THE GATO CODE: HOW TO RUN IT AND FIGURE OUT WHAT THE RESULTS MEAN

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IMD Project Lecture Series

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GATO Consists Of Four Separate Sources That Run Sequentially And Link Through Binary Data Files

***SMAP*:** Mapping from equilibrium to optimally packed flux surface grid

***SVAC*:** Construct matrices A (Potential energy matrix) and B (Kinetic energy matrix)
Construct wall and vacuum

$$AX = \lambda BX$$

SEIG*:** Solve eigenvalue equation for eigenvalue λ and eigenmode ***X

$$\begin{cases} \lambda = \omega^2 = \delta W / \delta K < 0 \Rightarrow \text{unstable } (\omega = i\gamma) \\ \lambda = \omega^2 = \delta W / \delta K > 0 \Rightarrow \text{stable} \end{cases}$$

SPLT*:** Reconstruct physical eigenmode from Finite element node values in ***X
Plotting and diagnostics

- Sources are generally labeled as: $\text{s}xxx\text{h}yyy\text{.f}$ with $xxx = \begin{Bmatrix} map \\ vac \\ eig \\ plt \end{Bmatrix}$ and $yyy = \begin{Bmatrix} 100 \\ 200 \\ 300 \\ 400 \end{Bmatrix}$

where: $N_\psi = yyy \quad N_\chi = 2yyy$ are the precompiled mesh sizes

Input And Output Is Provided Mostly Through ASCII Files

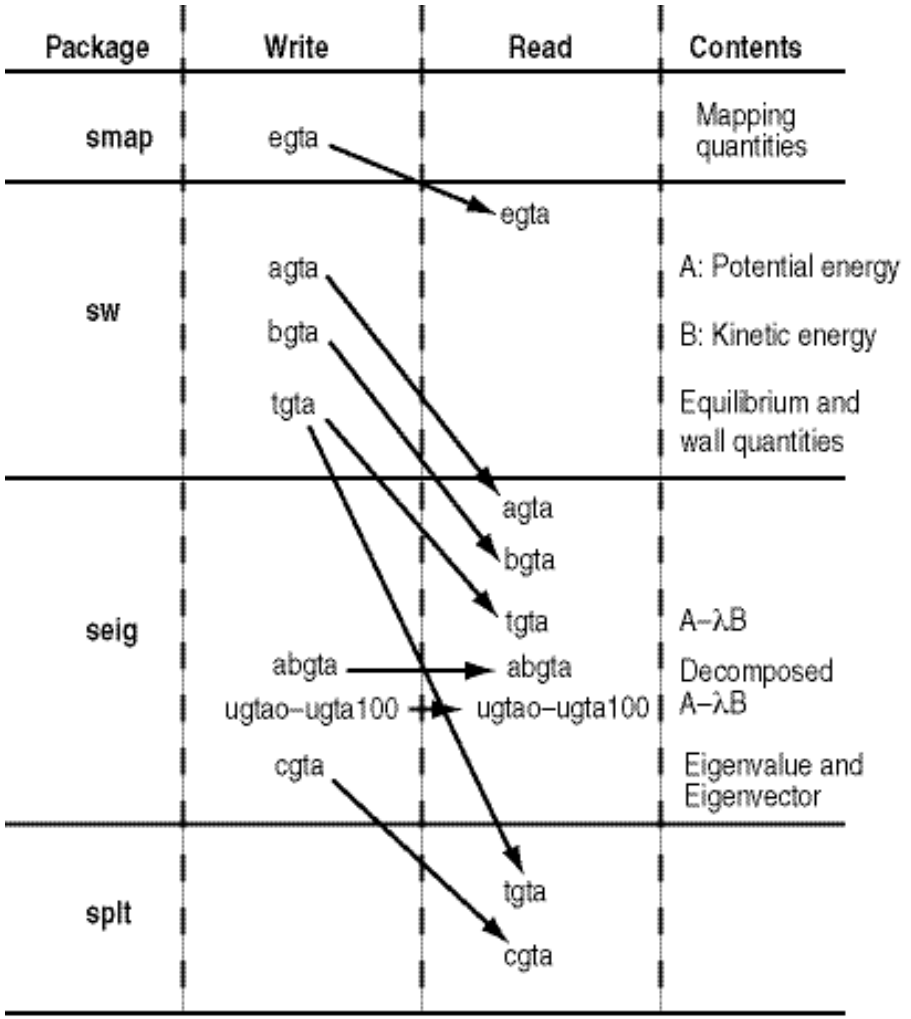
- **Input is from three files:**
  - '*eqgta*': Equilibrium (EFIT 'g file', TOQ 'dskgato file' or JSOLVER 'u-file'
  - '*ingta*': Namelist input
  - '*inwgta*': Namelist input for wall data
- **Namelist input '*ingta*' is read by all four sources:**
  - Equilibrium '*eqgta*' file is read by mapping SMAP only
  - Namelist input '*inwgta*' is read by SVAC only and only if a finite wall
- **Output is to ASCII and to cgm or PostScript files:**
  - ASCII files labeled '*okgta*' for *k = 1, 2, 3, 4* from the four sources
  - Graphics output from SMAP and SPLT labeled '*gato1.cgm*' and '*gato4.cgm*'
  - Graphics metafiles (cgm) can be converted to PostScript
- **Additional equilibrium and eigenmode output produced by SPLT:**
  - '*vacuum*': Input for Vacuum analysis (ASCII)
  - '*diagnostics*' Limited input for graphics analysis (ASCII)
  - '*nimrod*': Limited input for code benchmarks (Binary)
  - '*o4dump*': Complete input for graphics analysis (ASCII)

