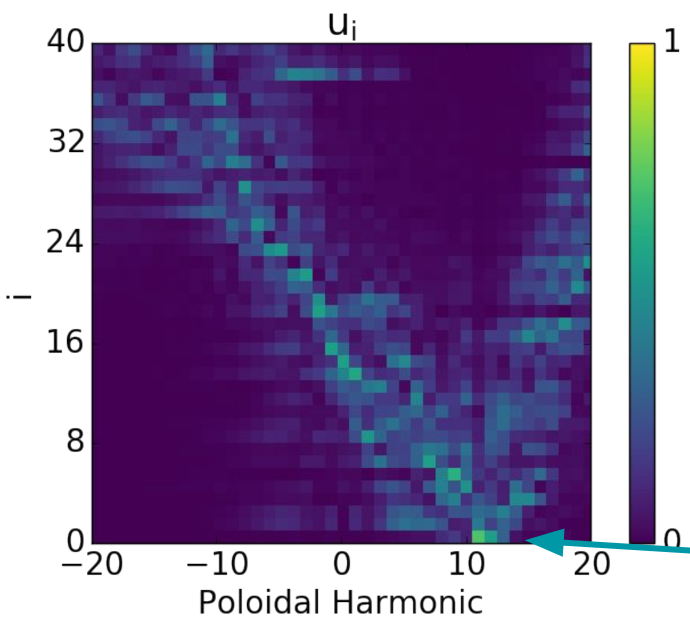

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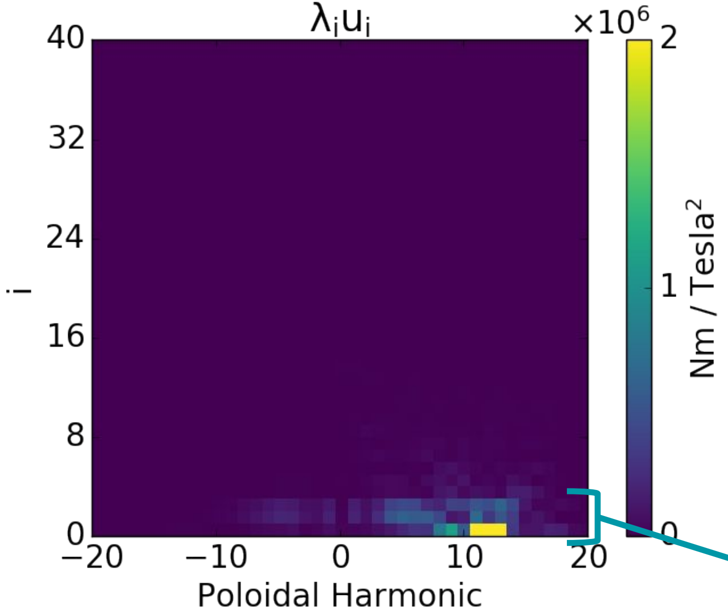

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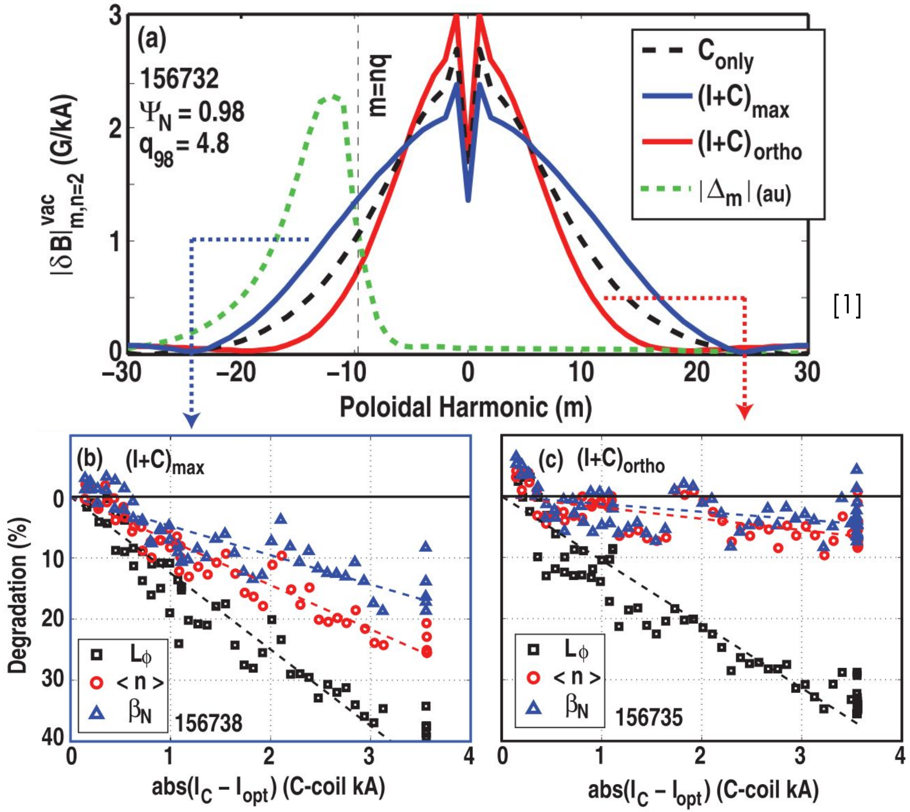
Torque Eigenmodes Provide Insight into Decoupled Transport



[1]

- **Coil combinations orthogonal to the "dominant" (resonant) mode cause little pumpout but still cause momentum transport**
- **Torque matrix reveals low m≲nq of coils couples well to 2nd and 3rd torque eigenmodes**
- **Would need to null out all 15>m>4 to avoid breaking**

[1] Paz-Soldan, C., et al., *Nuclear Fusion* 55, 083012 (2015).

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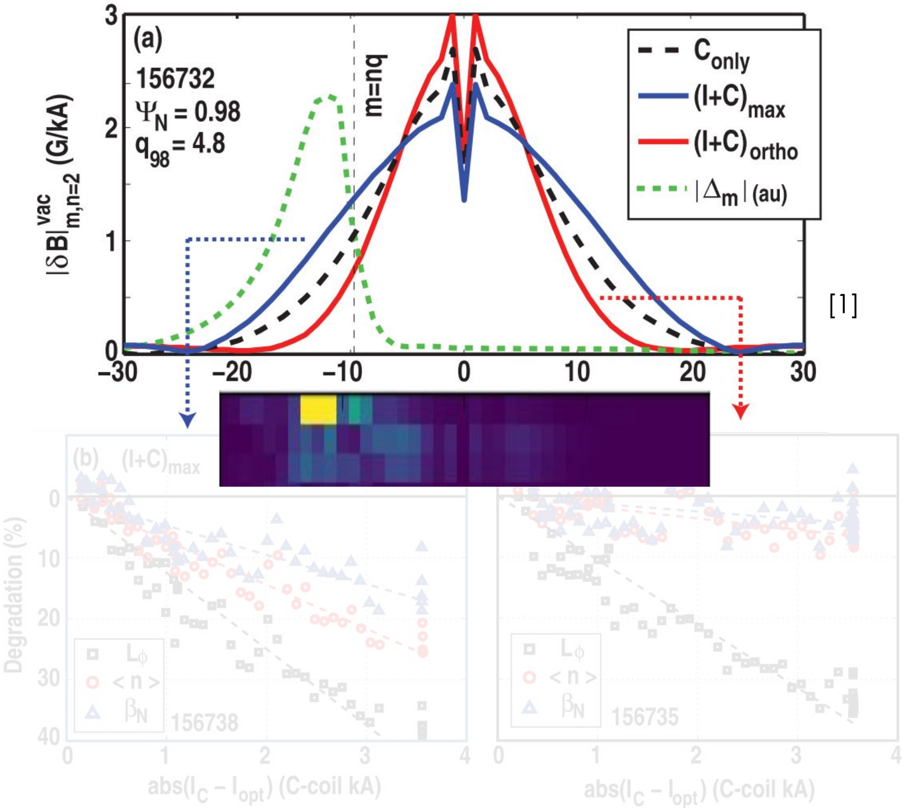

