Energetic Particle Stability and Confinement Issues in 3D Configurations

D. A. Spong

Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA

Collaborators: Y. Todo, M. Osakabe, B. Breizman, D. Brower, C. B. Deng, L. A. Berry

Energetic Particle Group Working Session

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Three main topics will be discussed (work in progress):

- Discrete shear Alfvén mode structure in 3D configurations
- Coupled Alfvén/sound continua for stellarators - application to HSX observations
- Self-consistent ripple calculations for ITER and Monte Carlo alpha confinement simulation

Motivations for 3D Alfvén mode structure calculations (AE3D):

- **Collaboration with Y. Todo (NIFS)**
- **Applications**:
  - Alpha/beam/ICRF/ECH driven AE instabilities
  - Stellarators (both high R/<a> and compact)
  - Tokamaks with ripple, RWM coils, etc.
- **Useful in development of reduced models**
  - Perturbative linear stability models
  - Limited mode/particle nonlinear models
- **Improve understanding of instability drive, sideband coupling effects**
  - more complex in 3D systems
- **Optimization target function**
- **Useful for interpretation of experimental measurements**
  - Mode identification
  - synthetic diagnostics
- **Input for particle orbit confinement studies in presence of Alfvén instabilities** (collaborations at NIFS with Isobe, Okasabe)
- Other efforts: A. Könies, Phys. Plasmas 7 (2000) 1139.

Reduced MHD description leads to a single shear Alfvén eigenmode equation vs. the 3 coupled equations that characterize full MHD

$$\begin{cases} \vec{\nabla}\bullet\left(\dfrac{\mu_0\rho_{ion}}{B}\dfrac{\partial}{\partial t}\dfrac{\vec{\nabla}\phi}{B}\right) = \left(\vec{B}\bullet\vec{\nabla}\right)\dfrac{J_{\|}}{B} \\ \\ \dfrac{\partial\psi}{\partial t} = \dfrac{\vec{B}}{B}\bullet\vec{\nabla}\phi \end{cases}$$

$$Setting \quad \frac{\partial}{\partial t} = -i\omega \quad and \quad combining \quad equations:$$

$$\omega^2 \ \vec{\nabla}\bullet\left(v_A^{-2}\ \vec{\nabla}\phi\right) + \vec{B}\bullet\vec{\nabla}\left[\frac{1}{B}\nabla^2\left(\frac{\vec{B}}{B}\bullet\vec{\nabla}\phi\right)\right]$$

Reduced MHD shear Alfvén model [based on “Generalized reduced MHD Equations,” by S. E. Kruger, et al., Phys. of Plasmas **5** (1998) 4169]

Variational form of 3D shear Alfvén equation:

- Multiply by weighting function $\tilde{\phi}$ and reduce to surface terms plus symmetric terms:
- Inertial term:

$$\tilde{\phi}\,\omega^2\,\vec{\nabla}\bullet\left(v_A^{-2}\,\vec{\nabla}\phi\right)=\omega^2\,\vec{\nabla}\bullet\left[v_A^{-2}\tilde{\phi}\,\vec{\nabla}\phi\right]-\frac{\omega^2}{v_A^2}\vec{\nabla}\tilde{\phi}\bullet\vec{\nabla}\phi$$

- Bending energy term:

$$\tilde{\phi}\,\vec{B}\bullet\vec{\nabla}\left[\frac{1}{B}\nabla^2\left(\frac{\vec{B}}{B}\bullet\vec{\nabla}\phi\right)\right]=\vec{\nabla}\bullet\left[\frac{\tilde{\phi}}{B}\vec{B}\,\nabla^2\left(\frac{\vec{B}\bullet\vec{\nabla}\phi}{B}\right)\right]-\vec{\nabla}\bullet\left[\left(\frac{\vec{B}\bullet\vec{\nabla}\tilde{\phi}}{B}\right)\vec{\nabla}\left(\frac{\vec{B}\bullet\vec{\nabla}\phi}{B}\right)\right]+\vec{\nabla}\left(\frac{\vec{B}\bullet\vec{\nabla}\tilde{\phi}}{B}\right)\bullet\vec{\nabla}\left(\frac{\vec{B}\bullet\vec{\nabla}\phi}{B}\right)$$

- Ignoring terms (indicated in blue) that will lead to surface integrals, this results in the following symmetric, self-adjoint equation:

$$-\frac{\omega^2}{v_A^2}\vec{\nabla}\tilde{\phi}\bullet\vec{\nabla}\phi+\vec{\nabla}\left(\frac{\vec{B}\bullet\vec{\nabla}\tilde{\phi}}{B}\right)\bullet\vec{\nabla}\left(\frac{\vec{B}\bullet\vec{\nabla}\phi}{B}\right)=0$$

Reduced MHD shear Alfvén model: solution method

- Solution procedure
- Galerkin method (weighted residuals with trial function = weight function)

$$\phi = \sum_{m,l} f_m(\rho)\cos(m_l\theta - n_l\zeta) \qquad (based\ on\ stellarator\ symmetry)$$

mode selection rules (e.g., for $n = 1$ family):

$n = 1,\ \ m = 0,1,2,3,\ldots$

$n = -1,\ m = 1,2,3,\ldots$

$n = 1 \pm N_{fp},\ m = 1,2,3,\ldots$

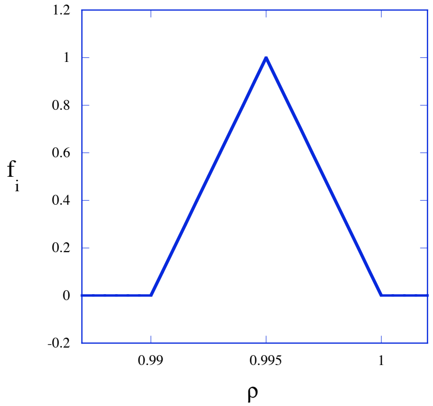
$n = -1 \pm N_{fp},\ m = 1,2,3,\ldots$

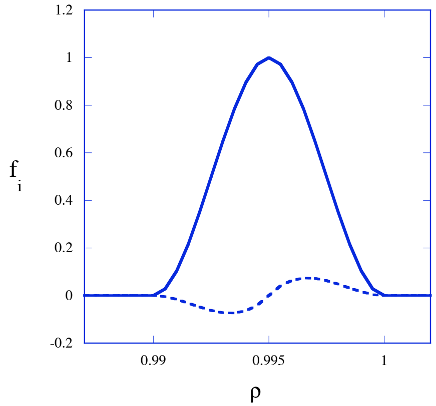
etc.

Up to 200 Fourier modes have been used with 40 - 60 radial elements

$\rho, \theta, \zeta = Boozer\ magnetic\ coordinates$

$$f_m(\rho) = \left\{ \begin{matrix} linear \\ or \\ cubic\ Hermite \end{matrix} \right\}\ finite\ element$$





The Alfvén equation is solved in ρ, θ, ζ Boozer coordinates:

Inertial term:

$$\frac{\omega^2}{v_A^2}\left[g^{\rho\rho}\frac{\partial\tilde{\phi}}{\partial\rho}\frac{\partial\phi}{\partial\rho}+g^{\theta\theta}\frac{\partial\tilde{\phi}}{\partial\theta}\frac{\partial\phi}{\partial\theta}+g^{\zeta\zeta}\frac{\partial\tilde{\phi}}{\partial\zeta}\frac{\partial\phi}{\partial\zeta}+g^{\rho\theta}\left(\frac{\partial\tilde{\phi}}{\partial\rho}\frac{\partial\phi}{\partial\theta}+\frac{\partial\tilde{\phi}}{\partial\theta}\frac{\partial\phi}{\partial\rho}\right)+g^{\theta\zeta}\left(\frac{\partial\tilde{\phi}}{\partial\zeta}\frac{\partial\phi}{\partial\theta}+\frac{\partial\tilde{\phi}}{\partial\theta}\frac{\partial\phi}{\partial\zeta}\right)\right]$$

Bending energy term:
$$P\tilde{P}g^{\rho\rho}+Q\tilde{Q}g^{\theta\theta}+R\tilde{R}g^{\zeta\zeta}+(P\tilde{Q}+Q\tilde{P})g^{\rho\theta}$$
$$+(P\tilde{R}+R\tilde{P})g^{\rho\zeta}+(Q\tilde{R}+R\tilde{Q})g^{\zeta\theta}$$

where

$$P=\frac{\partial}{\partial\rho}\left[\left(\frac{\psi'}{B\sqrt{g}}\right)\left(\frac{\partial\phi}{\partial\zeta}+\iota\frac{\partial\phi}{\partial\theta}\right)\right]$$

$$Q=\frac{\partial}{\partial\theta}\left[\left(\frac{\psi'}{B\sqrt{g}}\right)\left(\frac{\partial\phi}{\partial\zeta}+\iota\frac{\partial\phi}{\partial\theta}\right)\right]$$

$$R=\frac{\partial}{\partial\zeta}\left[\left(\frac{\psi'}{B\sqrt{g}}\right)\left(\frac{\partial\phi}{\partial\zeta}+\iota\frac{\partial\phi}{\partial\theta}\right)\right]$$

and $\tilde{P},\ \tilde{Q},\ \tilde{R}=P,\ Q,\ R$ *with* ϕ *replaced by* $\tilde{\phi}$

Remaining steps:

- Final step: integrate inertia and bending energy terms over plasma volume to obtain matrix equations:

$$\iiint \sqrt{g}\, d\rho\, d\theta\, d\zeta \left(\cdots\right) \;\Rightarrow Ax = \omega^2 Bx$$

- Involves convolution integrals of the form:

$$I_{ijk}^{(ccc)} \equiv \int_{-\pi}^{\pi} d\zeta \int_{-\pi}^{\pi} d\theta\, \cos\left(n_i\zeta - m_i\theta\right)\cos\left(n_j\zeta - m_j\theta\right)\cos\left(n_k\zeta - m_k\theta\right)$$

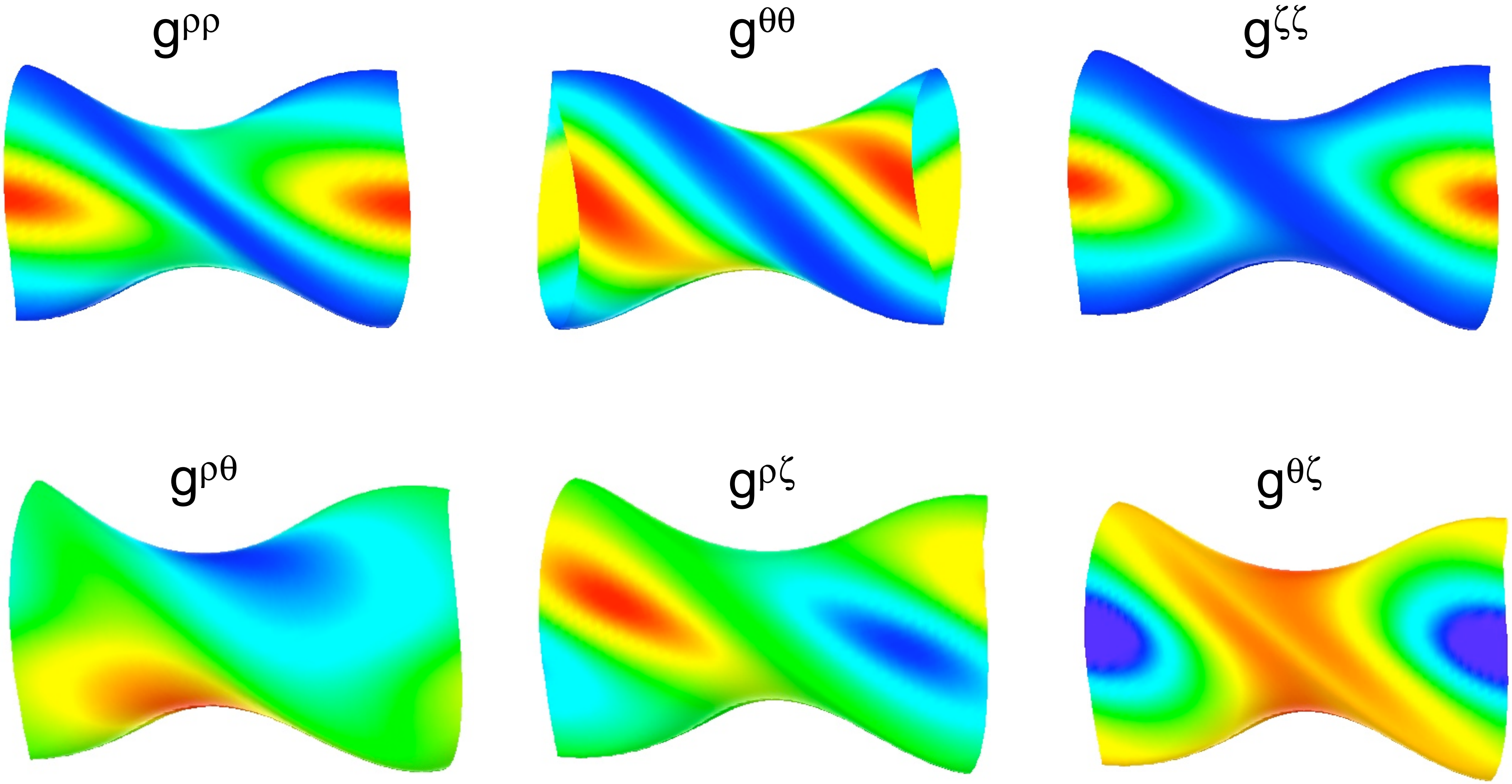
$$I_{ijk}^{(ssc)} \equiv \int_{-\pi}^{\pi} d\zeta \int_{-\pi}^{\pi} d\theta\, \sin\left(n_i\zeta - m_i\theta\right)\sin\left(n_j\zeta - m_j\theta\right)\cos\left(n_k\zeta - m_k\theta\right)$$