Four Options For Running GATO: With UNIX Scripts (Two Versions), By Hand, Or Through FusionGrid

- **Two Scripts differ mainly in file handling:**
  - *runGATO_job_label.batch (runs runGATO_platform.script)*
  - ~/gato/script/create_converge
  - First script will find input and sources from elsewhere as specified
    - First option runs in home directory with temporary files in /temp/
    - Second option runs in /temp/ and copies files over to home afterward
- **Run all four sources sequentially by hand:**
  - Set up a directory with required input files and run executables
- **Run on FusionGrid (with GUI interface):**
  - Requires only application for an account
  - Runs on GA star12 (a bit slow)
- **To use *runGATO_job_label.batch*:**
  - Edit file and set options using 'set option = value':
    - Directories for sources, input files, and place to run
    - Size label and the option to compile, load, and/or run as needed
    - Input file names to be used
    - Script file name for *runGATO_platform.script for the platform*
  - Exit and run the script

Link Between Sources Is Through Binary Data Files

- **Sources are linked by a set of binary files labeled '*xgta*' where:**
  - x = *a,b,e,c,t,u*
- **Scripts typically rename the ASCII and cgm output files:**
  - Generally as '*outpt*.xxxx'



Maximum Dimensions Set in Sources By Parameter Statements

- **Four sources labeled as:**
  - smaphxxx.f swnwhxxx.f seighxxx.f and splthxxx.f
  - xxx = the maximum mesh size: say 100 or 200 or 300
  - Generally the maximum number of flux surfaces used in the stability mesh
  - Number of poloidal angles is set at twice this by default
- **Maximum stability mesh size set in the source by:**
  - '*parameter (npx = xxx, ncx = 2*npx)*'
  - To change this one needs to edit all four sources with a global change:
    - "*npx = xxx*" → "*npx = yyy*"
- **Equilibria can be either:**
  - Direct: $\psi(r,z)$ in EFIT 'g-file' format
  - Inverse: $r(\psi,\chi)$, $z(\psi,\chi)$ from TOQ or JSOLVER
- **Maximum input equilibrium mesh size set in the source by:**
  - '*parameter (nxx = xxx)*' (Direct equilibria: Number of radial and vertical mesh points)
  - '*parameter (npp = xxx, ntt=2*npp-1)*' (Inverse equilibria: Number of equilibrium flux surfaces and angles)
  - To change this one needs to edit only smaphxxx.f with a global change:
    - "*nxx = xxx*" → "*nxx = yyy*"
    - "*npp = xxx*" → "*npp = yyy*"

Some Restrictions Apply To The Allowed Dimensions And Meshes

- **For Direct equilibria:**
  - Equilibrium mesh must be uniform in r and z
  - *nxx* must equal both the number of radial and vertical points
  - *nxx* must exactly equal these numbers - not just exceed them
- **For Inverse equilibria:**
  - *npp* and *ntt* need only exceed the input
  - Flux mesh can be arbitrarily and nonuniformly spaced
  - But the poloidal angle must be a uniformly spaced equal arc
- **Additional source requirements:**
  - The sources swnwhxxx.e and seighxxx.e require additional I/O routines for fast word addressable binary files 'agta' and 'bgta' (and 'abgta' internal file)
  - These are provided presently as a group of c routines in a file:
    - *'dmroutines_cio.c'*
  - These must be compiled in C and loaded with the executables
- **Additional graphics requirements:**
  - The sources smaphxxx.e and splthxxx.e require additional graphics libraries:
    - TV80: Provided as a source with GATO
    - NCARGKS: Free graphics library
  - These must also be loaded with the executables
  - An alternative interface with PGPLOT library exists
    - Requires compilation of '*pgplotdriver.f*' and loading along with PGPLOT