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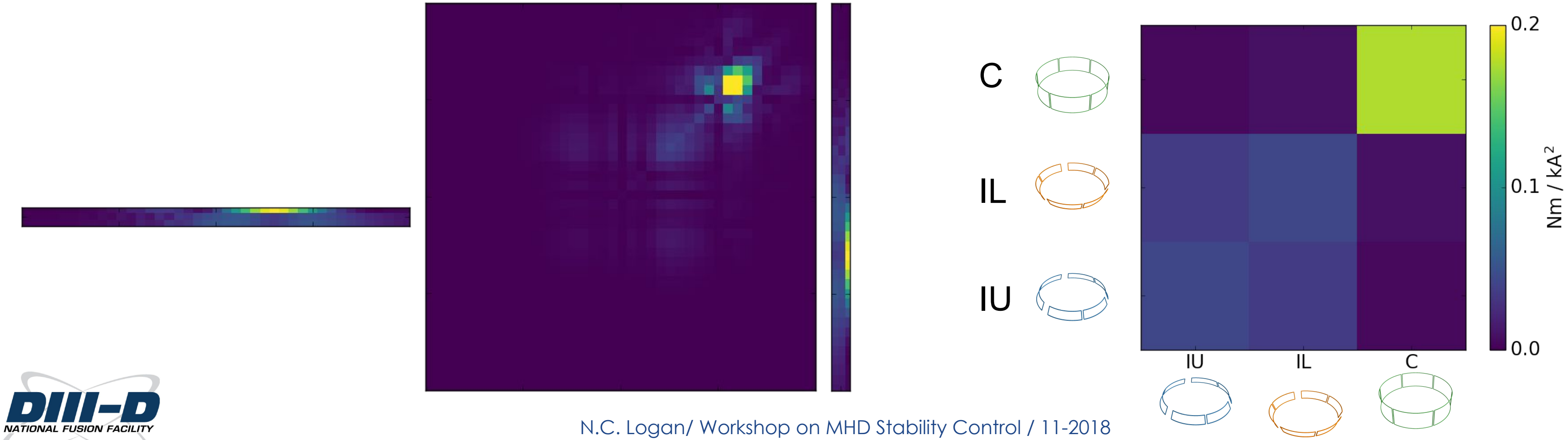
Now We Couple All This Powerful Formalism to the Experiment

- We know the spectra made by each coil array, $\Phi_{xc} = M \cdot I_c$
- For DIII-D, we get a 3x3 complex Coil NTV matrix
- Eigenvectors immediately identify optimal coil configurations

$$M^\dagger \cdot T \cdot M = T_{coil}$$



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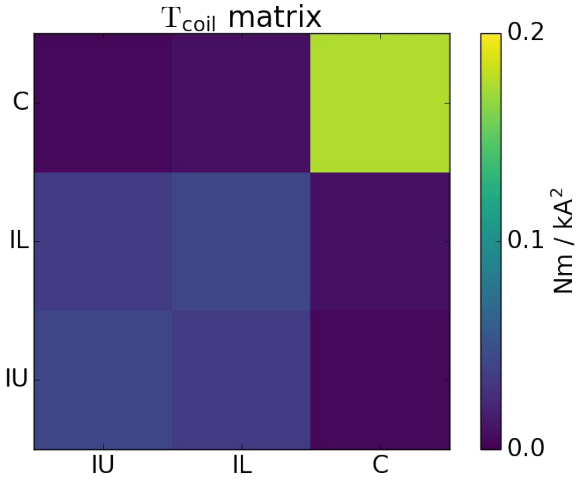

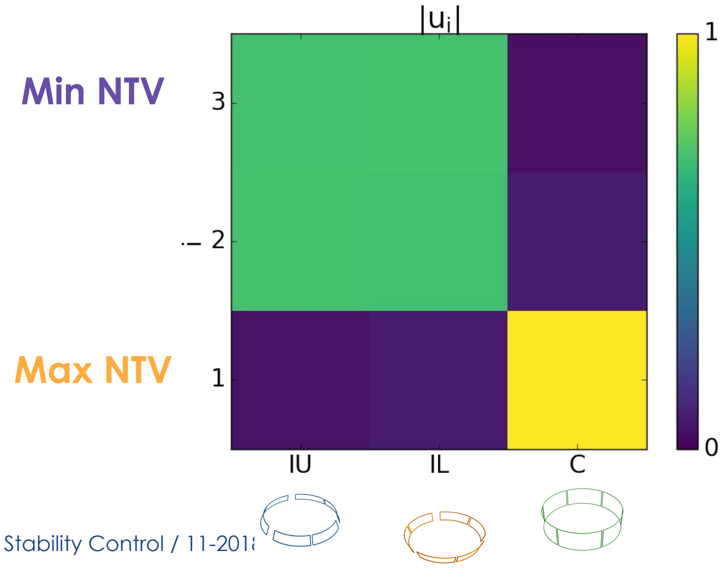

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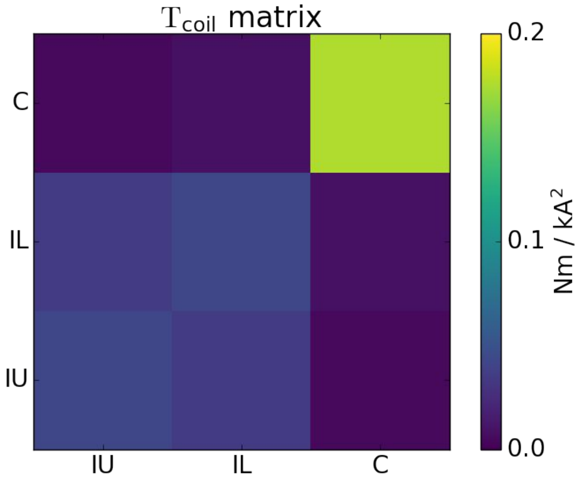

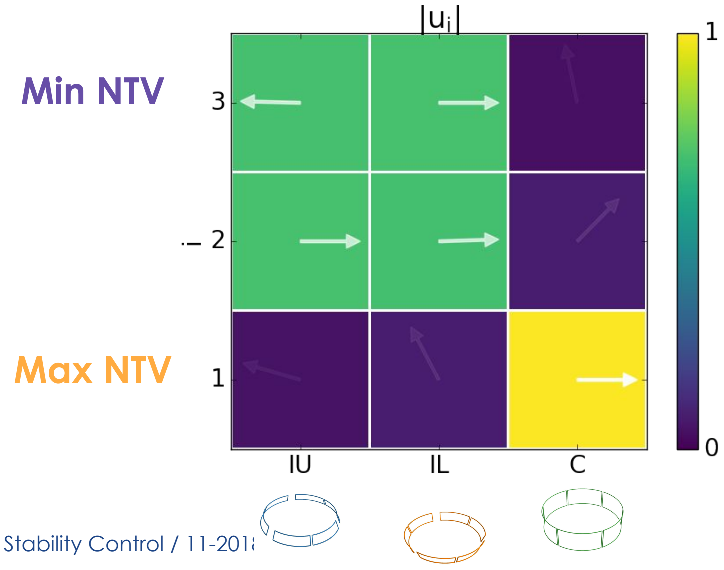

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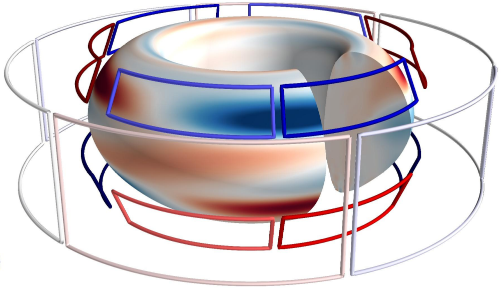
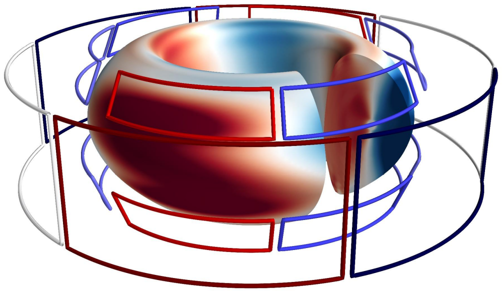
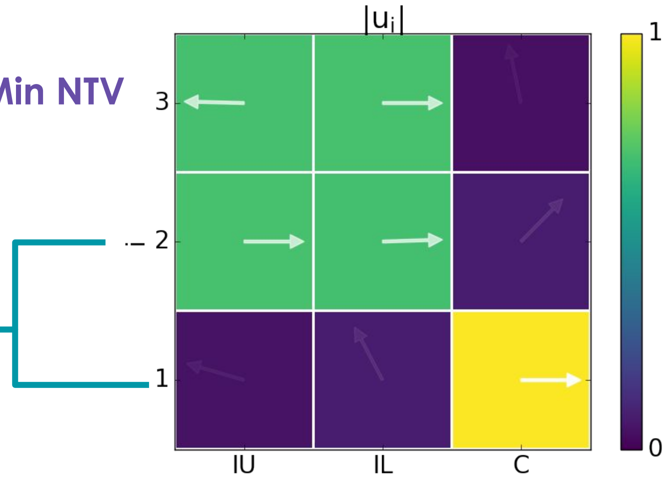
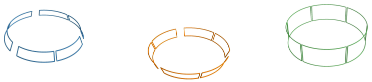