- these can be done exactly/analytically
- Setting coefficient of highest order ρ derivatives to zero leads to continuum equation:

$$\frac{\omega^2}{v_A^2}\frac{g^{\rho\rho}}{B^2}\frac{\partial\phi}{\partial\rho} + \vec{B}\bullet\vec{\nabla}\left\{\frac{g^{\rho\rho}}{B^2}\left(\vec{B}\bullet\vec{\nabla}\right)\frac{\partial\phi}{\partial\rho}\right\} = 0$$

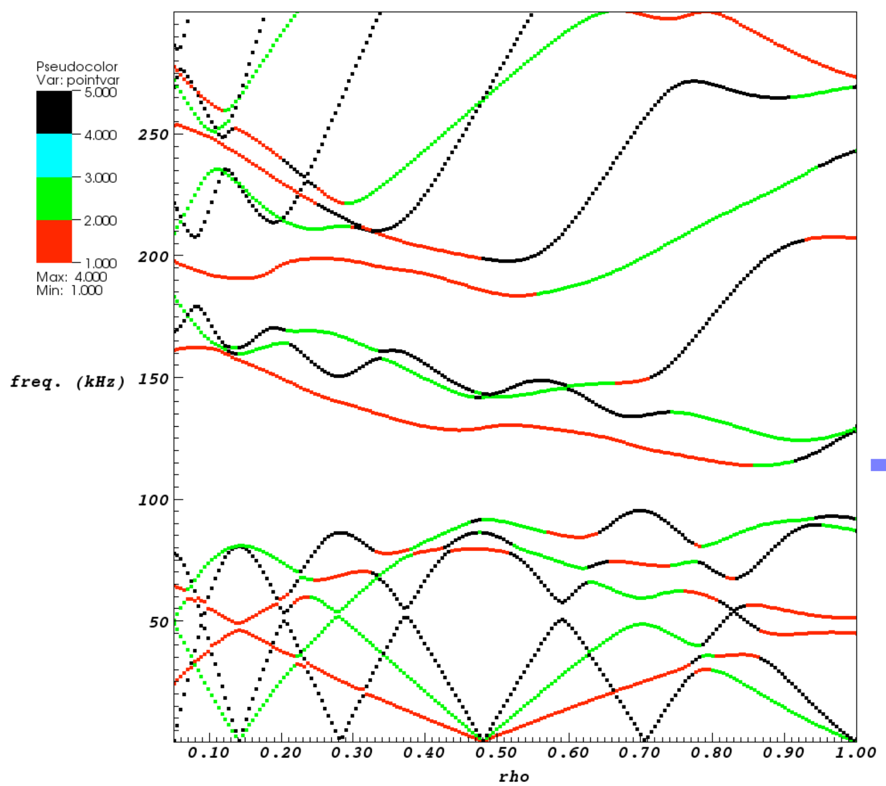
- [previously studied in D. A. Spong, R. Sanchez, A. Weller, Phys. of Plasmas **10** (3217) 2003]

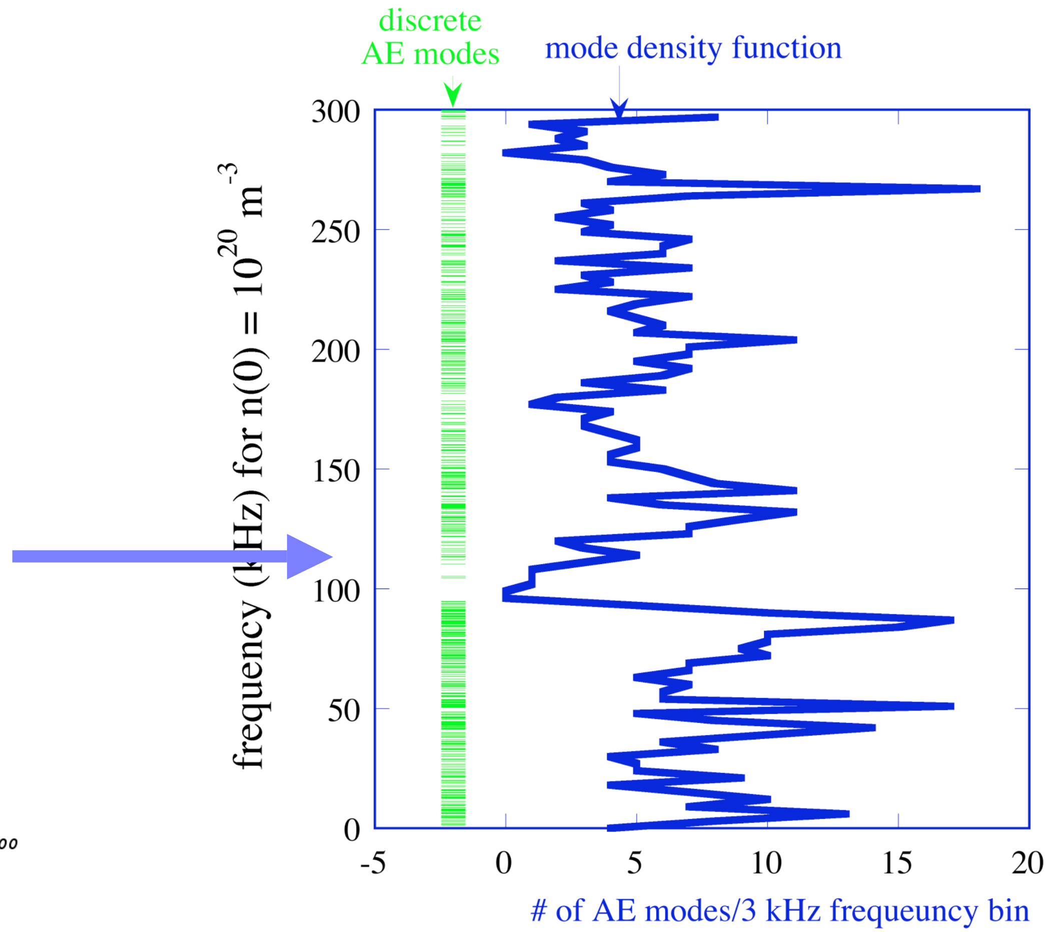
Metric elements (variation on a flux surface shown here for LHD)



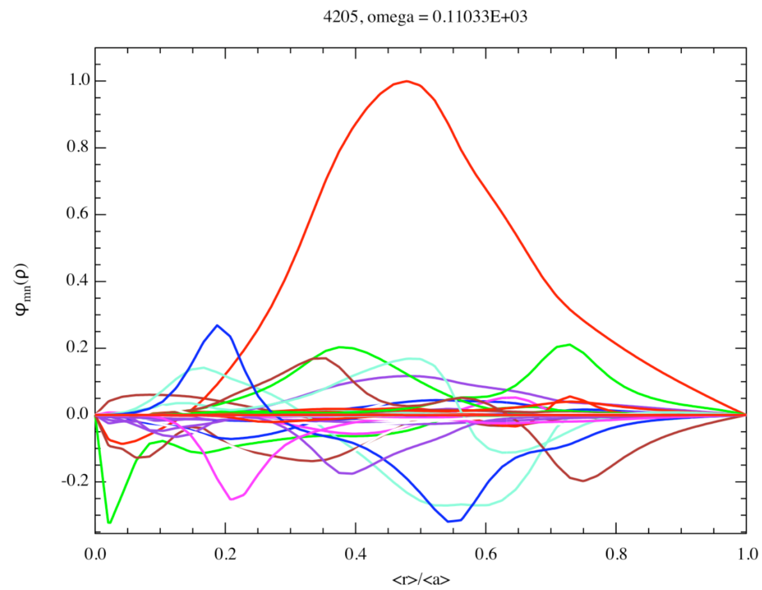
- Coefficients that involve these matrix elements are calculated on a θ, ζ grid on each surface
- This data is then Fourier transformed

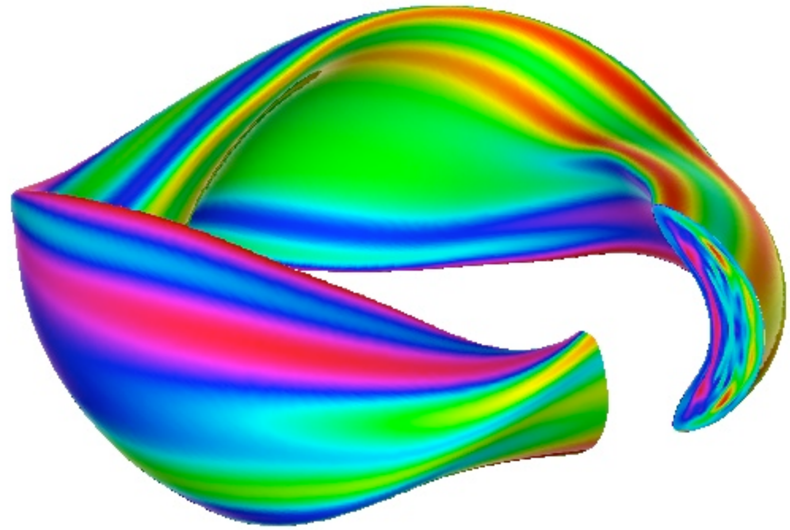
NCSX (quasi-toroidal stellarator)