Run By Hand Requires Running Four Sources

- **Compile and load the four sources:**
  ⇒ Executables: smaphxxx.e swnwhxxx.e seighxxx.e and splthxxx.e
- **Set up a directory to run in**
- **Copy over the three required input files:**
  - Name as ‘eqgta’ ‘ingta’ and ‘inwgta’
- **Run executable smaphxxx.e:**
  - Produces output ‘o1gta’ ‘gato1.cgm’
  - Produces file ‘egta’ for swnwhxxx.e
- **Run executable swnwhxxx.e:**
  - Produces output ‘o2gta’
  - Produces files ‘agta’ and ‘bgta’ for seighxxx.e
  - Produces file ‘tgta’ for splthxxx.e
- **Run executable seighxxx.e:**
  - Produces output ‘o3gta’
  - Produces file ‘cgta’ for splthxxx.e
- **Run executable splthxxx.e:**
  - Produces output ‘o4gta’ ‘gato4.cgm’

Namelist File 'ingta' Controls The Run

- **Namelist parameters control:**
  - Physics parameters
  - Equilibrium mapping
  - Mesh sizes and packing
  - Tolerances
  - Wall and vacuum options
  - Eigenvalue search and solution
  - Eigenvector diagnostic output
  - General diagnostic output
- **Roughly a half dozen parameters specify the physics case:**
  - *Key parameters are ntor, ncase, qxin, idnsty*
- **Generally these and a few other parameters need be changed from their default:**
  - A few mesh and packing options
  - Wall and vacuum options
  - A few eigenvector diagnostic output options
- **Occasionally some mapping parameters need to be changed if the mapping fails in a specific case:**
  - Generally very few
  - Mapping is often self correcting when it detects a problem

Key Physical Parameters Can Be Reset in Namelist Input As Needed

- **Namelist parameters are reset in 'ingta' file as:**
  variable = value
  - Default value remains if not set

Type	Variable	Default	Definition (* *Option is not yet implemented)*
Physical Case	*ntor*	1	Toroidal mode number
	ncase	0	Set = 0 for full compressible; = 1 for incompressible
	norm	0	Set = 0 for full KE norm; > 0 for normal displacement only
	nlt	0	Set = 0 for ideal wall boundary conditions; =1 for line tying
	nmod	0	Set = 1 to force $\xi(q<1)=0$; Set = 2 for floating boundary condition
	nlim	0	Set limiter boundary condition at *nlim'th* poloidal angle
	idnsty	0	Read density profile if = 2 or set density profile if ≤ 0
	ndnxp0	0	Set density profile for *idnsty* < *0* as $\psi^{ndnxp0}(1-\psi^{ndnxp1})^{ndnxp2}$
	ndnxp1	2	Set density profile for *idnsty* < *0* as $\psi^{ndnxp0}(1-\psi^{ndnxp1})^{ndnxp2}$
	ndnxp2	2	Set density profile for *idnsty* < *0* as $\psi^{ndnxp0}(1-\psi^{ndnxp1})^{ndnxp2}$
	bfieldf	1.0	Set overall normalization for magnetic field
	qxin	0.0	Rescale q_0 to *qxin* and B_ϕ if non zero. Otherwise use input q_0
	btdes	0.0	Rescale B_ϕ to *btdes* and q_0 if non zero. Otherwise use input q_0
	qsurf	2.0	Calculate q_0 value required to obtain $q_{lim} = qsurf$
	gamma	5/3	Adiabatic factor C_p/C_v
	rmantl		Fraction of edge plasma treated as cold mantle in calculation of beta

Input Namelist Parameters Specify The Equilibrium Type And Stability Mesh

- **Equilibrium type (direct or inverse) set from *nmap*:**
  - Remaining equilibrium parameters are less important
- **Numerical correction to restabilize continuum set from *ncorr***
- **Grid size and type set from *jpsi, itht, isym, and igrid***