“$(I+C)_{ortho}$”

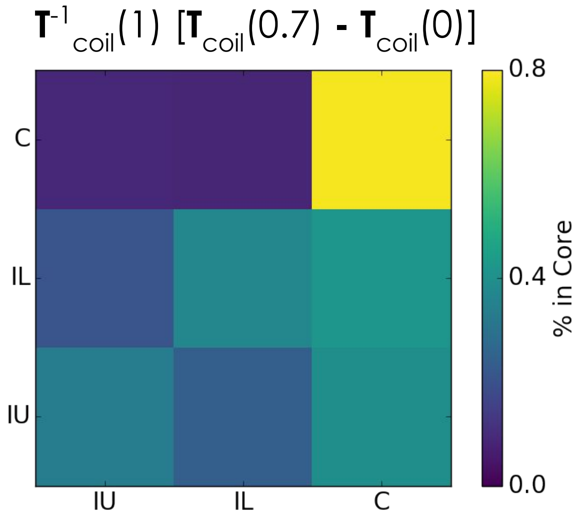






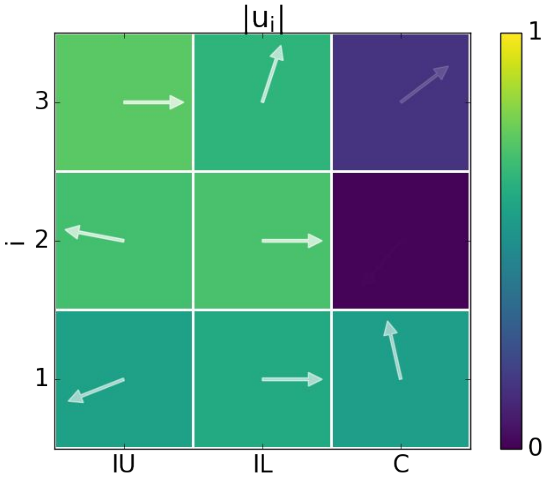
The Optimal Field Spectrum Changes if Localized Core or Edge NTV is Desired for Rotation Profile Control

- **There is a profile of these matrices, $T_{coil}(\psi)$, describing the torque within ψ**
- **The most localized NTV is simply u_1 of $T^{-1}_{coil}(\psi=1)\ [T_{coil}(\psi_1) - T_{coil}(\psi_2)]$**



Most edge concentrated

Most Core concentrated



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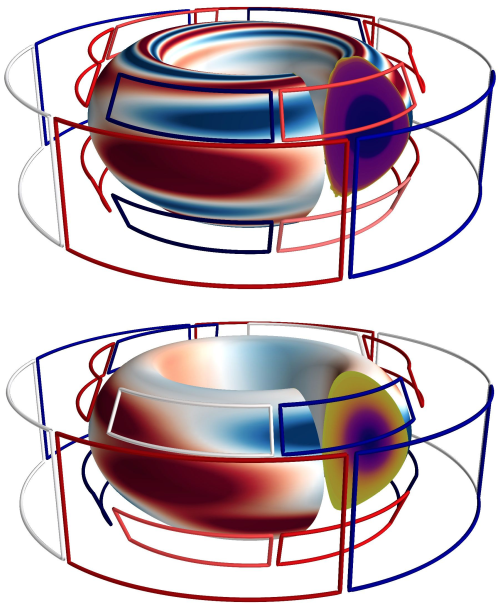
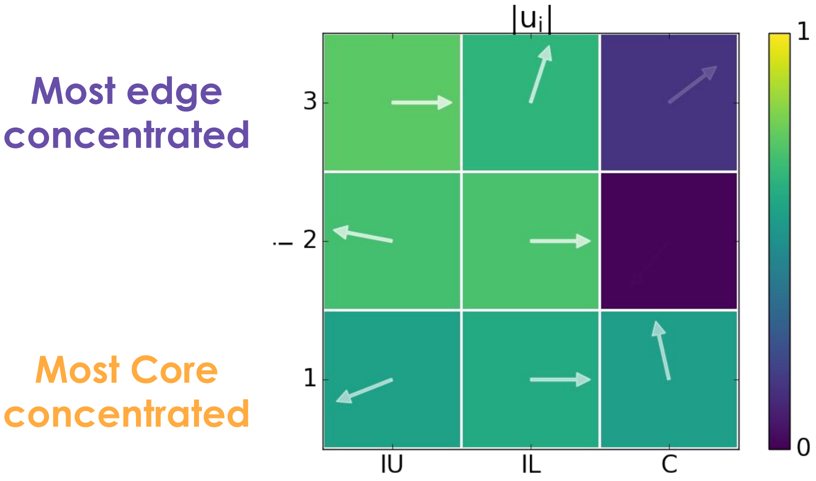
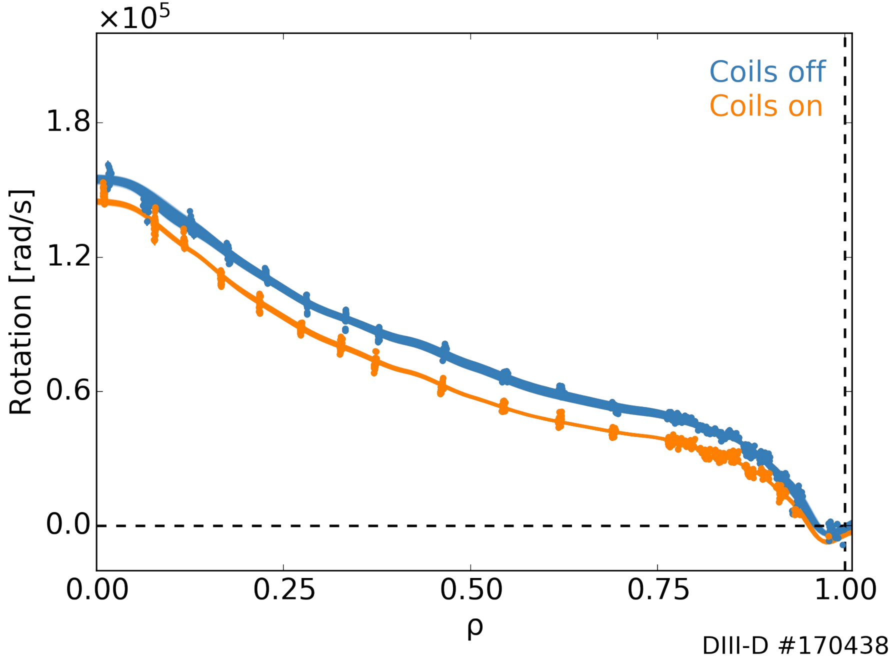