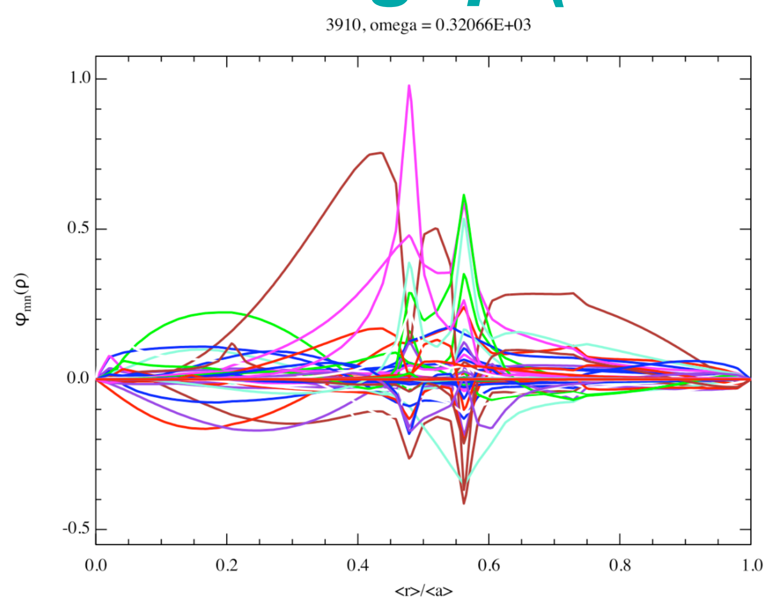


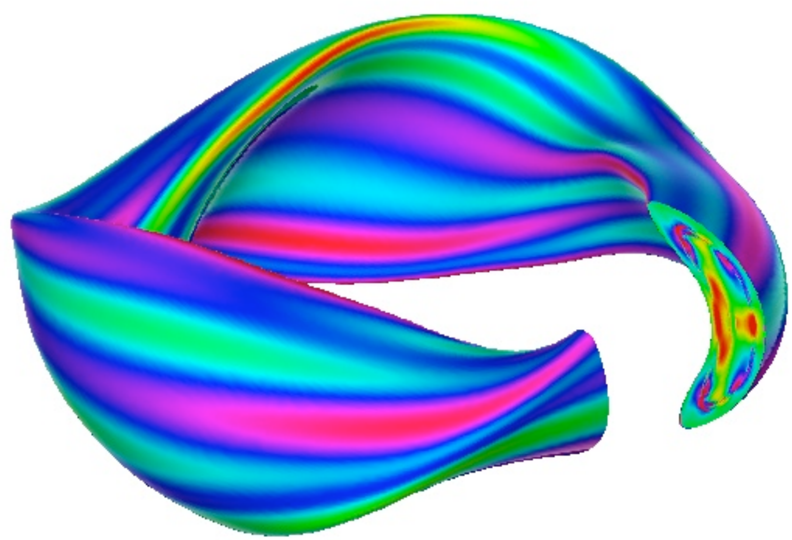
NCSX: Open gap mode





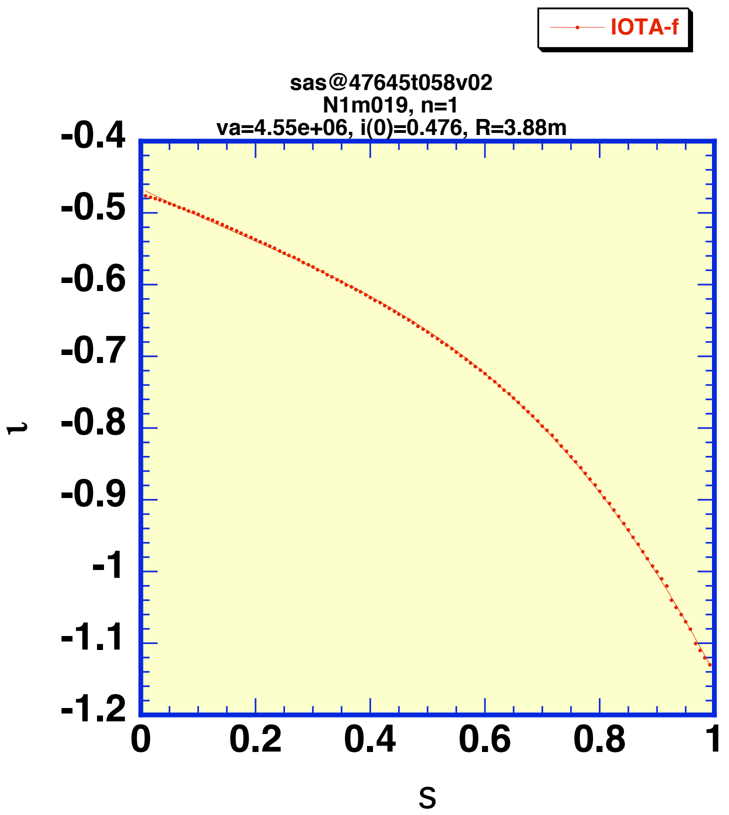
Closed gap (continuum damped) mode

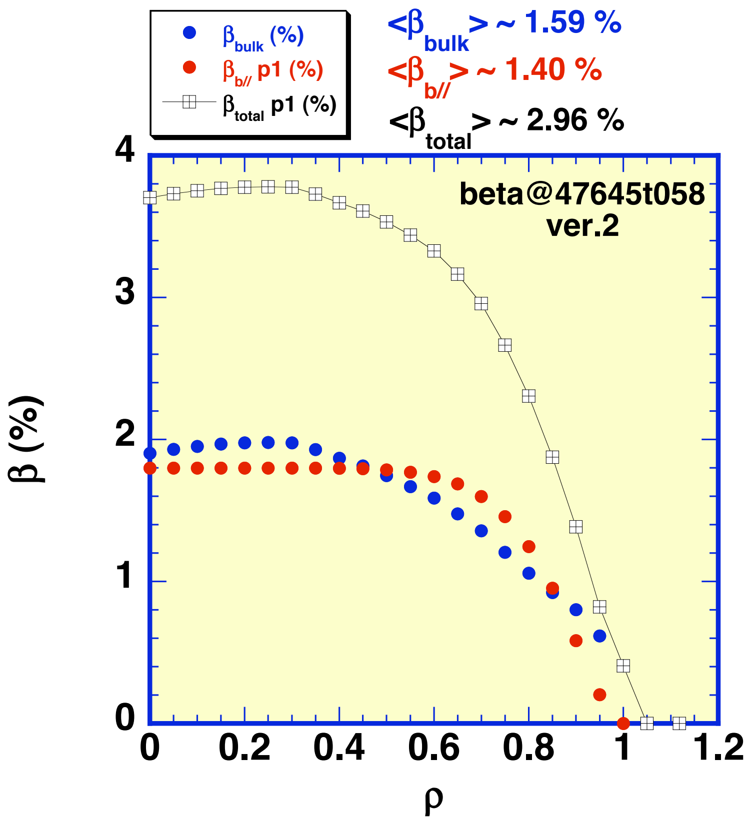




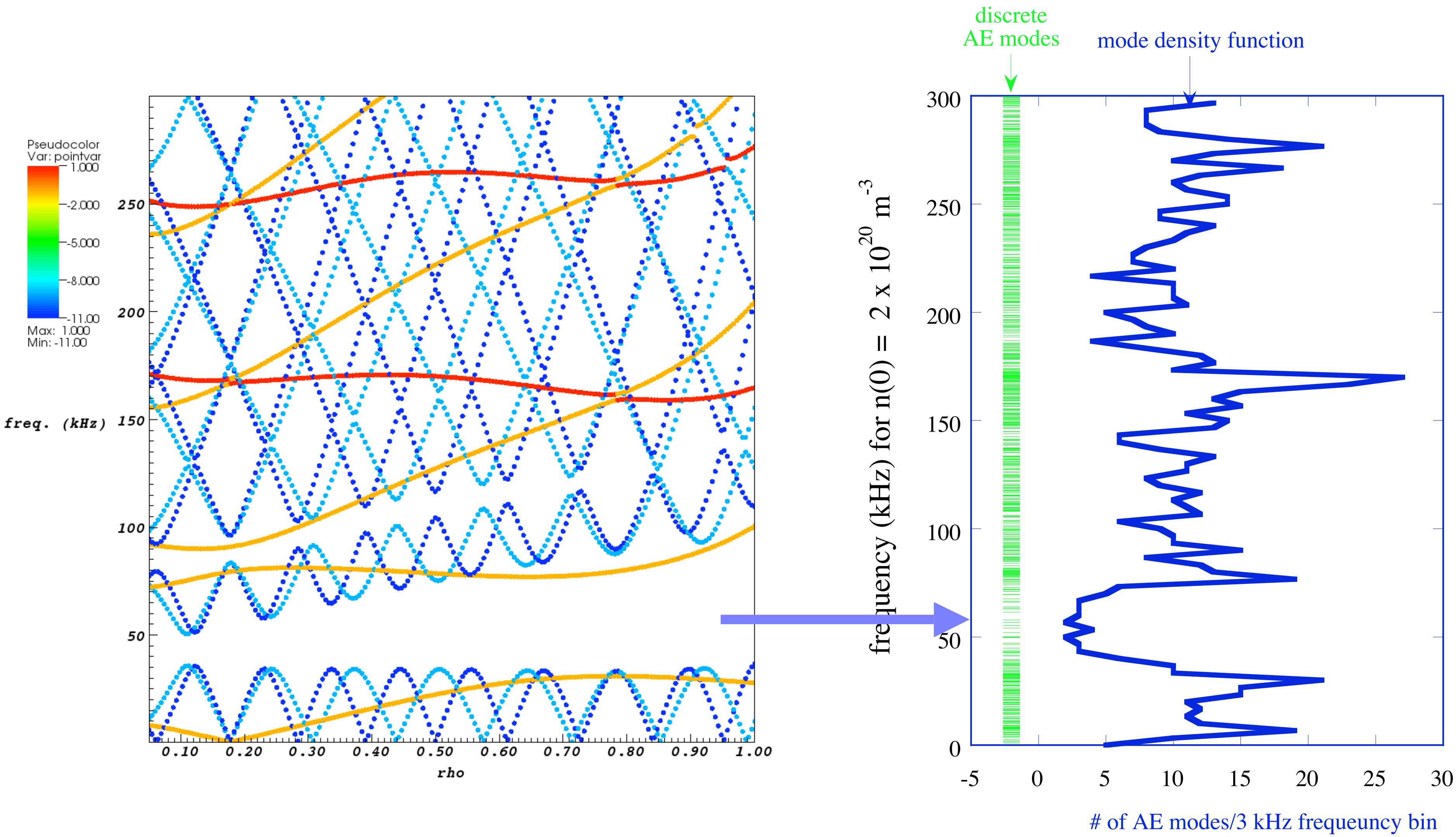
LHD Beam-driven AE's

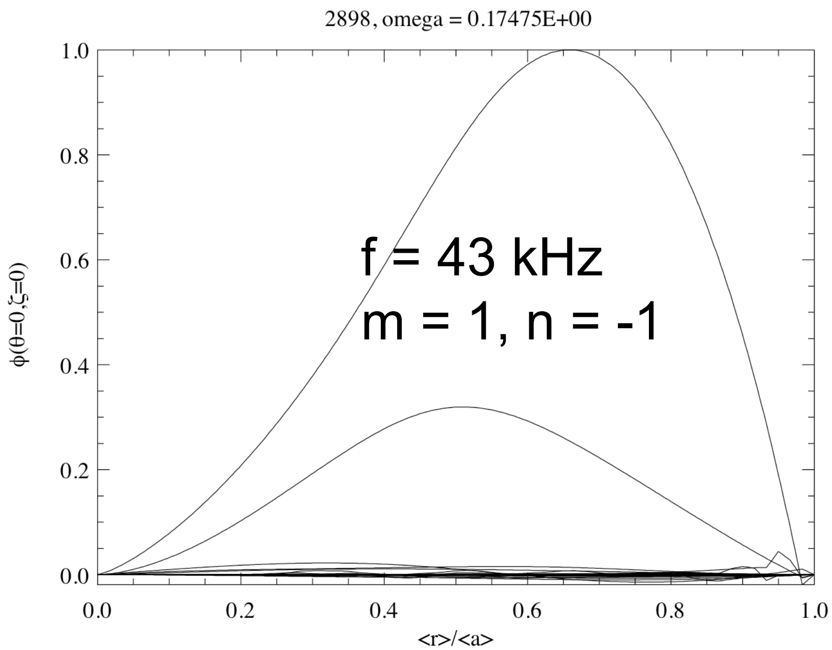
- Alfvén instabilities driven by beam ions have been studied in LHD shot #47645 [M. Osakabe, et al., Nucl. Fusion **46**, S911 (2006)]
- Hole-clump pairs in the fast ion distribution are observed with the neutral particle analyzer
- Shown to be consistent with AE3D Alfvén mode structures [Y. Todo, et al., Proc. Of ITC/ISHW2007]