Type	Variable	Default	Definition (* *Option is not yet implemented)*
Equilibrium Type	*nmap*	0	Set = 0 for direct equilibrium; set > 0 for TOQ and < 0 for JSOLVER
	neqtyp	1	Set = 0 for old style TOQ input; set = 1 for TOQ input with heading
	ndoublt	0	Set = 0 for Dee coding options; set = 1 if doublet equilibrium
	ndivert	1	Set = 0 for limiter edge; set = 1 for diverted edge
Numerical correction	*ncorr*	0	Set = 0 for standard FHE; set ≠ 0 for numerical restabilization
	corrfac	1.0	Factor for numerical restabilization corrections
Grid	*jpsi*	*npx*	Number of flux surfaces
	itht	*ncx*	Number of poloidal angles
	isym	0	Set = 0 for up-down asymmetry; set = 1 for symmetric
	igrid	0	Set = 0 for equalarc poloidal angle; = 1 for PEST angle
	nham1	0	Alternative output angle χ_H; $J(\psi,\chi_H,\phi)=r^{nham1}B_p^{nham2}B^{nham3}$
	nham2	0	Alternative output angle χ_H; $J(\psi,\chi_H,\phi)=r^{nham1}B_p^{nham2}B^{nham3}$
	nham3	+2	Alternative output angle χ_H; $J(\psi,\chi_H,\phi)=r^{nham1}B_p^{nham2}B^{nham3}$

GATO Has Great Flexibility In Specifying Options For Grid Packing Of Stability Mesh

- **Key parameters are *nmesh*, *npak*, and *nedge*:**
  - *sedg0* and *sedg1* control edge packing
  - *plpak* and *pspak* control special packing at specific q or ψ

Type	Variable	Default	Definition (* *Option is not yet implemented)*
	nmesh	1	Set = 1 for repacking mesh; set = 0 for no packing; set < 0 to read mesh
	npak	0	Number of q values for additional packing; set < 0 to pack in nq
	mpak	0	Number of ψ values for additional packing; set ≠ 0 to pack in ψ^{cspak}
	nedge	+4	Set > 0 to force edge packing; = 0 for no edge packing; < 0 to include search for rational surfaces near edge
	npkmax	*npx*	Maximum number of rational surfaces packed
	nrat	*1 +npx/3*	Number of flux surfaces reserved for packing
Mesh	*nrepeat*	0	Set > 0 (< 0) to pack only (skip) the *nrepeat*'th occurrence of q in *plpak*
Packing	*nppack*	1	Set > 0 (< 0) to eliminate packing in negative (positive) shear
	nqpack	0	Set ≠ 0 to set weights evenly distributed in $s^{nqpack}q$
	psipak	2.0	Set initial ψ distribution evenly spaced in $\tilde{\psi}^{psipak}$
	cspak	0.5	Set distribution for underlying ψ mesh distribution to $\tilde{\psi}^{cspak}$
	pkfrac	2/3	Fraction of flux surfaces reserved for packing ($\times pkfrac$)
	qpfrac	1/3	Fraction of flux surfaces reserved for distributing in q
	sedg0	0.0	Inverse width of packing weight at edge
	sedg1	0.0	Amplitude of packing weight at edge
	plpak(k,l)	0.0	Additional q packing: k=(q value, width, weight); (l=1,$npak$)
	pspak(k,l)	0.0	Additional ψ packing: k=(ψ value, width, weight); (l=1,*mpak*)

Options For Wall And Vacuum Can Be Used To Specify A Variety Of Wall Types

- **Options are:**
  - Wall at infinity or on plasma or intermediate
  - Conformal or self similar wall construction
  - Input wall defined by:
    - (r,z) coordinates or
    - Fourier harmonics for r(θ) and z(θ)
- **Any given option can be expanded by a factor *rext***

Type	Variable	Default	Definition (* *Option is not yet implemented)*
Wall and Vacuum Parameters	*iwal*	0	Set = 0 to construct wall; = 1 to read (r_{wall}, z_{wall}); = 2 to read coefficients
	iwalsym	0	Option to read in symmetric (= 0) or asymmetric (≠ 0) wall
	irext	0	Wall option to set center of wall; set = 0 for conformal wall
	norign	0	Define origin of coordinates used to interpolate final wall points
	nwall	60	Number of points to construct (*iwal=0*) or read (*iwal=1,2*) wall
	nekdefn	0	Use elliptic integral expansion (< 0), iterative scheme (> 0); default (= 0)
	maxitek	10	Maximum number of iterations for iterative elliptic integral method
	rext	1.0	Expand default wall by factor *rext*; set = 1.0 for input wall from *iwal*
	rexmax	$+10^{+3}$	Maximum wall expansion for equivalent infinite vacuum
	rcutoff	$+10^{-3}$	Minimum major radius for inboard toroidal wall