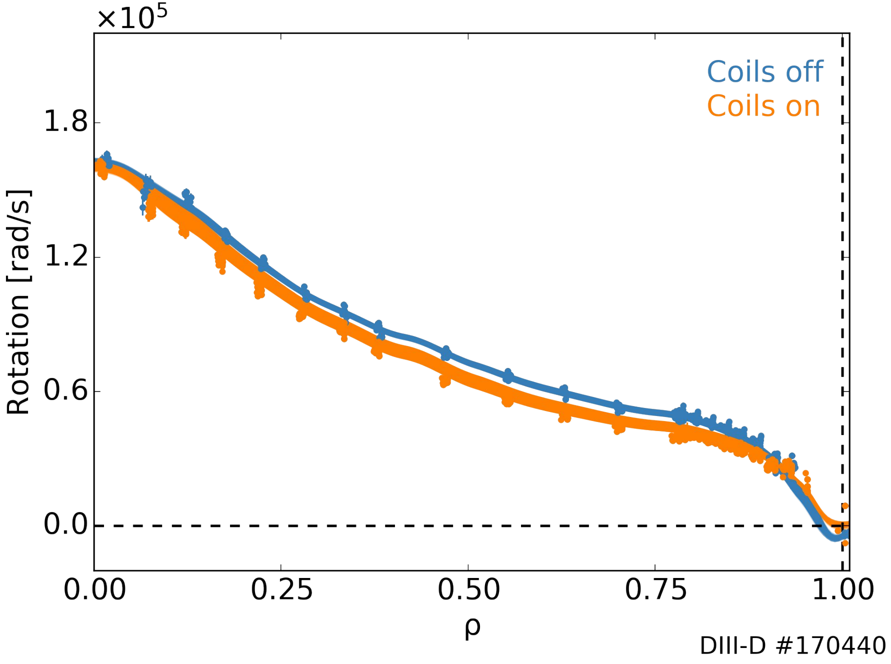


Both Core and Edge Resonant Fields Create Broad Rotation Damping, with Similar Final Results Across the Core Profile

Core

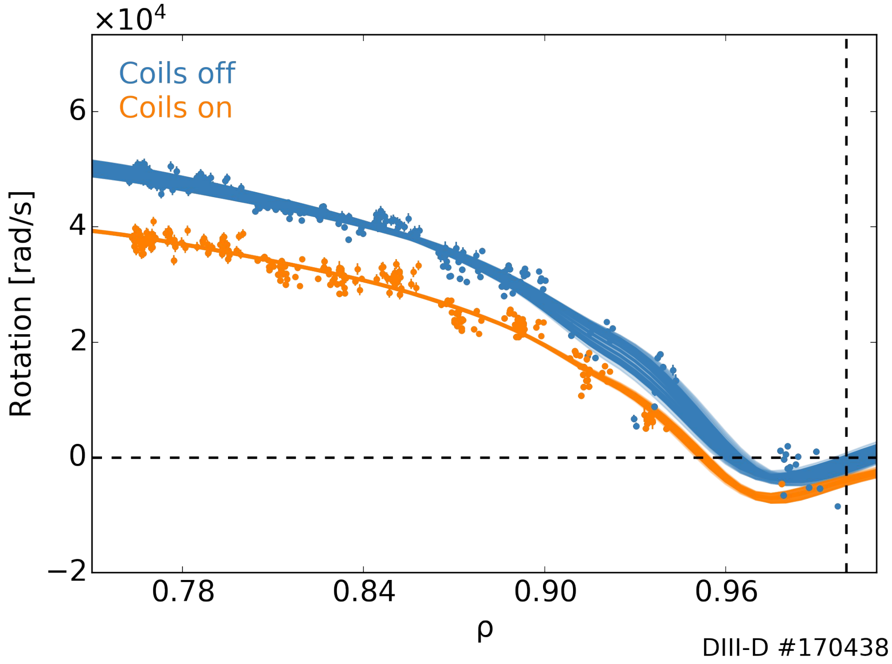


Edge

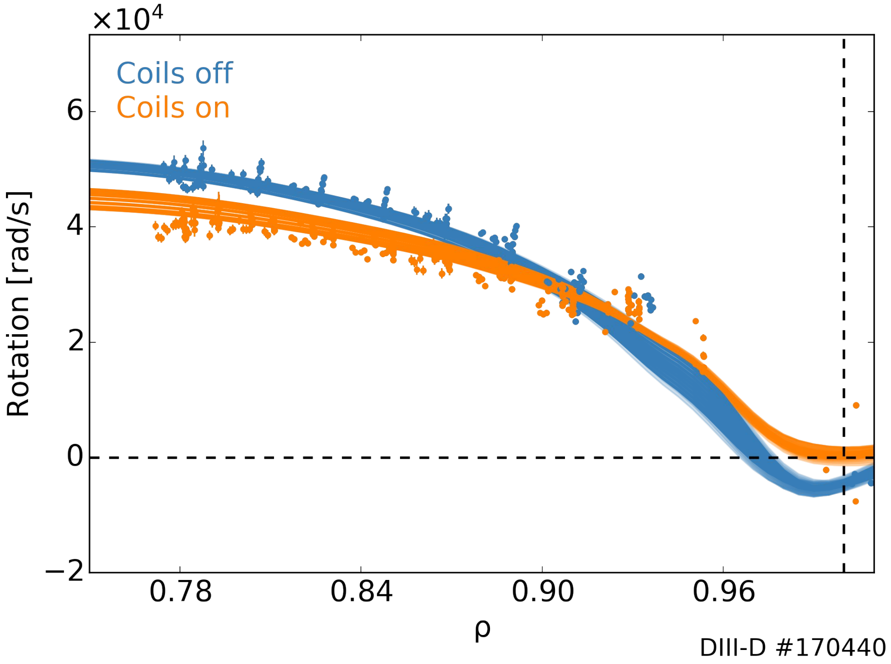


Edge Resonant Fields Changed the Local Edge Rotation Gradient, Broad Core Fields Do Not