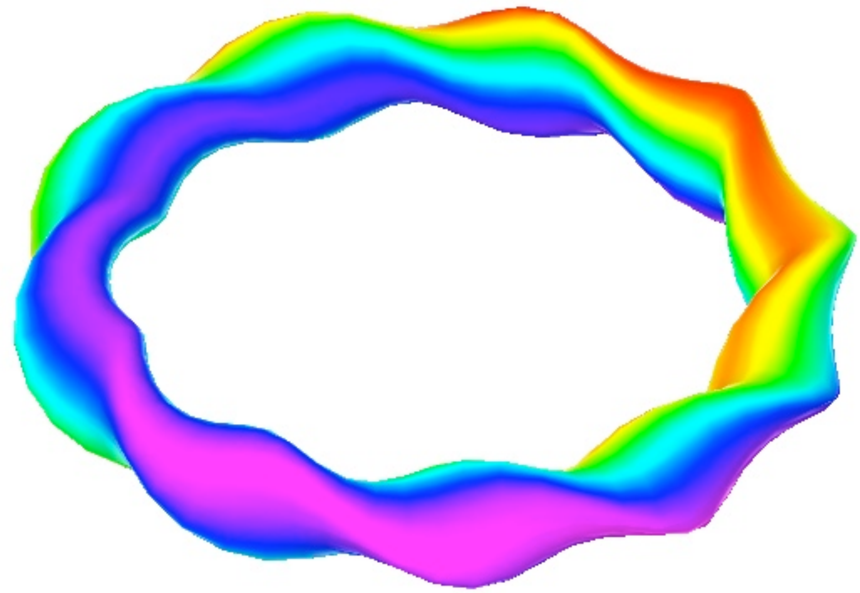


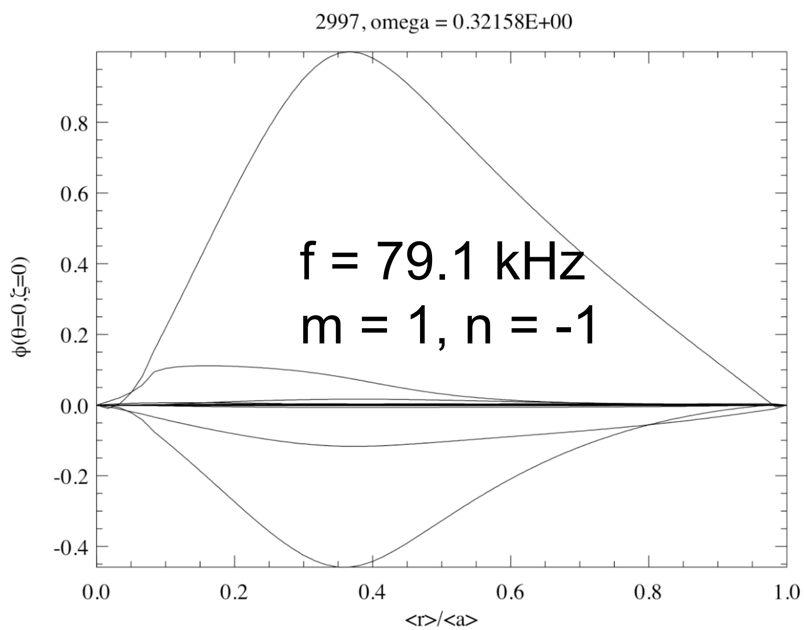


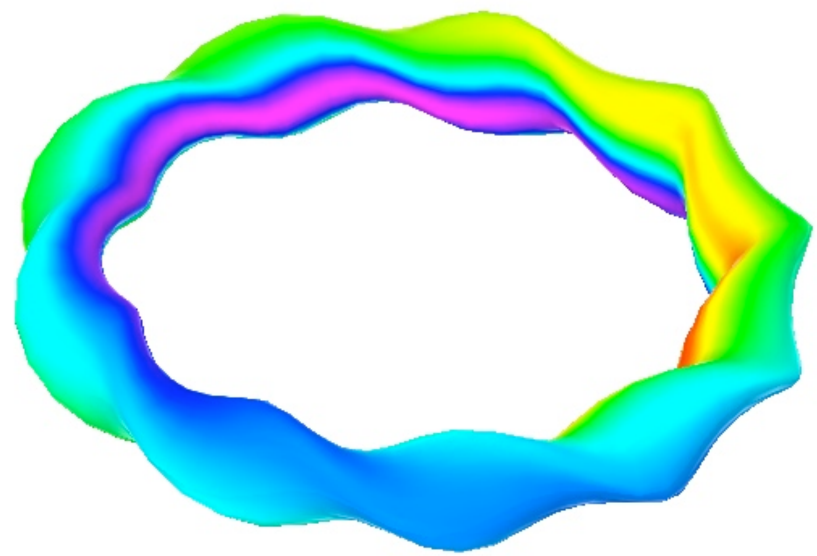
LHD gap structure

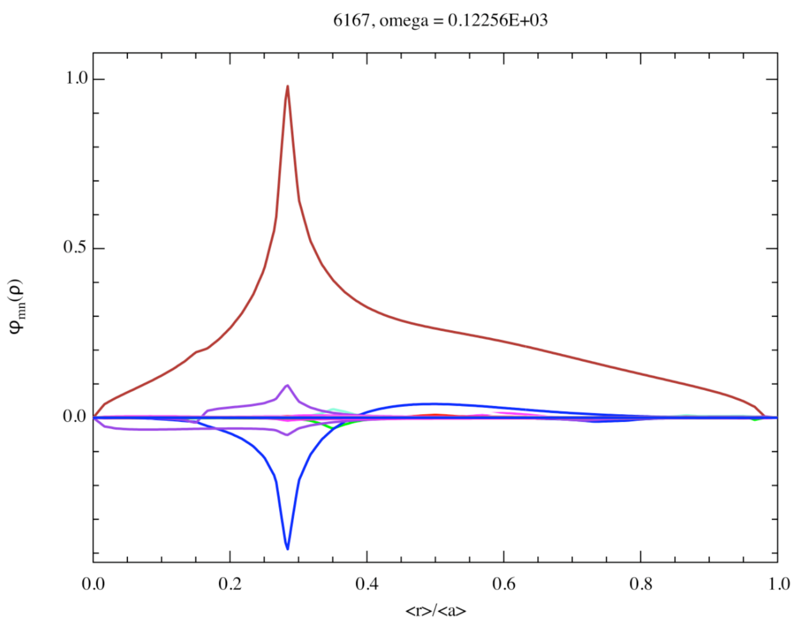


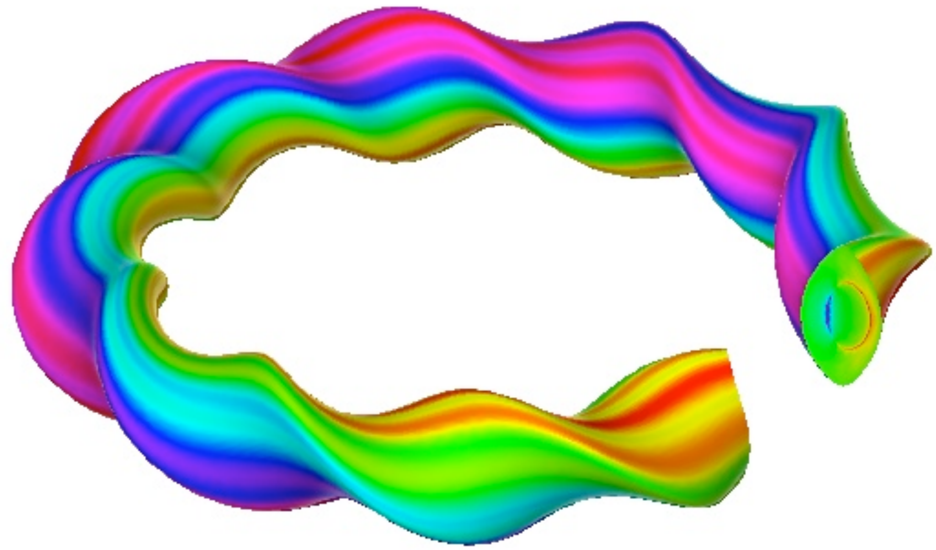

2898, omega = 0.17475E+00
f = 43 kHz
m = 1, n = -1
φ(θ=0,ζ=0)
<r>/<a>
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

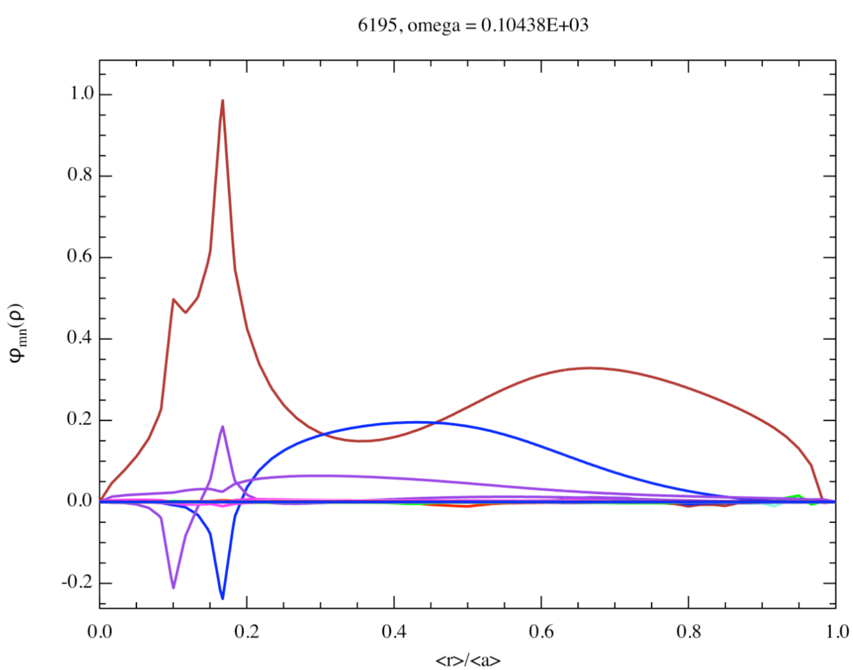


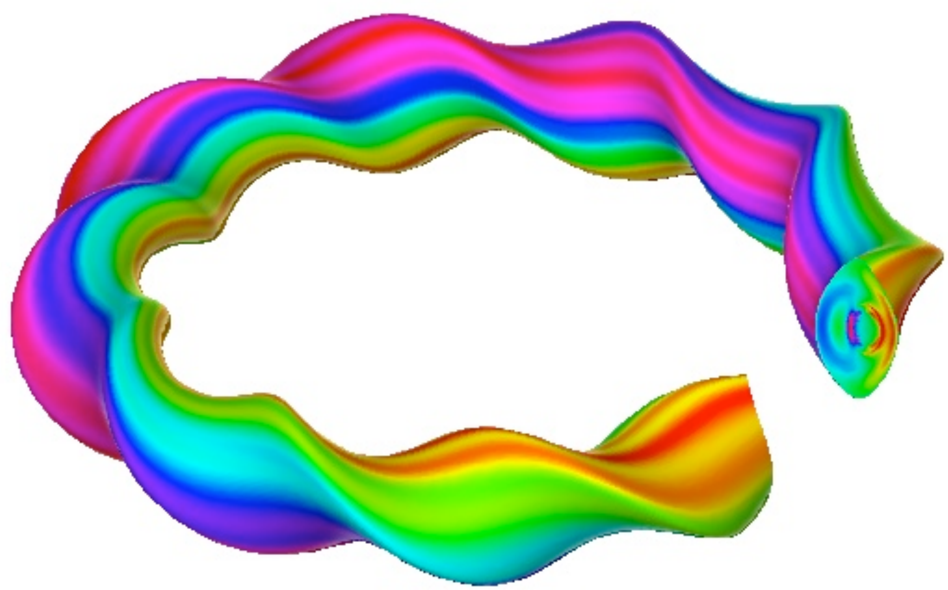

2997, omega = 0.32158E+00
f = 79.1 kHz
m = 1, n = -1
φ(θ=0,ζ=0)
<r>/<a>
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
0.0
0.2
0.4
0.6
0.8
1.0




6167, omega = 0.12256E+03
1.0
0.5
0.0
φmn(ρ)
0.0
0.2
0.4
0.6
0.8
1.0
<r>/<a>




6195, omega = 0.10438E+03
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
φmn(ρ)
0.0
0.2
0.4
0.6
0.8
1.0
<r>/<a>



Topics

- Discrete shear Alfvén mode structure in 3D configurations
- Coupled Alfvén/sound continua for stellarators
  - Collaborators: B. Breizman, D. Brower, C. B. Deng
  - Coherent modes observed in HSX with Alfvénic scaling
  - Alfvénic activity near 50 kHz (and sometimes 70kHz) is observed in HSX correlated with energetic electron tails
  - 1/q = iota ~ 1.05
  - Frequencies sometimes intersect continua near magnetic axis
  - Frequency insensitive to small iota variations
  - Motivated inclusion of sound continua
- Self-consistent ripple calculations for ITER and effects on alpha confinement

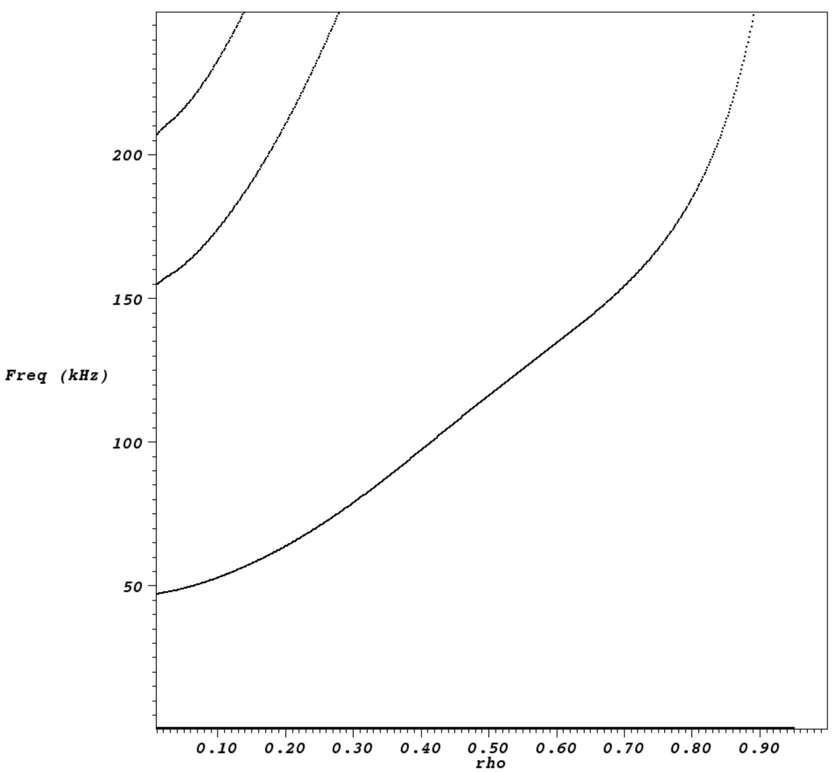
Solving the coupled Alfvén-sound continuum equations (derived by B. Breizman) provides a more complete picture of modes that may be available for energetic particle coupling (STELLGAP)