Eigenvalue Solver Robustly Converges To Any Desired Eigenvalue - Both Stable And Unstable

- **Parameter *nev* sets the desired eigenvalue**
- **Solver homes in on desired eigenvalue through a series of searches:**
  - Bracket desired eigenvalue (*nbrmax* iterations)
  - Isolate desired eigenvalue (*nismax* iterations)
  - Converge closer to desired eigenvalue (*ncymax* iterations)
  - Inverse iterations to find solution(*nitmax* iterations)

Type	Variable	Default	Definition (* *Option is not yet implemented)*
Eigenvalue Solver	*nev*	1	Compute *nev*'th eigenvalue
	neigmax	100	Maximum number of eigenvalues for any guess. Stop if exceeded.
	nforce	0	Set = ±1 to force convergence to one degenerate pair eigenvalue
	nreslv	0	Set = 0 ignore degenerate eigenvalues; = ±1 to resolve
	nbrmax	10	Maximum number of bracket iterations for eigenvalue search
	nismax	10	Maximum number of isolation iterations for eigenvalue search
	ncymax	2	Maximum number of Cholesky iterations for eigenvalue search
	nitmax	20	Maximum number of inverse iterations for eigenvalue search
	ncyfin	1	Set = 1 for Cholesky decomposition with final eigenvalue
	mxdcomp	20	Maximum total Cholesky decompositions; stop if exceeded.
	al0	-10^{-4}	Initial eigenvalue guess
	dal0	10.0	Scale factor for incrementing *al0* in bracket search
	al0bas	+0.0	Offset for scaling bracket search
	al0min	−1.0	Minimum allowed eigenvalue. Stop if exceeded.
	al0max	-10^{-9}	Maximum allowed eigenvalue. Stop if exceeded.
	epschy	$+10^{-5}$	Convergence criterion for Cholesky iterations
	epscon	$+10^{-5}$	Convergence criterion for inverse iterations

Large Range Of Options Control Eigenvector Output In All Major Representations

Type	Variable	Default	Definition (* *Option is not yet implemented)*
Plot Parameters	*lineplt*	0	Plot line plots versus flux surface (> 0), poloidal ray (< 0), or both (= 0)
	lampplt	0	Plot line plots of quantity (> 0), amplitude and phase (< 0), or both (= 0)
	njplot	0	Plot line plot of ξ versus θ of *njplot*'th surface or boundary (= 0)
	niplot	0	Plot line plot of ξ versus ψ of *niplot*'th poloidal angle or midplane (= 0)
	nskpi	+1	Skip every *nskpi*'th angle in displacement vector plot
	nskpj	+1	Skip every *nskpj*'th surface in displacement vector plot
	njedge	+1	Include or exclude plasma edge in plot normalizations
	ntphase	-4	Set toroidal phase option
	npowr	-2	Set transformation option for plot of logarithmically divergent quantities
	ncont	10	Number of contours in contour plots
	ncplot	10	Number of contours in perturbed flux surfaces
	mshpsi	12	Specify radial coordinate in Fourier and line plots
	mshchi	3	Specify poloidal angle in Fourier analysis and line plots
	nxisgn	+1	Reset sign of eigenvector (= ±1)
	nxiplt	+1	Set = +1 plot $\underline{\xi}\cdot\nabla\Psi/\lvert\nabla\Psi\rvert$; +2 add $\underline{\xi}\cdot\nabla\chi/\lvert\nabla\chi\rvert$; +3 add $\underline{\xi}\cdot\nabla\phi/\lvert\nabla\phi\rvert$
	nxuplt	+1	Set +1 to plot $\underline{X}=\underline{\xi}\cdot\nabla\psi$; +2 to add U; +3 to add $\underline{Y}=\underline{\xi}\cdot\nabla\phi$
	nxrplt	0	Set = 1,2,3 to plot radial, axial, toroidal ξ components
	nxpplt	0	Set = 1,2,3 to plot normal, perpendicular, and parallel ξ components
	nxdplt	0	Set = +1 plot $\partial X/\partial\psi$; +2 add $\partial X/\partial\theta$; +3 add $\partial U/\partial\theta$; +4 add $\partial Y/\partial\theta$
	ncphip	0	Set = +1 to plot perturbed electric potential contours
	nbiplt	0	Set +1 plot $\underline{\delta B}\cdot\nabla\psi/\lvert\nabla\psi\rvert$; +2 add $\underline{\delta B}\cdot\nabla\chi/\lvert\nabla\chi\rvert$; +3 add $\underline{\delta B}\cdot\nabla\phi/\lvert\nabla\phi\rvert$
	nbuplt	0	Set +1 to plot $\underline{\delta B}\cdot\nabla\psi$; +2 to add $\underline{\delta B}\cdot\nabla\chi$; +3 to add $\underline{\delta B}\cdot\nabla\phi$
	nbrplt	0	Set = 1,2,3 to plot radial, axial, toroidal δB components
	nbpplt	0	Set = 1,2,3 to plot normal, perpendicular, and parallel δB components
	naiplt	0	Set +1 plot $\underline{\delta A}\cdot\nabla\psi/\lvert\nabla\psi\rvert$; +2 add $\underline{\delta A}\cdot\nabla\chi/\lvert\nabla\chi\rvert$; +3 add $\underline{\delta A}\cdot\nabla\phi/\lvert\nabla\phi\rvert$
	nauplt	0	Set +1 to plot $\underline{\delta A}\cdot\nabla\psi$; +2 to add $\underline{\delta A}\cdot\nabla\chi$; +3 to add $\underline{\delta A}\cdot\nabla\phi$
	narplt	0	Set = 1,2,3 to plot radial, axial, toroidal δA components
	napplt	0	Set = 1,2,3 to plot normal, perpendicular, and parallel δA components
	nvfft	0	Specify number of Fourier harmonics as 2^{nvfft}; set = 0 for maximum
	torphase	0.0	Add *torphase* to default toroidal phase of computed eigenvector