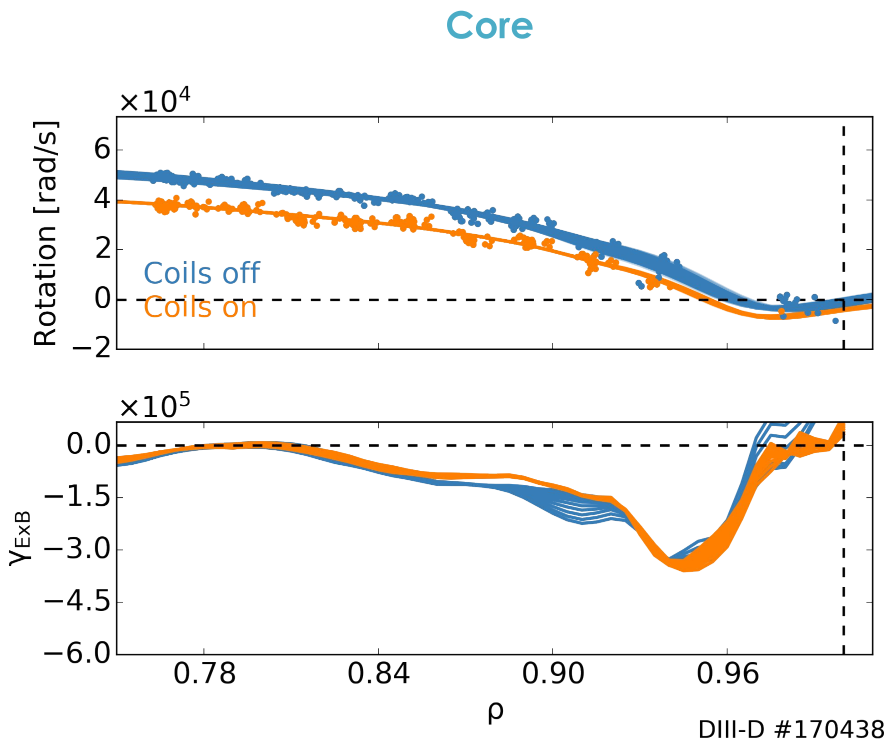
Core

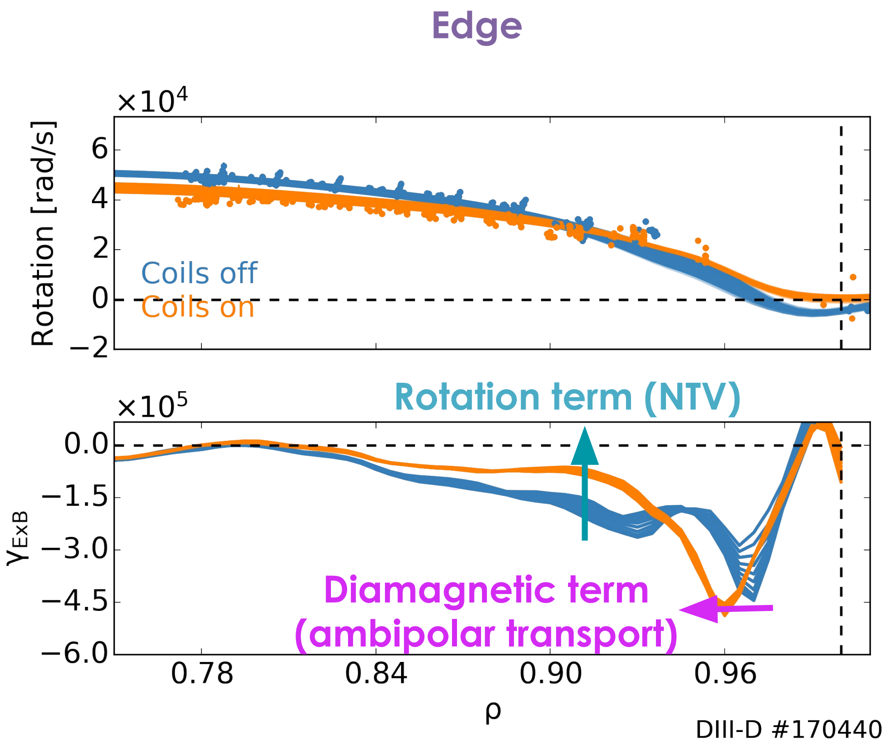


Edge



Edge Resonant Fields Changed the Local ExB shear even more - combines with ambipolar transport





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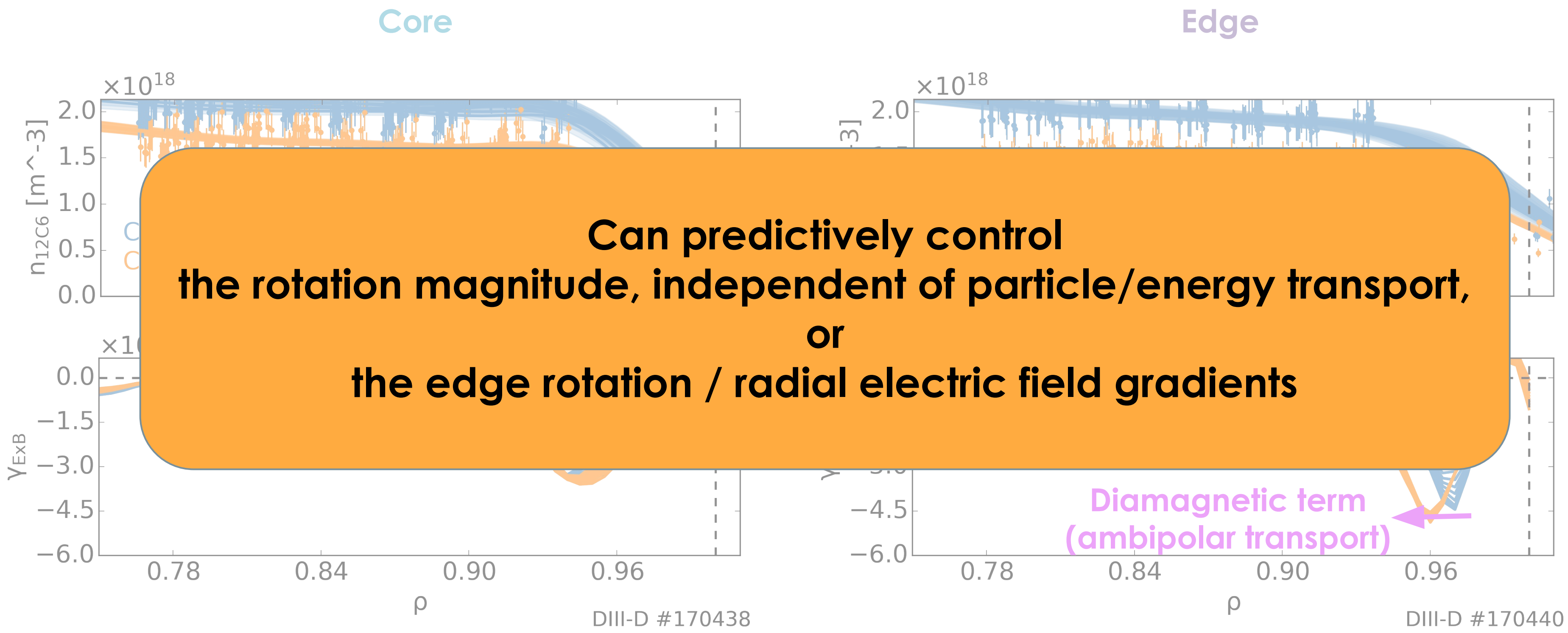
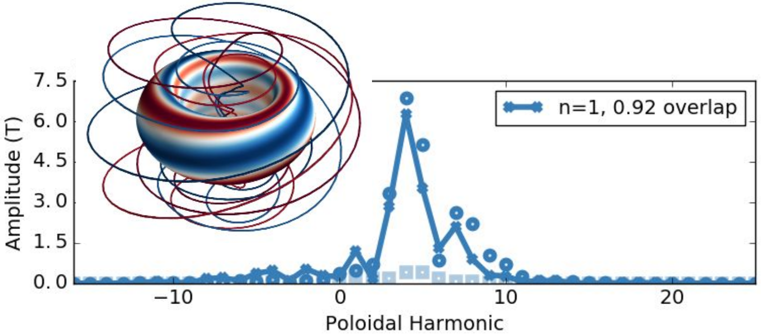


Can predictively control
the rotation magnitude, independent of particle/energy transport,
or
the edge rotation / radial electric field gradients

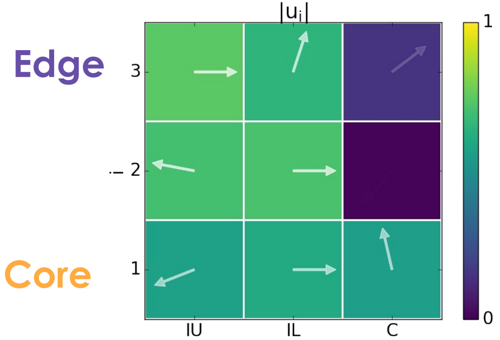
Takeaways and Talking Points

We have validated metrics for EFC, ELM control and rotation profile control

- **Matrix formalism enable us to calculate optimal spectra for each**
  - Useful in single dominant mode approximations
  - Provides targets for future coil designs

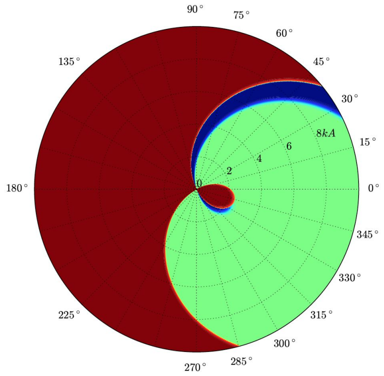


- **Can be related directly to existing coil sets**
  - Provides direct optimization of coil currents*
  - Brute force maps can be used to investigate phase space features

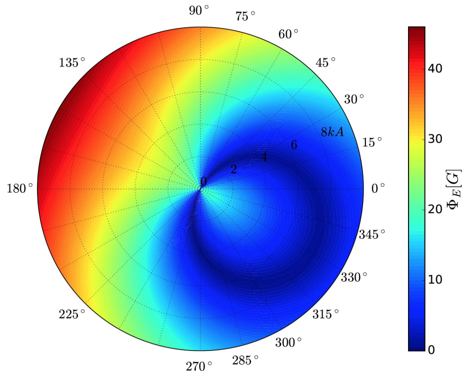


Next steps

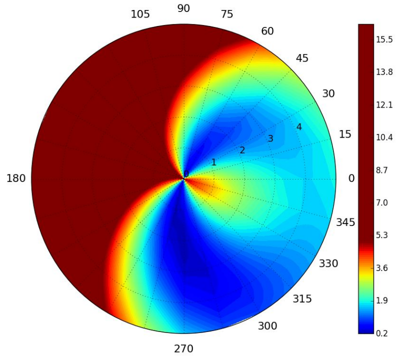
Thresholds



Continuous Edge RMP [G]



NTV [Nm]

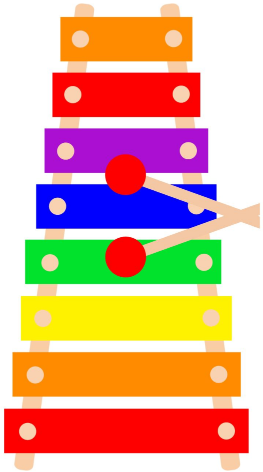


- **Projection of thresholds to ITER**
  - LM threshold database exists... can we do the same for ELMs?
- **Unaccounted for 3D field implications**
  - Particle Pumpout related to resonance in outer pedestal?
- **Multiple, continuous criteria**
  - How much edge RMP would you trade for every Nm of braking?