Shear Alfvén-sound continuum (B. Breizman)

$$-\frac{\omega^2 g}{B^2}\eta - \frac{1}{4\pi\rho}(\mathbf{B}\cdot\nabla)\frac{g}{B^2}(\mathbf{B}\cdot\nabla)\eta = C_s^2\left[(\mathbf{B}\cdot\nabla)\zeta - \eta G\right]G$$

$$-\omega^2 B^2\zeta - C_s^2(\mathbf{B}\cdot\nabla)(\mathbf{B}\cdot\nabla)\zeta = -C_s^2(\mathbf{B}\cdot\nabla)(\eta G)$$

Shear Alfvén only

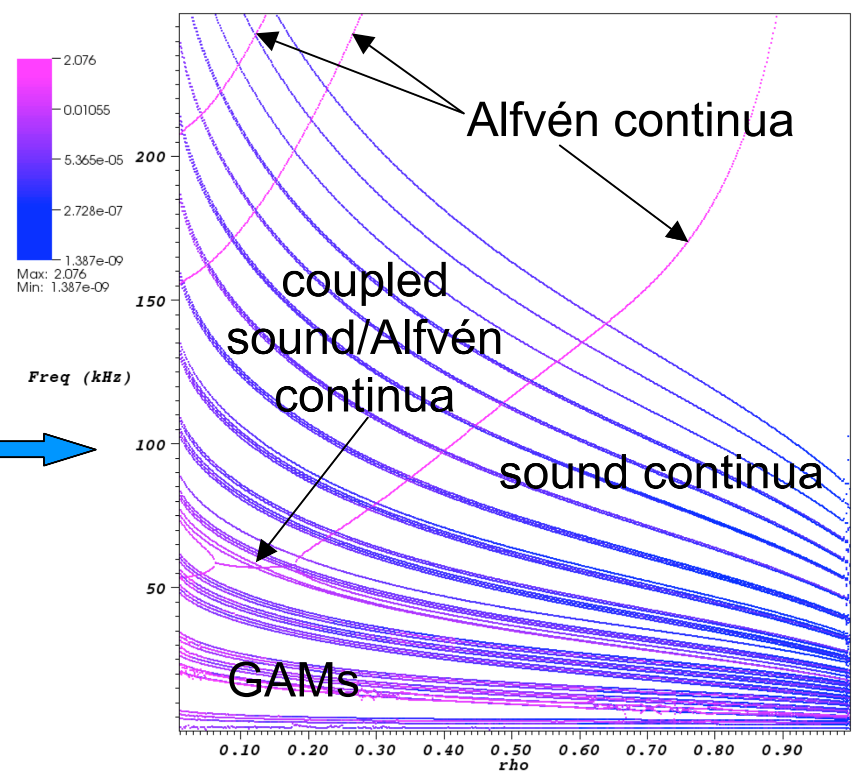


Coupled sound-shear Alfvén waves

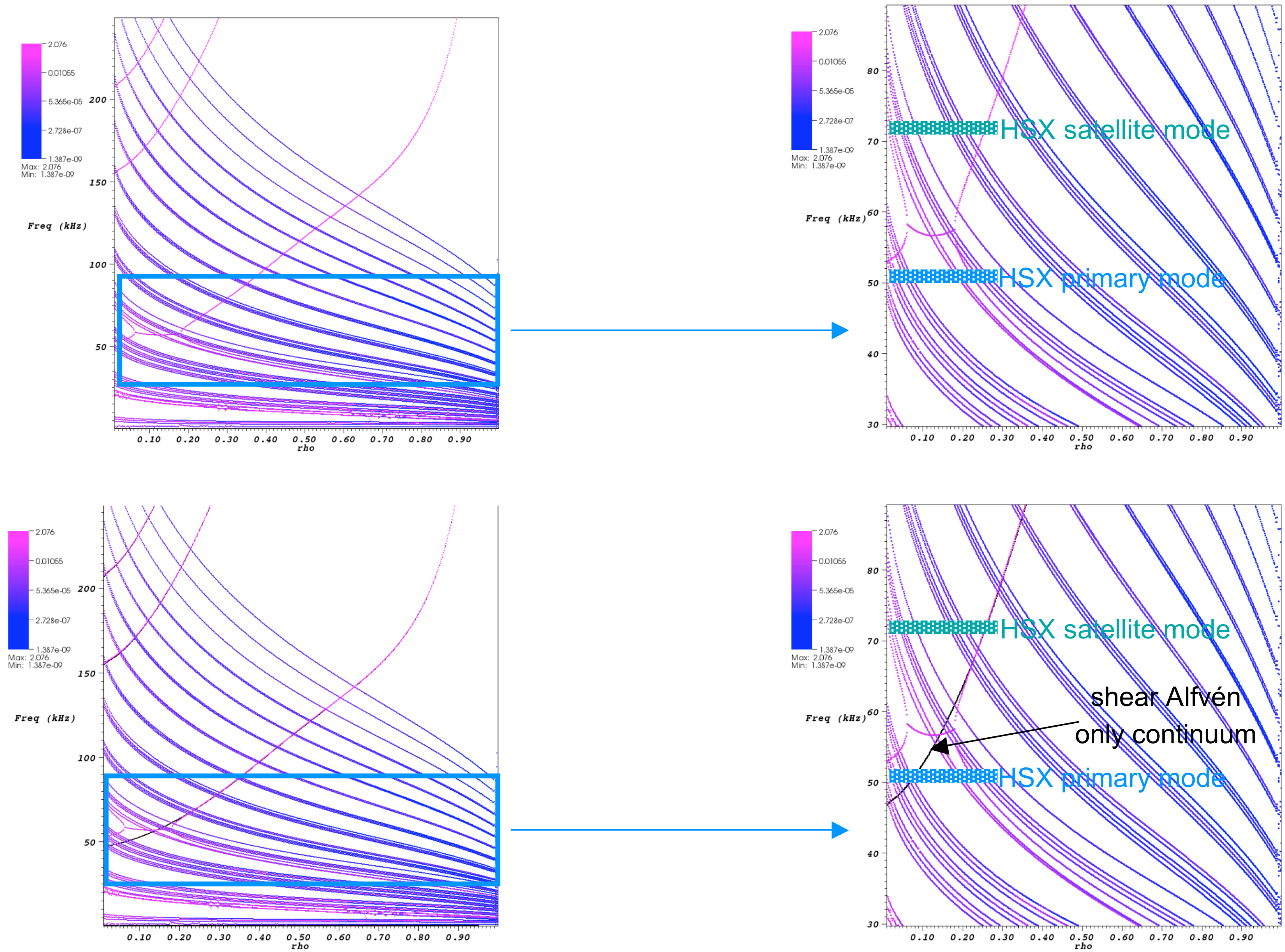
(color scale $\propto \sqrt{\sum_{m,n}\eta_{mn}^2}$

blue - sound wave,

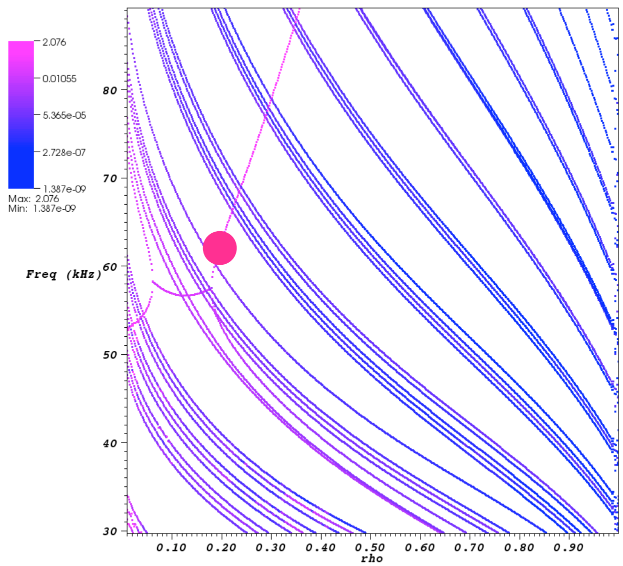
magenta - shear Alfvén wave)

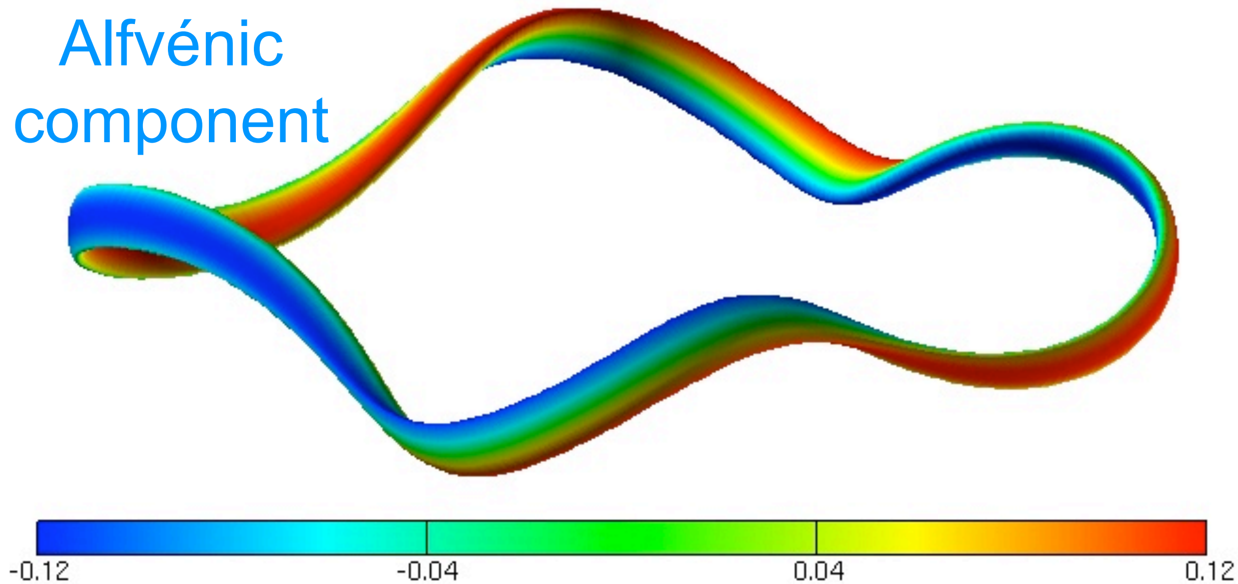


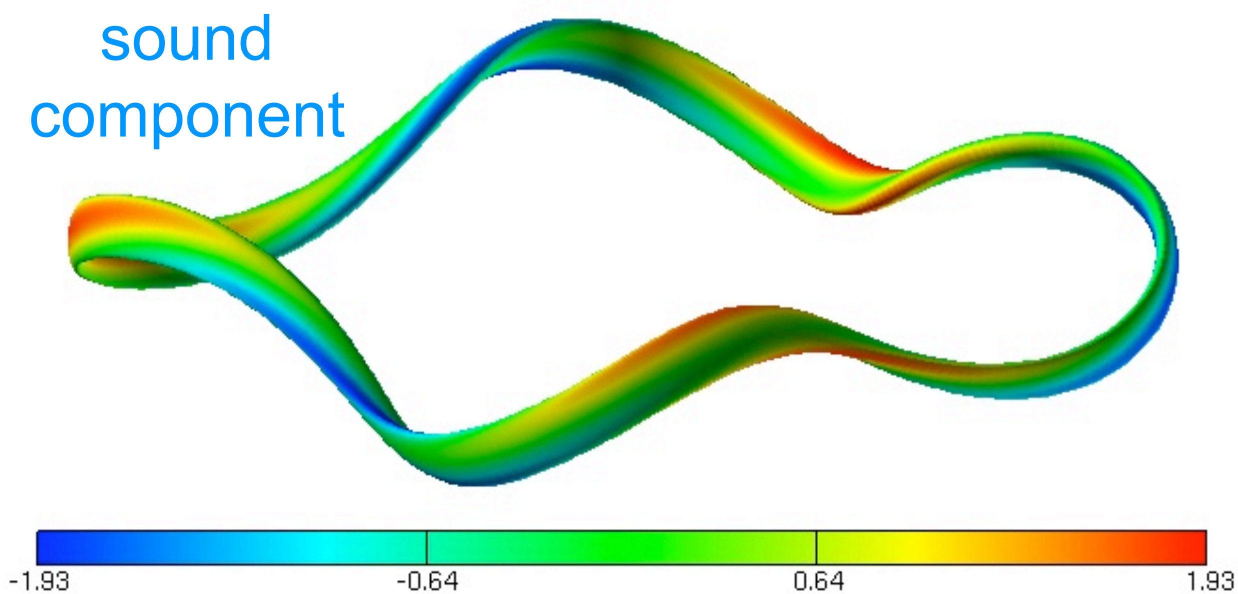
Upward frequency shift in coupled sound-shear Alfvén continua near magnetic axis is more consistent with HSX observations:



63.45 kHz eigenmode (at ρ = 0.2 surface)

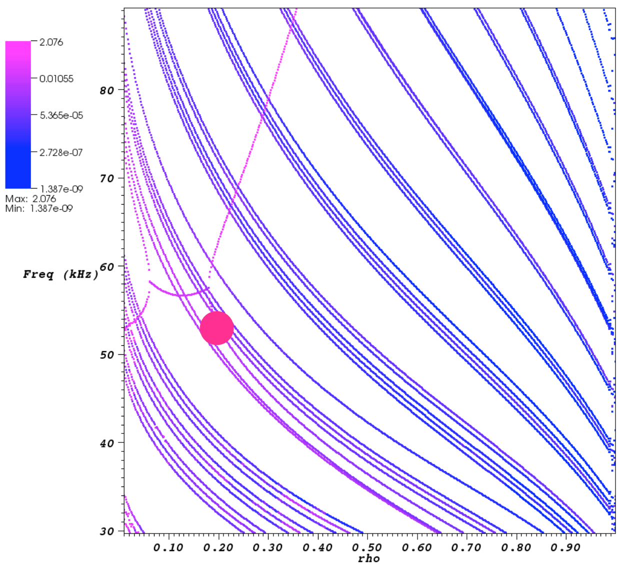




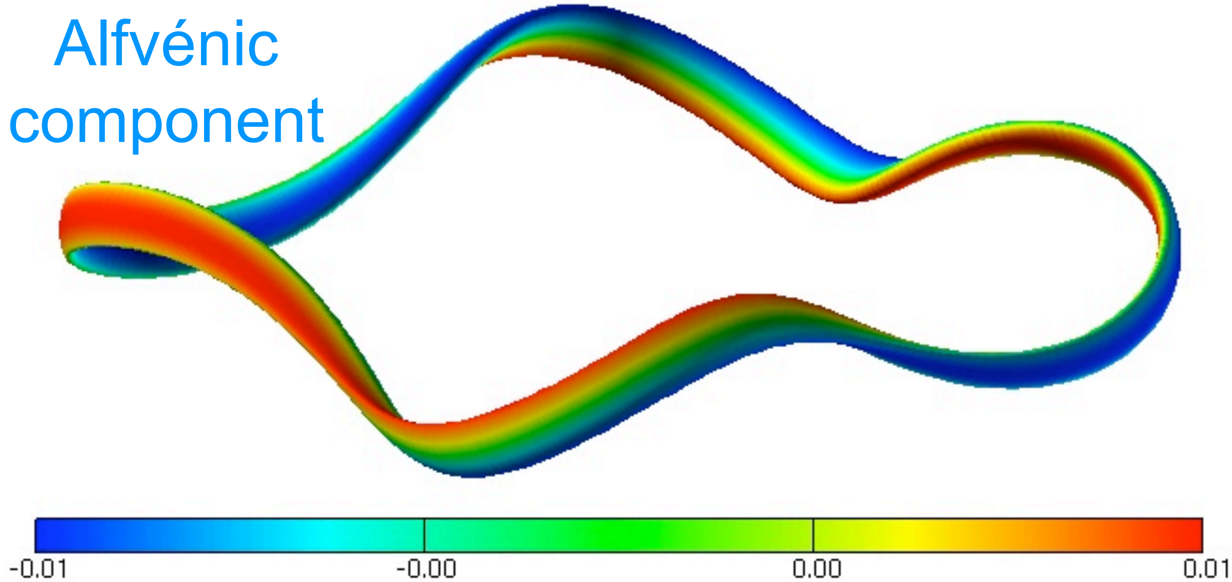


52.6 kHz eigenmode (at ρ = 0.2 surface)

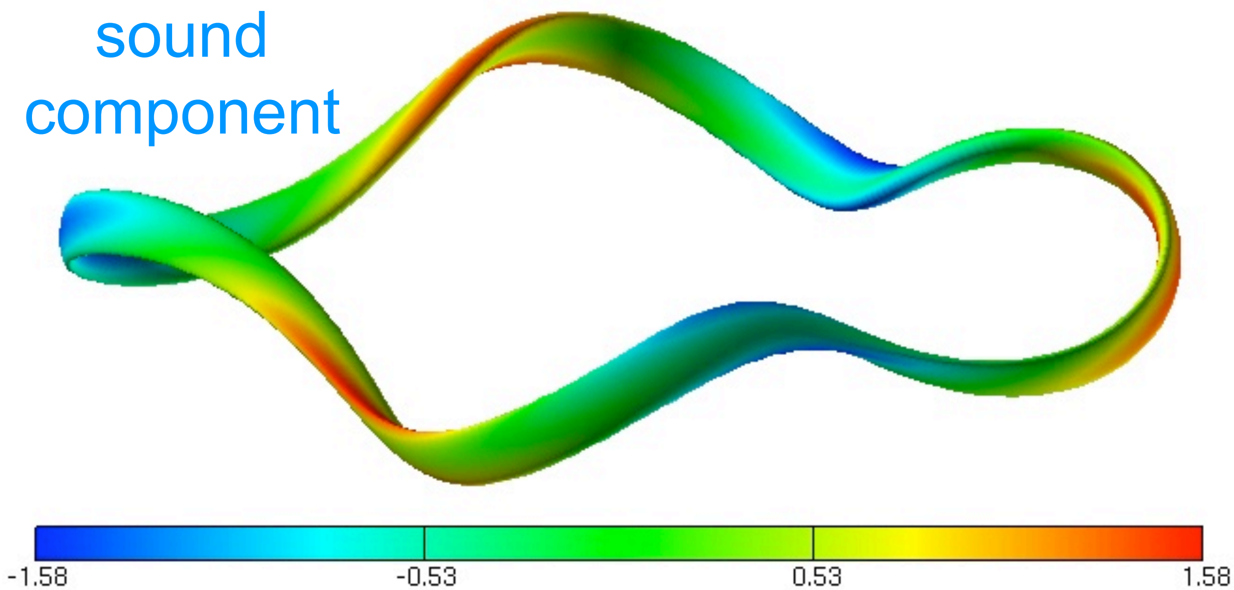
- sound and Alfvén continua meet with similar frequency and mode structure



Alfvénic component



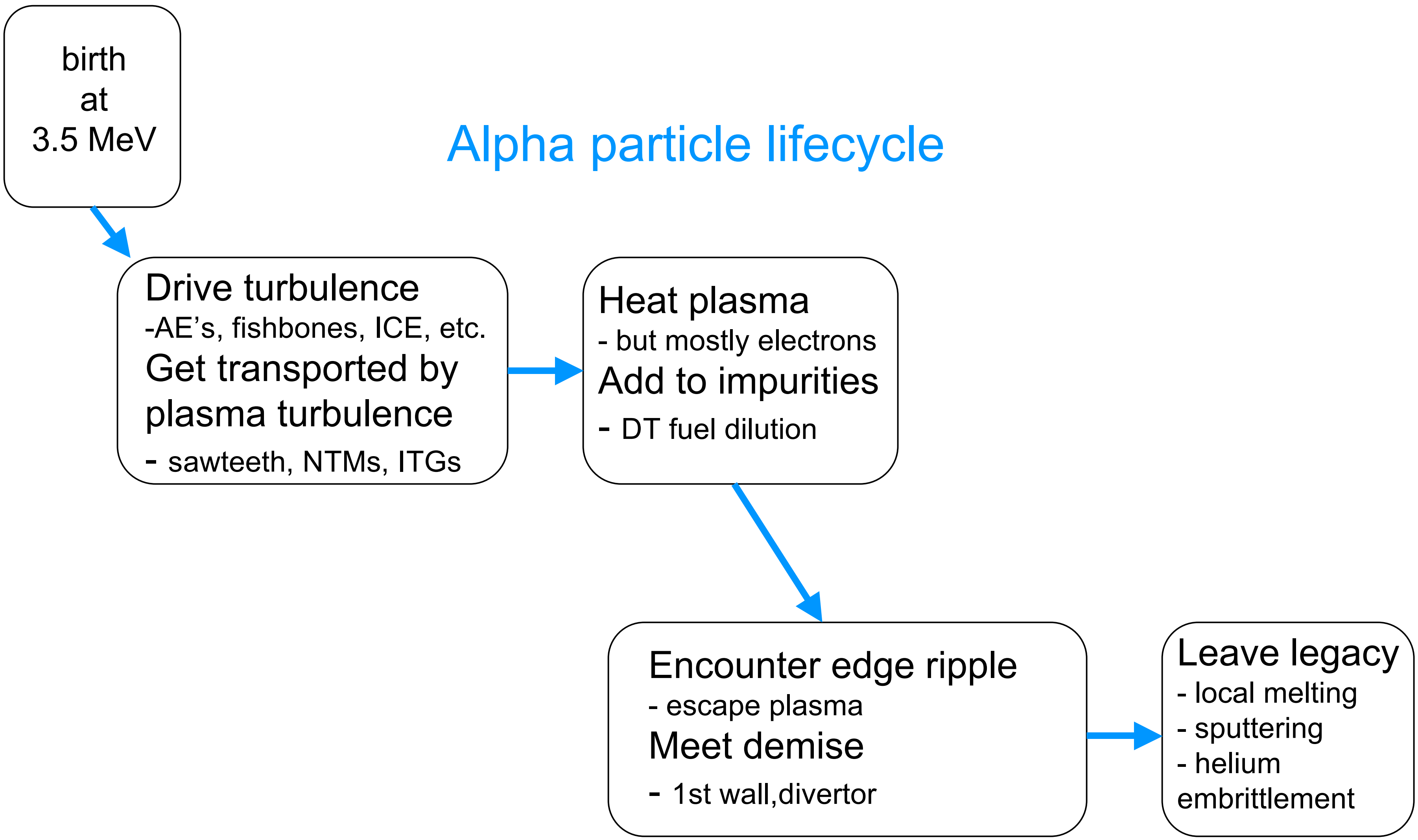
sound component



Topics

- Discrete shear Alfvén mode structure in 3D configurations
- Coupled Alfvén/sound continua for stellarators
- Self-consistent ripple calculations for ITER and effects on alpha confinement
  - Free boundary VMEC equilibria including both up-down asymmetry and TF ripple
  - Allows more self-consistent study of finite β, Shafranov shift, diamagnetic/Pfirsh-Schlüter currents on alpha confinement than previously possible
    - Y. Suzuki, Y. Nakamura, K. Kondo, "Finite beta effects on the toroidal field ripple in three-dimensional tokamak equilibria," Nucl. Fusion **43** (2003) 406.
    - J. L. Johnson, A. H. Reiman, "Self-consistent, three-dimensional equilibrium effects on tokamak magnetic field ripple," Nucl. Fusion **38** (1988) 1116.
  - DELTA5D + VMEC + MFBE* codes will be used modeling of alpha transport in ITER
    - *E. Strumberger, P. Merkel, et al., IPP Report 5/100 (May, 2002)

View of the future: an operating ITER or DEMO will need accurate "cradle-to-grave" modeling of alphas in order to assure protection the first wall/divertor/PFCs