Line plot control

Toroidal phase

Plotting mesh

Eigenvector representations

δB representations

δA representations

GATO Diagnostic Output Available In All Major Representations And Coordinates

Plotting Mesh Options:

Radial coordinates: $\rho_{midplane}$, ψ_{pol} , Ψ_{tor} , V $\rho^2_{midplane}$, $\sqrt{\psi_{pol}}$, $\sqrt{\Psi_{tor}}$, $\sqrt{V}$

Poloidal coordinates: $\theta_{midpoint}$, $\theta_{\substack{magnetic \\ axis}}$, $\chi_{arclength}$, χ_{PEST}

Mode diagnostic representations:

Normal orthogonal	$\xi_\psi = \xi\cdot\nabla\psi \big/ \lvert\nabla\psi\rvert^2$ $\xi_p = r\xi\cdot(\nabla\psi\times\nabla\phi) \big/ \lvert\nabla\psi\rvert$ $\xi_t = r\xi\cdot\nabla\phi$
GATO (X, U, Y)	$X = \xi\cdot\nabla\psi$ $U = \xi_p \big/ B_p - Y + J\beta_\chi X$ $Y = r \big/ B_\phi\, \xi\cdot\nabla\phi$
Cylindrical	$\xi_r = \xi\cdot\nabla r$ $\xi_z = \xi\cdot\nabla z$ $\xi_\phi = r\xi\cdot\nabla\phi$
Field line	$\xi_n = \xi\cdot\nabla\psi \big/ \lvert\nabla\psi\rvert^2$ $\xi_\perp = \xi\cdot(\nabla\psi\times B) \big/ (\lvert B\rvert\lvert\nabla\psi\rvert)$ $\xi_B = \xi\cdot B \big/ \lvert B\rvert$