Backup

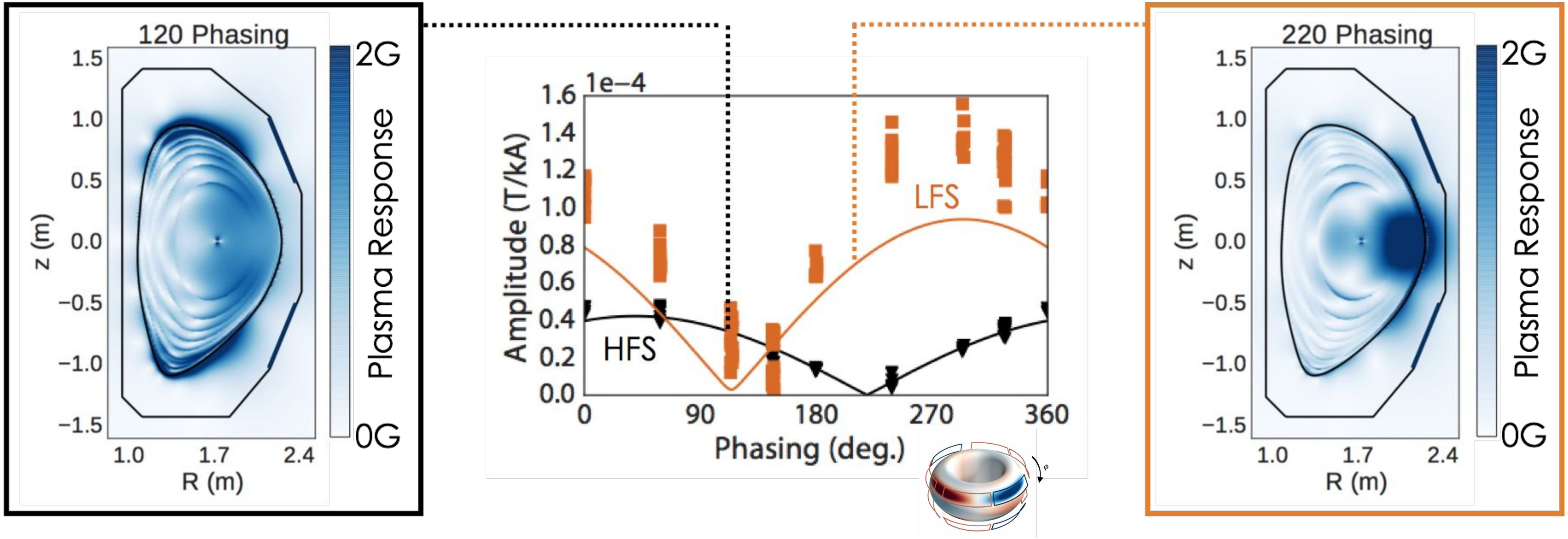
Let's Be Clear About "Single-mode" vs "Multi-modal"

- **Single-mode: A single, coherent radial and poloidal structure dominates 3D perturbations (tuning fork)**
  - Implication: EFC only needs to correct 1 poloidal spectrum for stability and transport
- **"Mode" does not refer to single m/n**
  - Everything here is single n (decoupled toroidally)
- **Multi-mode: The radial and poloidal structure of 3D perturbations depends on the driving field**
  - Implication: EFC must better match EF, separation of resonant and nonresonant effects, seperation of radial impact