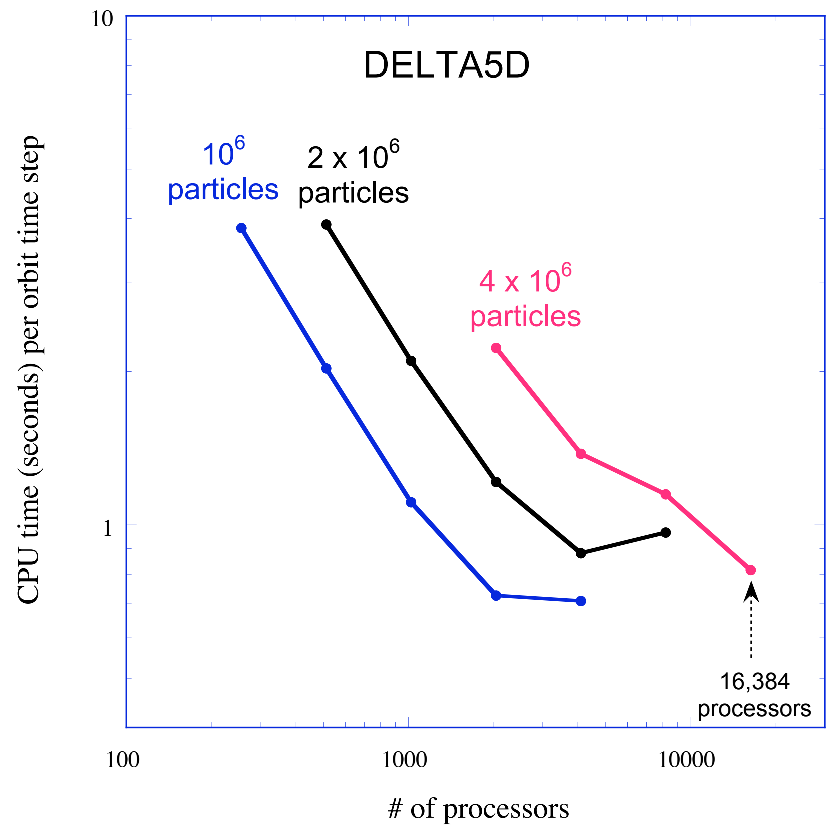


Accurate modeling of the edge region and large numbers of simulation particles are needed to obtain good alpha loss statistics at the 1st wall

Codes:

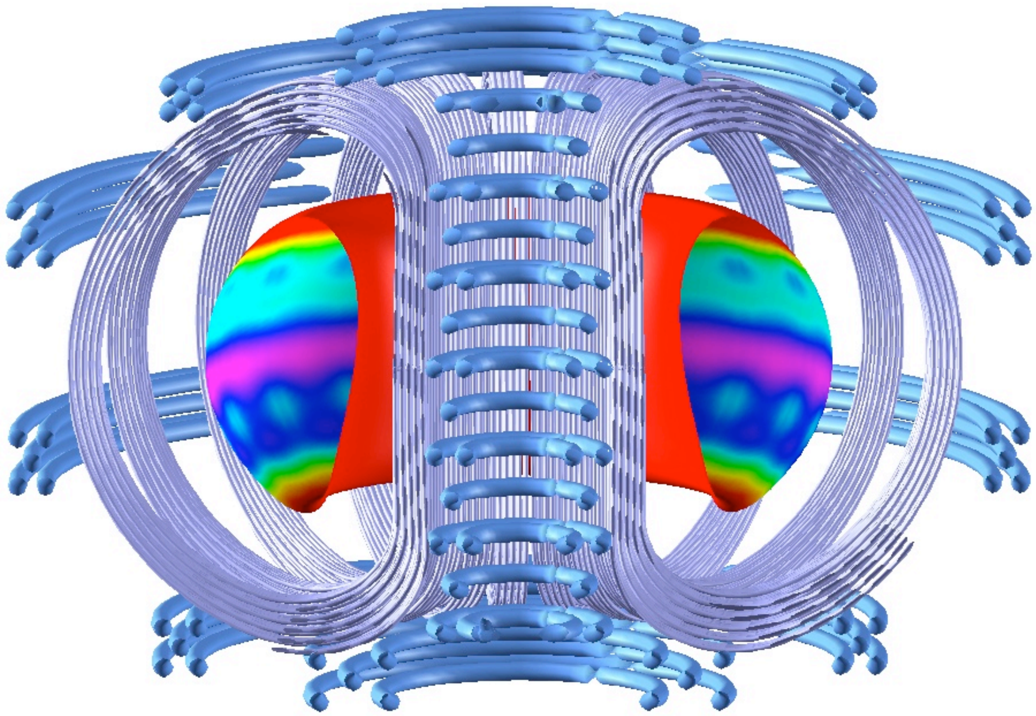
- **VMEC** - 3D rippled equilibria
- **MFBE**
  - E. Strumberger, P. Merkel, et al., IPP Report 5/100 (May, 2002)
  - Accurate magnetic field (virtual casing method) evaluation in vacuum region between plasma and coils
  - Follow alphas to their demise at the wall -> predict localized wall heating
- **DELTA5D-Magcoords**
  - Monte Carlo orbits in Boozer coordinates
  - R. H. Fowler, J. A. Rome, J. F. Lyon, Phys. Fl. **28**, (1985) 338.
- **DELTA5D-CylSVD**
  - Monte Carlo orbits in VMEC and cylindrical coordinates using SVD compression methods
  - Can include islands/AE modes
  - “Compression of magnetohydrodynamic simulation data using singular value decomposition,”del-Castillo-Negrete, D., Hirshman, S. P., Spong, D. A., D'Azevedo, E. F., JOURNAL OF COMPUTATIONAL PHYSICS **222**, 265 (2007)

Recent parallel scaling on Cray XT3

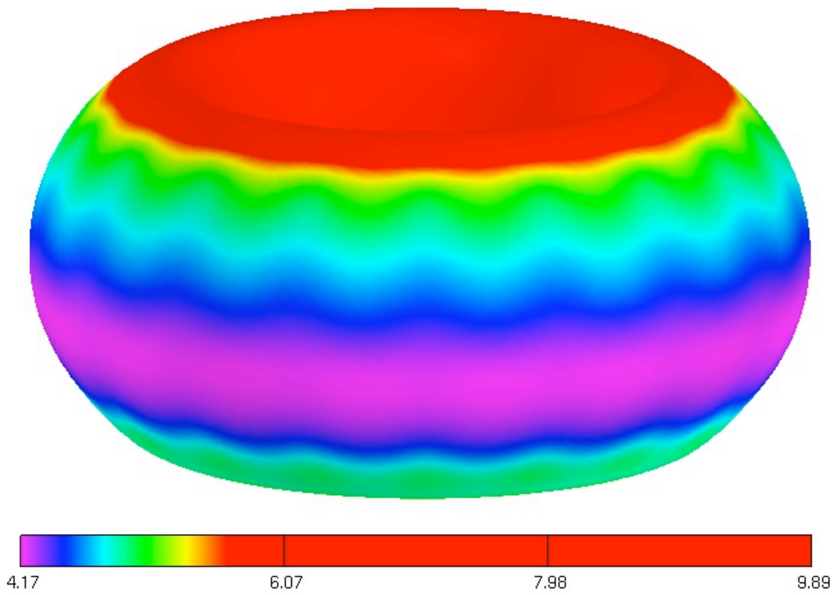


ITER free boundary VMEC equilibria have been calculated using a filamentary coil model:

Coil model: 18 TF's with 25 filaments each 5 filaments per PF



|B| contours on outer flux surface (compressed color map)

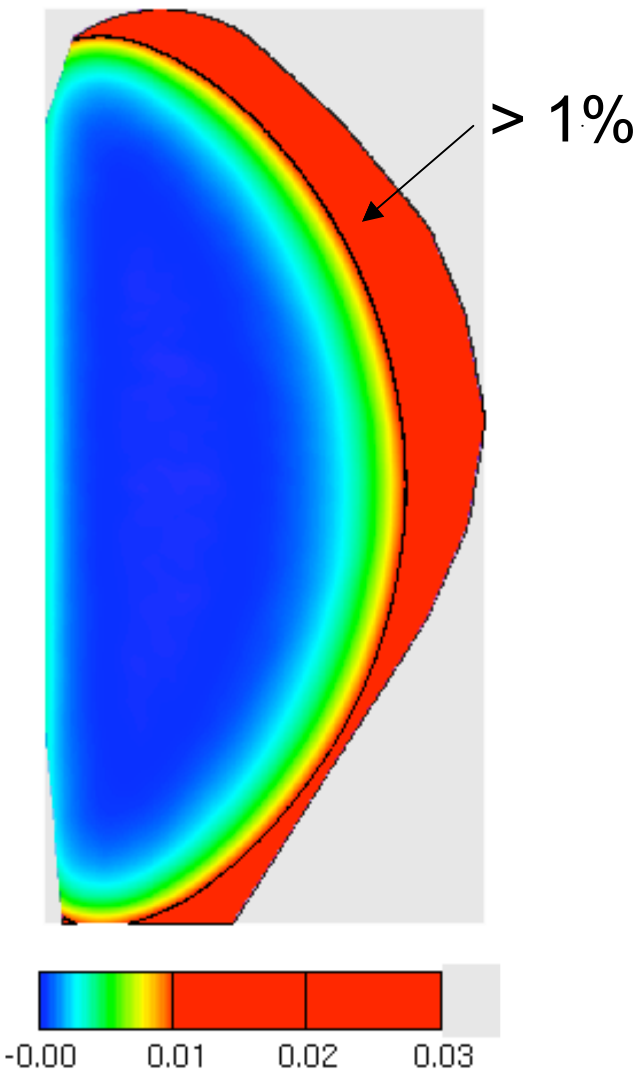


- Next step: complete connections between VMEC's external Green's function and ANSYS vacuum field calculation
  - Allows direct incorporation of the effects of ferritic inserts, TBM's, finite volume coil currents, RWM's etc. into VMEC equilibrium

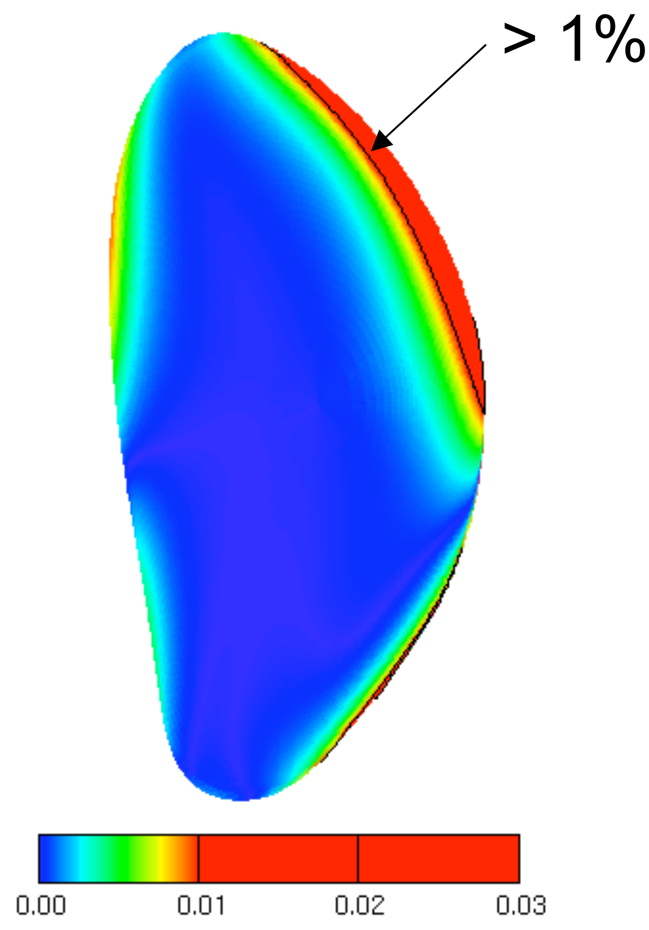
Contours of the main n = 18 ripple harmonic have been benchmarked against vacuum data from ANSYS:

Contours of $B_{n=18}(R,Z)$ where $B(R,\phi,Z) = \sum_n B_n(R,Z)\cos(n\phi)$

ANSYS vacuum n = 18 ripple



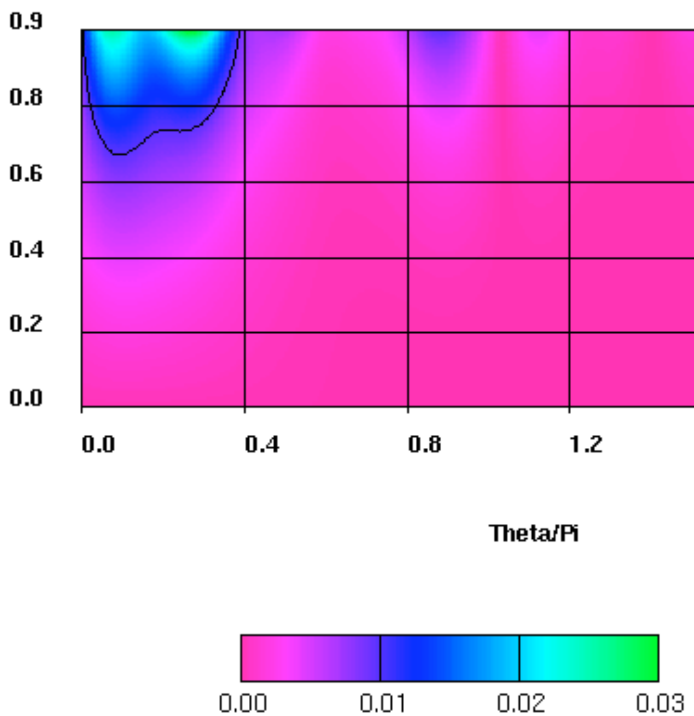
VMEC β = 2.4% n = 18 ripple



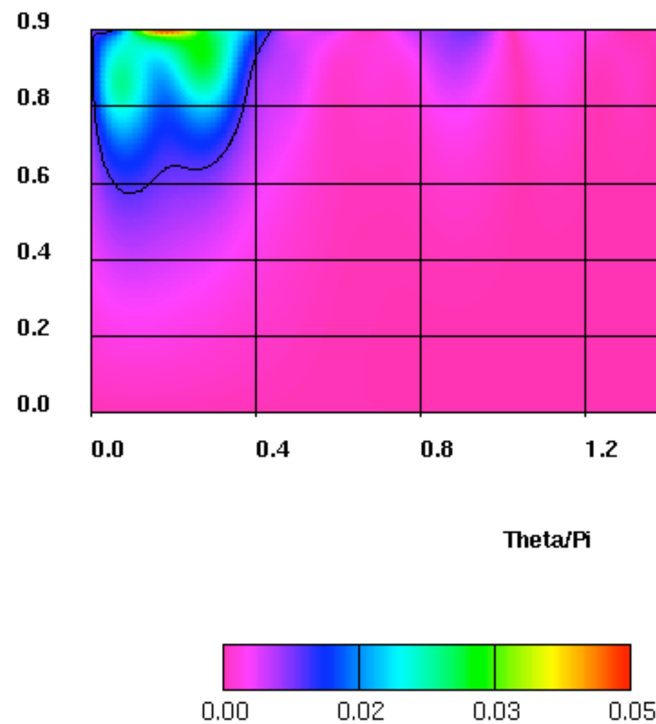
At finite β's ripple contours permeate somewhat further into core
(i.e., ripple amplification by diamagnetic currents)

note: edge ripple(δ) ~ $B_{n=18}/5$ ~ 0.2 - 1%

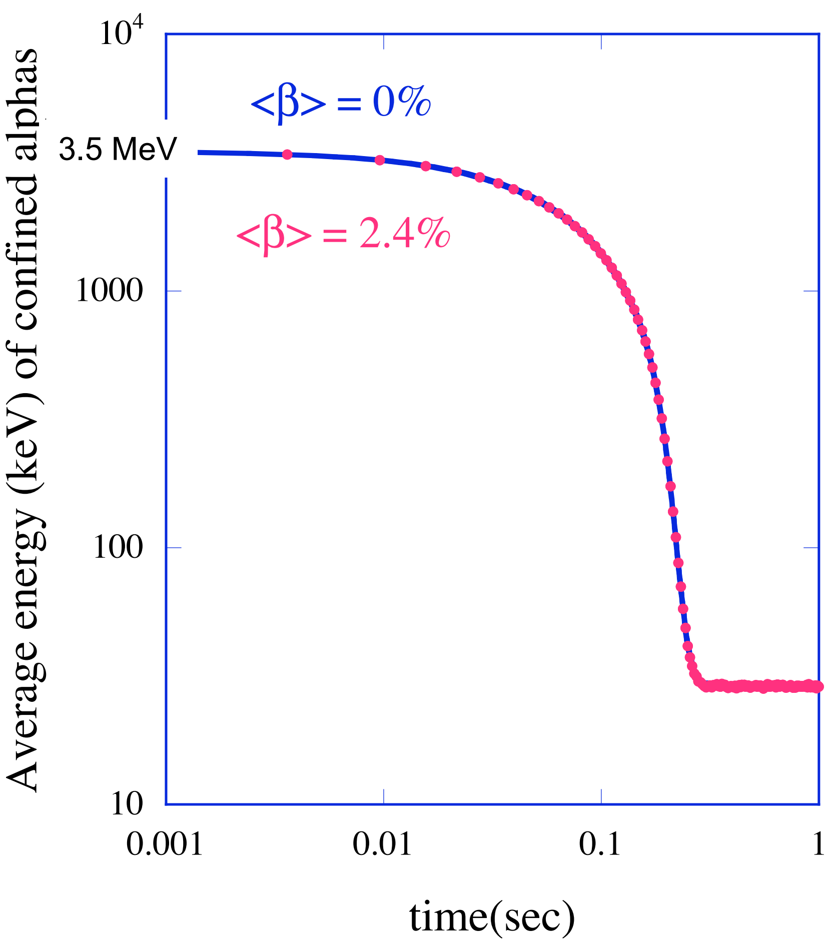
Contours of $|B_{n=18}(\theta,s)|$ for $\langle\beta\rangle = 0\%$

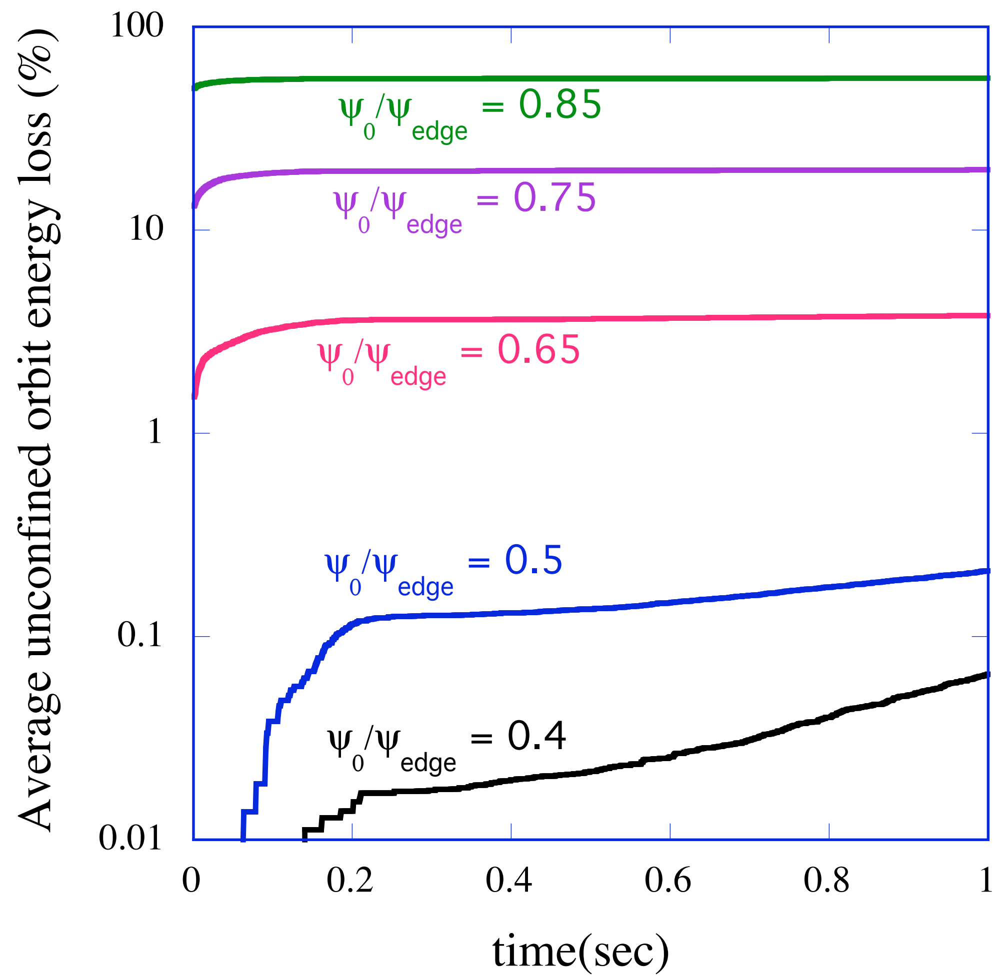


Contours of $|B_{n=18}(\theta,s)|$ for $\langle\beta\rangle = 2.4\%$

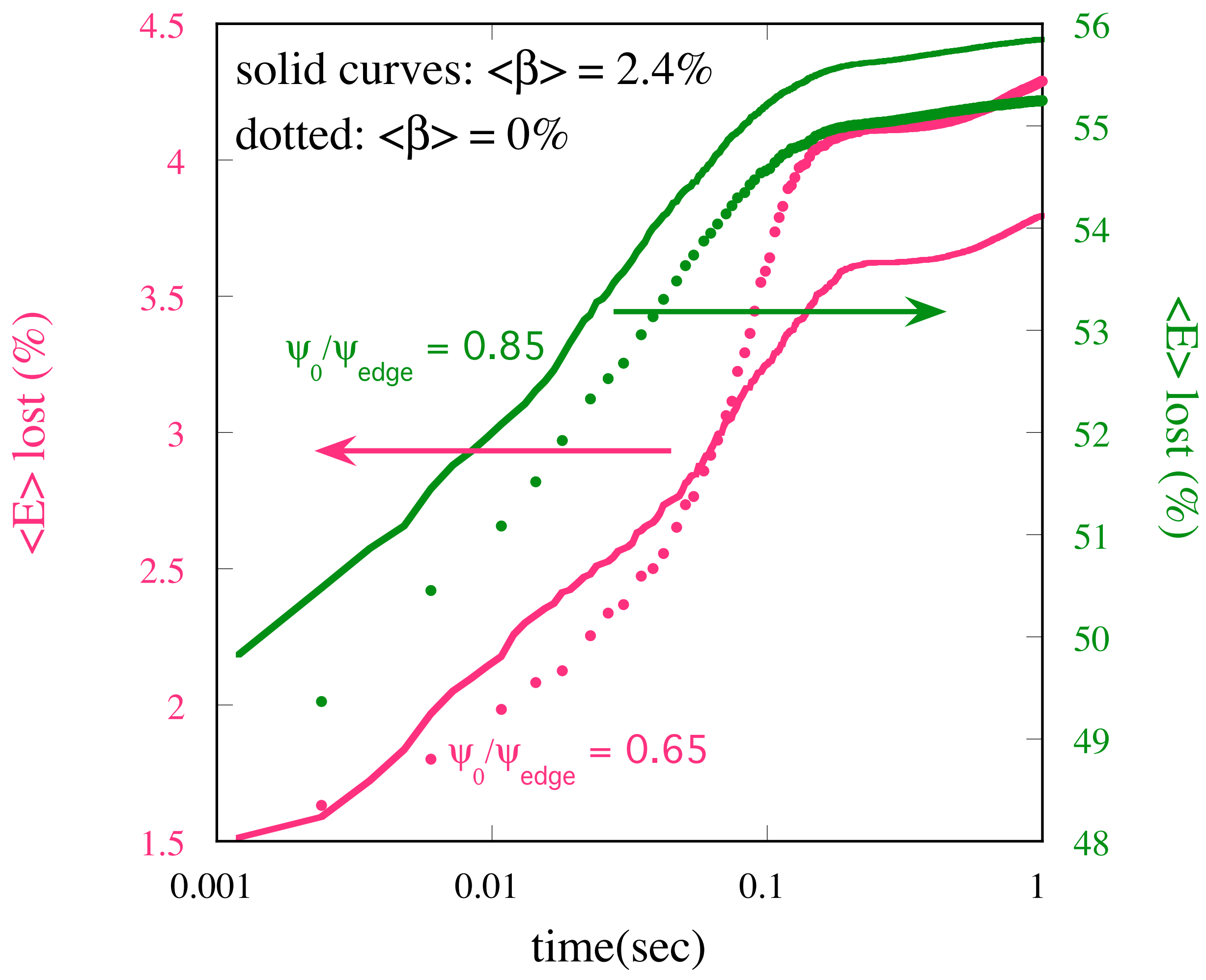


Test problem: follow slowing-down of alpha populations launched at various flux surfaces for $n(0) = 10^{20}\ m^{-3}$, $T_i(0) = T_e(0) = 20$ keV (more realistic centrally peaked profiles show zero losses)
10,240 alphas followed for 1 second physical time
up-down symmetric ITER fixed boundary equilibrium





Variation of losses with equilibrium <β>



Summary

- **New code (AE3D) developed for Alfven spectral analysis and mode structure calculations in stellarators**
  - Can calculate all or a subset of Alfvén spectrum
    - Up to 8000 eigenmodes have been kept per configuration
  - Mode density function vs. frequency
    - Density minima associated with more open gaps
  - 3D mode structure visualization
- **STELLGAP code has been upgraded for Alfvén-sound wave continua**
  - Regions identified where Alfvén and sound continua couple
  - Application to HSX shows that minima of coupled continua is somewhat higher in frequency
  - Further experimental information needed to identify observed mode
- **ITER rippled equilbria calculated with VMEC and used for alpha loss calculations**
  - Self-consistent finite β 3D model including ripple
    - future upgrades to include effects of ferritic steel inserts, RWM coils, etc.
  - Coupled to Monte Carlo alpha loss code (DELTA5D)
  - Can be extended to include turbulence/follow alphas to the 1st wall