Options Control Quantity Of Eigenvector And Other Diagnostic Output

Type	Variable	Default	Definition (* *Option is not yet implemented)*
Plot Control	*iomshp*	0	Force on (+2) or off (–2) or leave (0) mesh plots
	ioeqlp	0	Force on (+2) or off (–2) or leave (0) equilibrium plots
	iowalp	0	Force on (+2) or off (-2) or leave (0) wall and plasma surface plots
	ioeigp	0	Force on (+2) or off (-2) or leave (0) ξ vector plots
	iodbvp	0	Force on (+2) or off (-2) or leave (0) δB vector plots
	iodavp	0	Force on (+2) or off (-2) or leave (0) δA vector plots
	iopsip	0	Force on (+2) or off (-2) or leave (0) perturbed surface plots
	iolinp	0	Force on (+2) or off (-2) or leave (0) ξ line plots
	iolnbp	0	Force on (+2) or off (-2) or leave (0) δB line plots
	iolnap	0	Force on (+2) or off (-2) or leave (0) δA line plots
	iofftp	0	Force on (+2) or off (–2) or leave (0) ξ fourier analysis plots
	ioffbp	0	Force on (+2) or off (–2) or leave (0) δB fourier analysis plots
	ioffap	0	Force on (+2) or off (–2) or leave (0) δA fourier analysis plots
	ioconp	0	Force on (+2) or off (-2) or leave (0) ξ contour plots
	iodlbp	0	Force on (+2) or off (-2) or leave (0) δB contour plots
	iodlap	0	Force on (+2) or off (-2) or leave (0) δA contour plots
	iodlbw	0	Force on (+2) or off (-2) or leave (0) δW contour plots
Diagnostic Output Control	*iplotm*	9	Specify default number of plots from ***smap.f***; set = 9 for all plots
	ioutm	0	Specify debug output from ***smap.f***; set = 0 for minimum output
	ioutw	0	Specify debug output from ***swnw.f***; set = 0 for minimum output
	iouta	0	Specify matrix pattern output from ***swnw.f***; set = 0 for minimum output
	ioute	0	Specify debug output from **seig.f**; set = 0 for minimum output
	ioutp	17	Specify output from ***splt.f***; set = 17 for all plots

Interpretation Of Any MHD Stability Code Results Is Always Complicated

- **One or more of following techniques should be used to avoid misinterpretation of results:**
  - Especially critical when numerical correction is not applied
  - None is a guarantee

In order of increasing reliability:

- **For given finite mesh choose small negative λ^* as marginal stability criterion:**
  - For $N\psi$ x $N\chi$ = 100 x 200: $\lambda^* \sim -10^{-4}$
    - Eigenvalues: $\lambda < \lambda^* \Rightarrow$ **unstable** $\lambda > \lambda^* \Rightarrow$ **stable**
  - Cutoff λ^* chosen by observing structure of most unstable mode:
    - Strongly localized: $\Rightarrow$ Numerically destabilized continuum mode
    - Global extended: $\Rightarrow$ Physically unstable mode
    - Mixture of localized plus small extended part: $\Rightarrow$ Ambiguous
- **Continuously vary physical stabilizing parameter:**

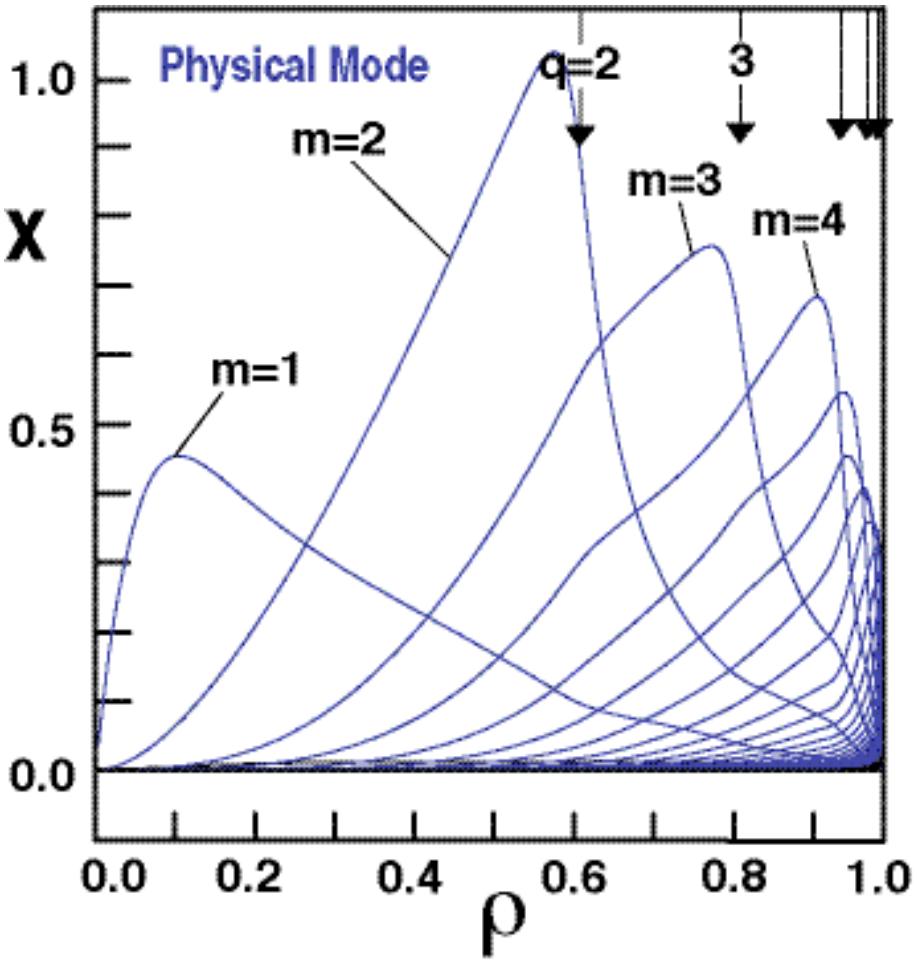
  $\Rightarrow$ Eigenvalue generally approaches zero fairly rapidly with marginal stability