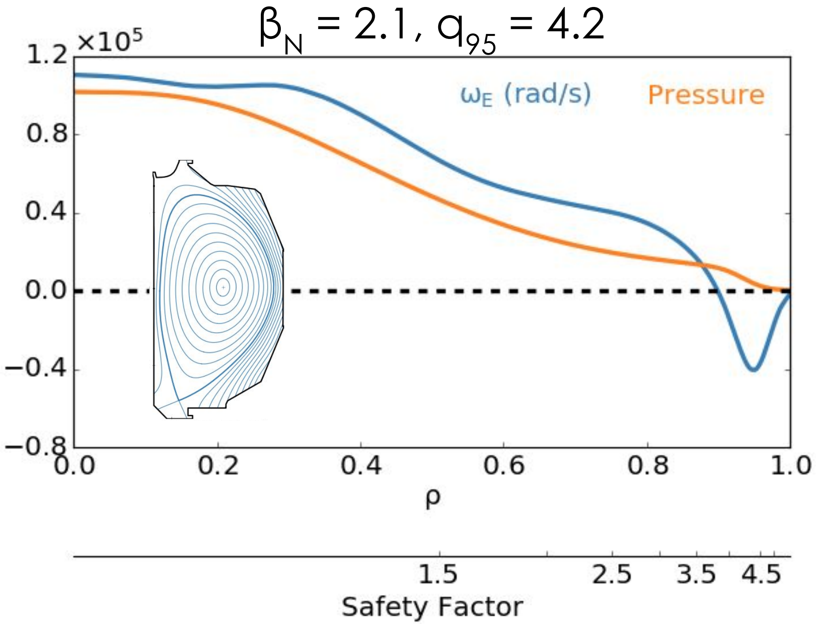


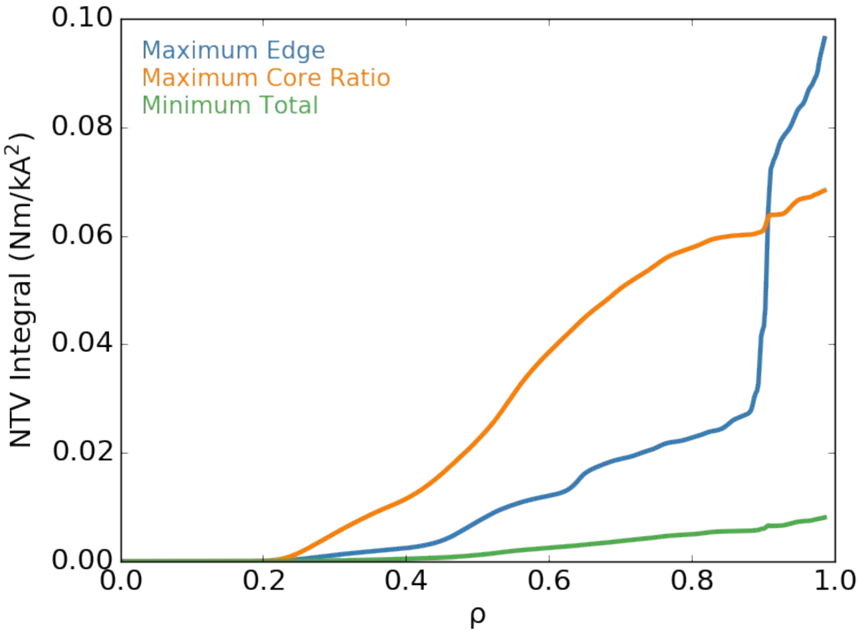
DIII-D NATIONAL FUSION FACILITY

The Multi-modal Plasma Response in DIII-D

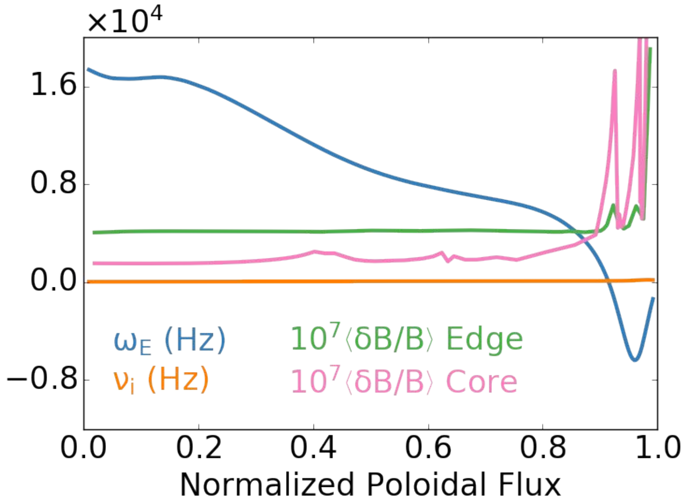


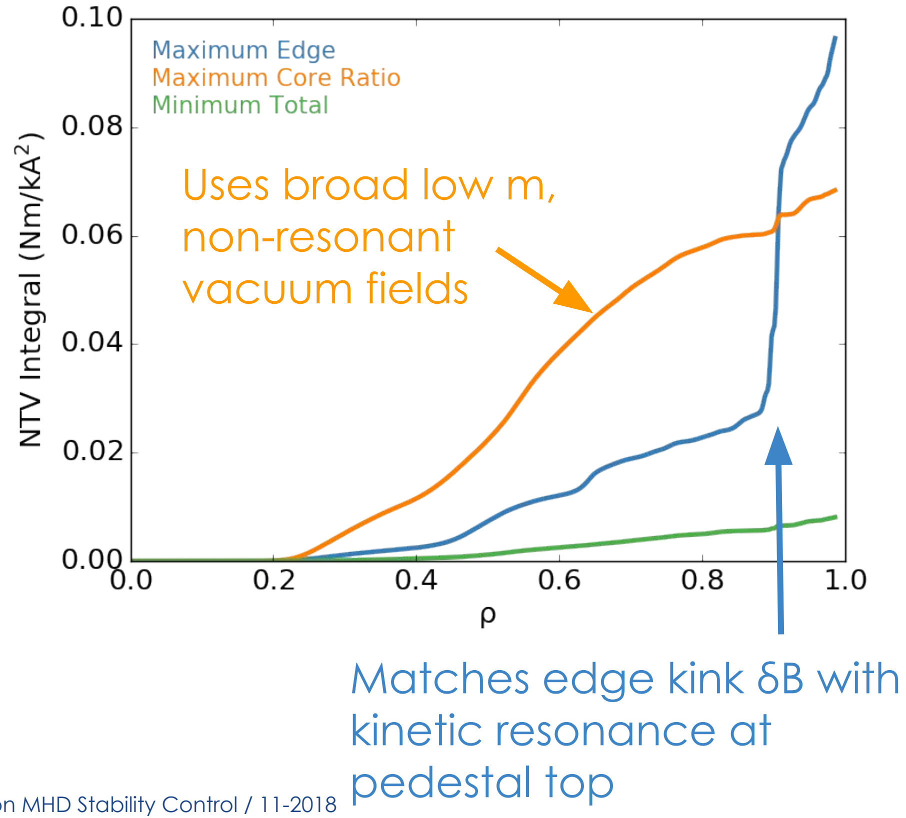
These Optimized Spectra for n=2 NTV in the DIII-D ITER Similar Shape Scenario Have Clear Distinctions



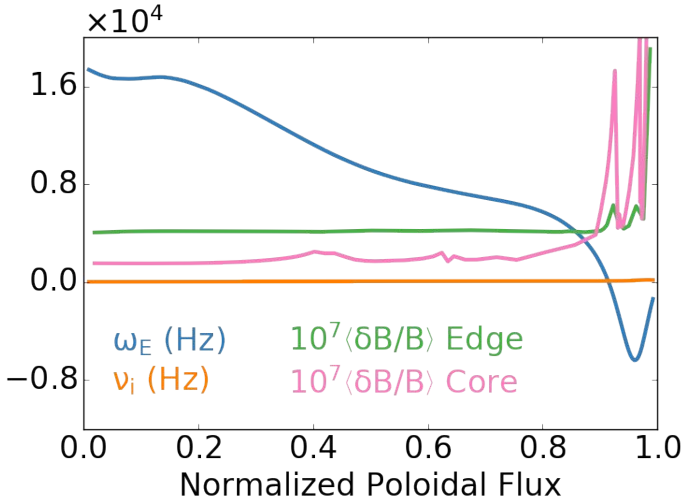


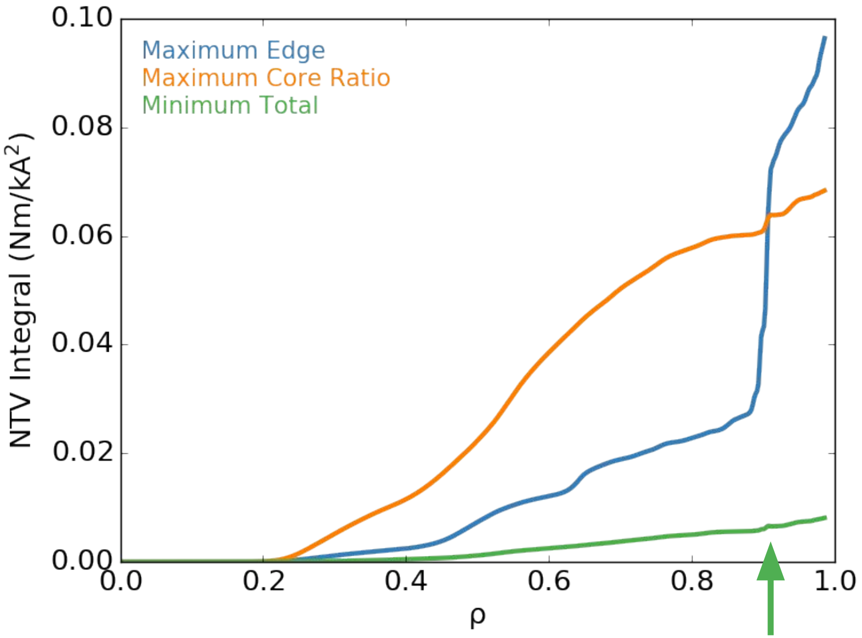
Optimized Spectra for n=2 NTV in the DIII-D ITER Similar Shape Scenario Look Promising





Optimized Spectra for n=2 NTV in the DIII-D ITER Similar Shape Scenario Look Promising

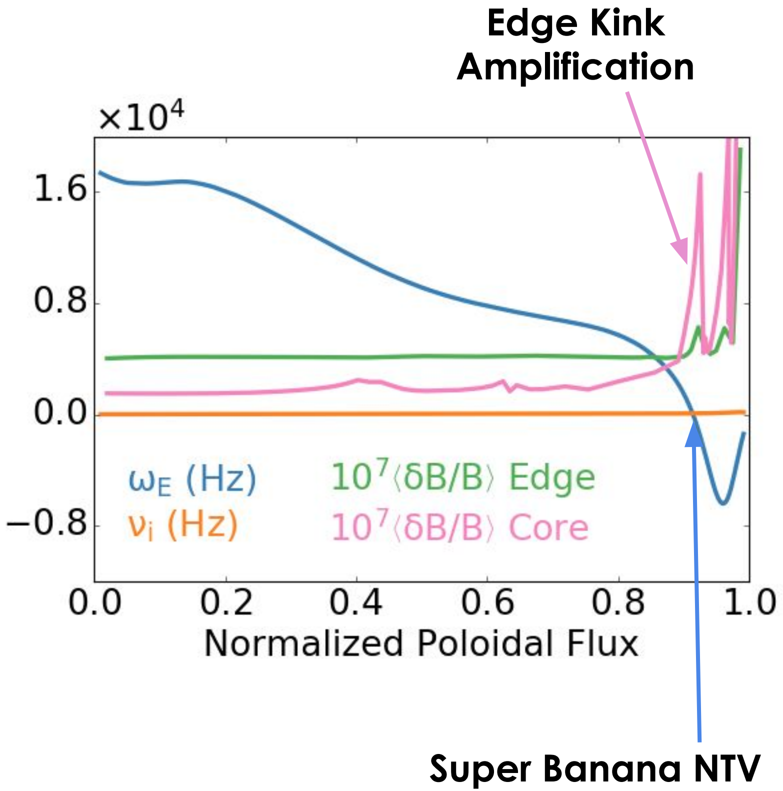




Large coil currents predicted to have no braking effect!

The 10's Saw Great Advancements In The Details Of Ntv Theory That Hint At "Resonant" Nature

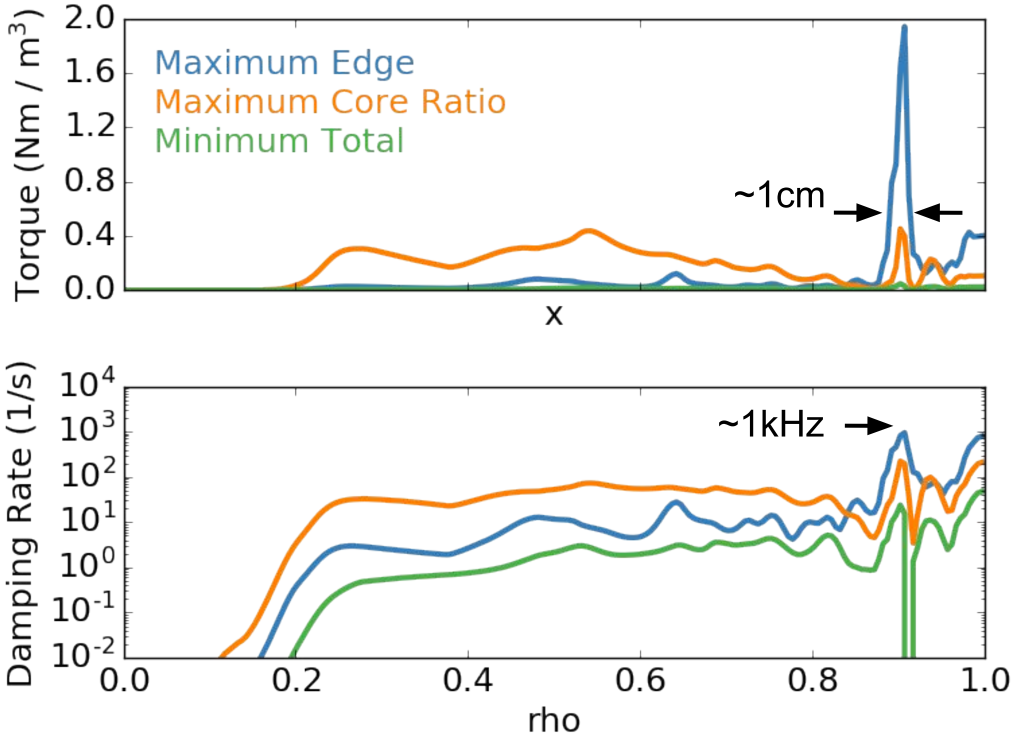
- **Many new regimes described by both reduced and numerical models [1]**
- **"Super banana" plateau → NTV largest when ω_E~0**
- **This means large spikes at E_r=0 near the top of H-mode pedestals**
  - Requires accurate kinetic equilibrium reconstruction!



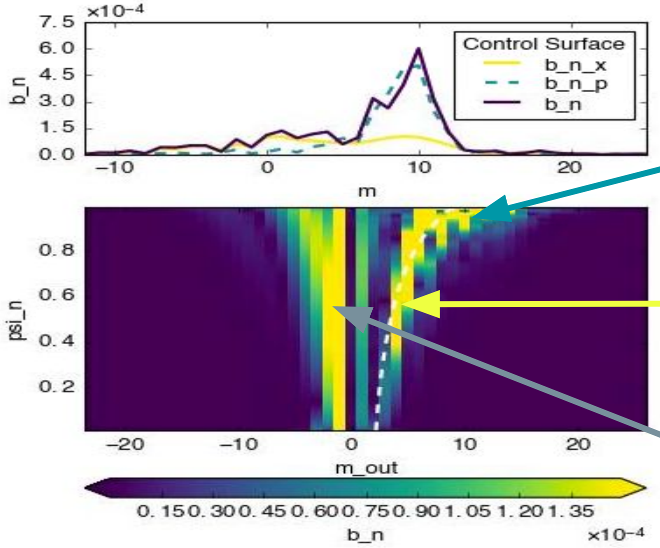
[1] Shaing, Ida, & Sabbagh, *Nucl. Fusion* 55, (2015).
[2] Callen, Nucl. Fusion 51, (2011).
[3] Wang, *et al.*, *Phys. Plasmas* 21, (2014).

Time Scale of Edge Evolution Challenges Diagnostic Capabilities for Direct Measurement of Localized Torque

- **Momentum confinement time τ_φ ~ 80ms**
- **Assuming T = ν u_φRmn, predicted local damping rate spikes to ~kHz**
- **Damping → measurement rate ≪ coil ramp rate**



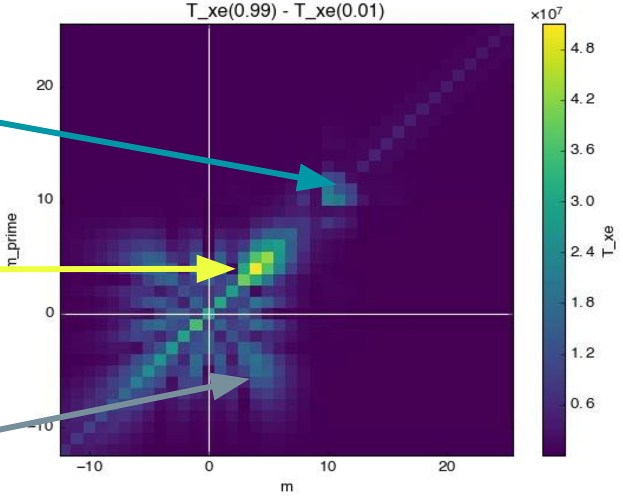
Torque Matrix Provides Immediate Insight into How Best to Induce Nonambipolar Transport in the Core or Edge



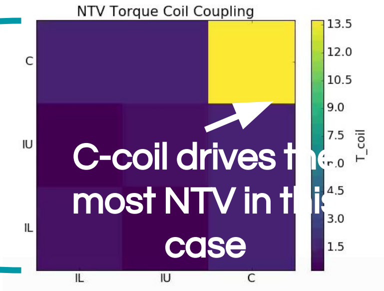
Edge:
m > nq Kink

Core:
q=4/2 response

Non-resonant:
vacuum drive



Simple Biot-Savart reduces MxM to IxI where I is the # of coils



C-coil drives the most NTV in this case

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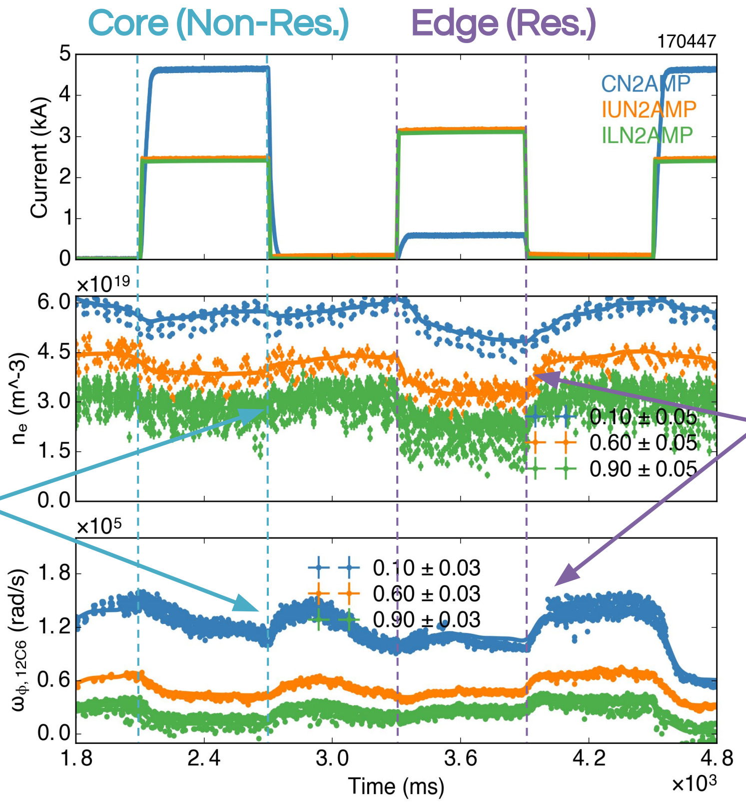
Exp. Revealed Obvious Spectral Dependencies: Pumpout vs Rotation



Fast rotation, slow density response

Fast rotation & density response