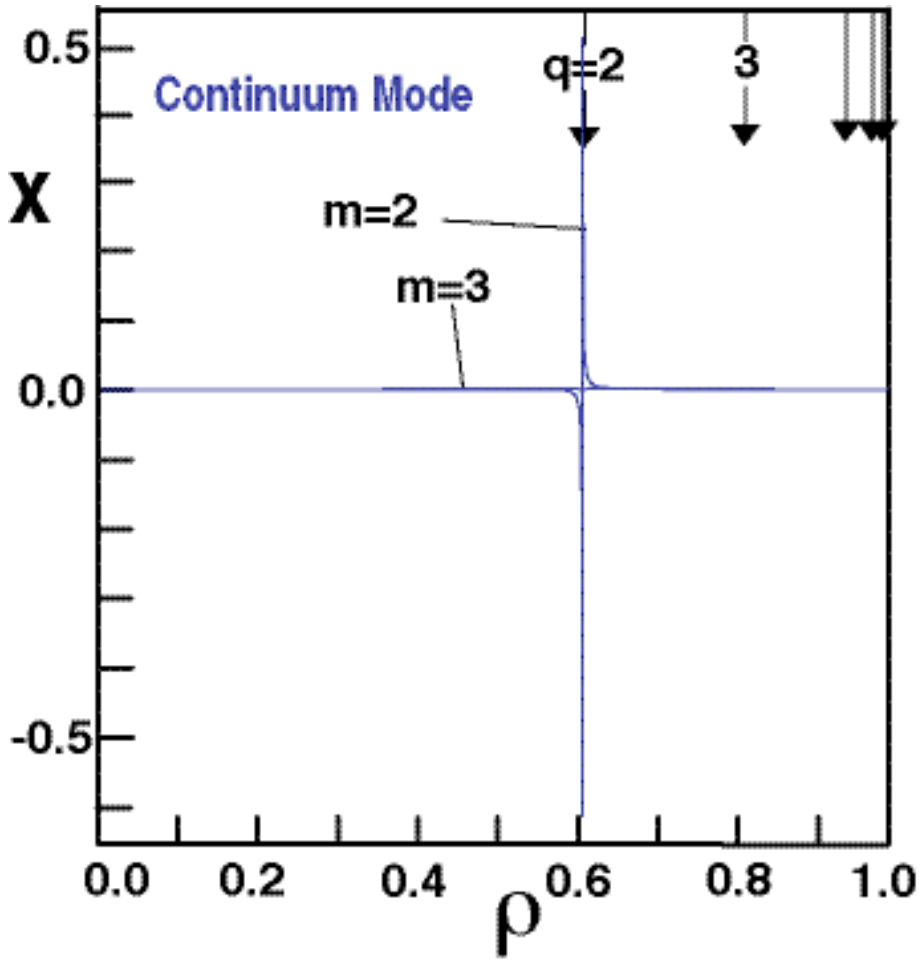
  $\Rightarrow$ Points on either side are usually clearly stable or unstable
    - Mode structure changes from global to strongly localized continuum-like
    - A reasonable marginal point can be identified
- **Perform a partial or full convergence study:**
  - *Vary mesh keeping fixed $N\psi/N\chi$ and ploteigenvalue λ against $(N\psi \times N\chi)^{-1}$*
  - *Extrapolate to $(N\psi \times N\chi)^{-1} = 0$*
    - *If extrapolation is negative the mode is physically unstable*
    - *Otherwise λ should extrapolate to zero*

      $\Rightarrow$Eigenfunction becomes increasingly singular

Physically Unstable And Continuum Modes Distinguished By Structure

- Equilibrium with single physically unstable mode plus stable continua





With Numerical Restabilization Option Continua Now Converge To Marginal Stability From Stable Side

- **All modes converge to same point as without correction:**
  - Convergence remains quadratic