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Computation of Ideal Perturbed Equilibria and its Applications

Jong-kyu Park,
Jonathan E. Menard, Allen H. Boozer[a], Michael J. Schaffer[b]

Princeton Plasma Physics Laboratory, Princeton, NJ08543
[a]Department of Applied Physics and Applied Mathematics, Columbia University, New York, NY10027
[b]General Atomics, San Diego, CA92186

I.I. Computation of ideal perturbed equilibria

- Ideal perturbed force balance equation

Three-component equations of plasma displacement $\vec{\xi}$

$$\vec{f}_{ideal}(\vec{\xi}) = \vec{0} = \vec{\nabla}\delta p + \delta\vec{J} \times \vec{B} + \vec{J} \times \delta\vec{B}$$

$$\vec{\nabla}\delta p = -\vec{\xi} \cdot \vec{\nabla} P - \gamma P(\vec{\nabla} \cdot \vec{\xi}) \qquad \delta\vec{B} = \vec{\nabla} \times (\vec{\xi} \times \vec{B}) \qquad \delta\vec{J} = \vec{\nabla} \times \vec{\nabla} \times (\vec{\xi} \times \vec{B}) / \mu_0$$

Can be obtained by the minimization of the perturbed potential in ideal MHD

$$\delta W = -\frac{1}{2}\int dx^3 \vec{f}_{ideal}(\vec{\xi}) \cdot \vec{\xi}$$

- Euler-Lagrange equation for $\vec{\xi} \cdot \vec{\nabla}\psi$ is solved in DCON

$$(\overleftrightarrow{F} \cdot \vec{\Xi}_\psi' + \overleftrightarrow{K} \cdot \vec{\Xi}_\psi)' - (\overleftrightarrow{K}^\dagger \cdot \vec{\Xi}_\psi' + \overleftrightarrow{G} \cdot \vec{\Xi}_\psi) = \vec{0}$$

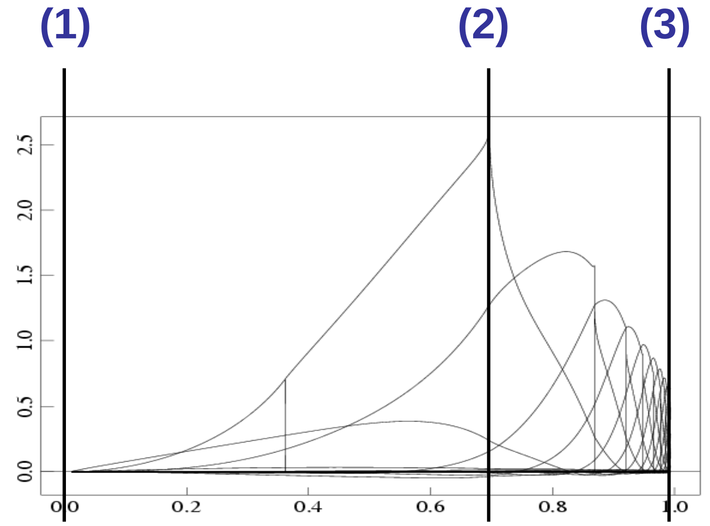
$$\vec{\Xi}_\psi \equiv \left\{ \xi^\psi_{mn}(\psi) \right\}$$

Boundary conditions are

(1) $\vec{\Xi}_\psi = \vec{0}$

(2) *Continuous, but* $(\delta\vec{B} \cdot \hat{n})_{mn} = 0$

(3) *Given* $(\vec{\xi} \cdot \hat{n}_b)$ *or* $(\delta\vec{B} \cdot \hat{n}_b)$

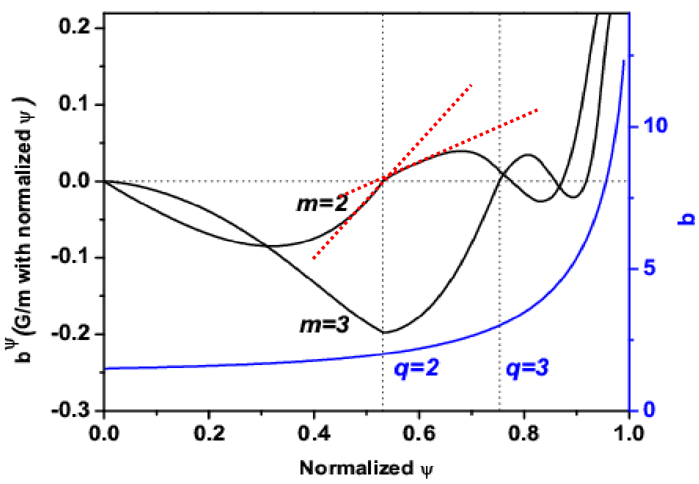


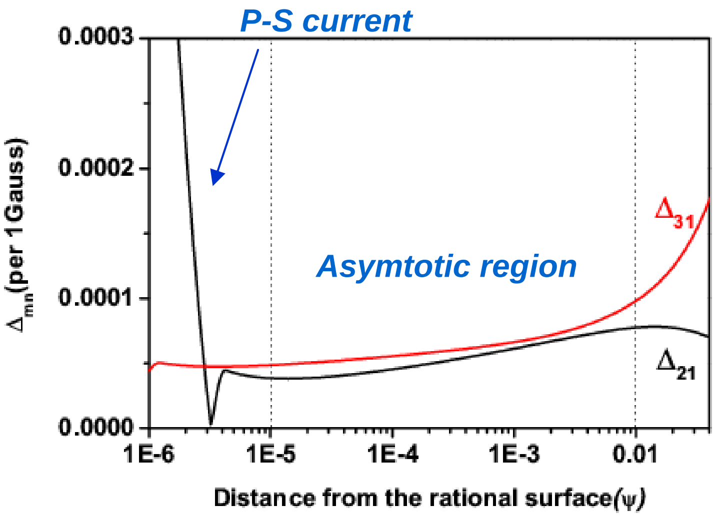
I.II. Boundary conditions at rational surfaces

- Ideal MHD constraint does not allow islands, so produces a singular current to preserve magnetic topology - $\left(\delta\vec{B}\cdot\vec{\nabla}\psi\right)_{mn} = 0 \;\; at \;\; q = m/n$

$$\Delta_{mn} = \left[\frac{\partial}{\partial\psi}\left(\frac{\delta\vec{B}\cdot\vec{\nabla}\psi}{\vec{B}\cdot\vec{\nabla}\varphi}\right)\right]_{mn} \Rightarrow \vec{j}_s = \frac{\Delta_{mn} i m e^{i(m\theta - n\varphi)}}{\mu_0 n^2\left(\oint dSB^2/\left|\vec{\nabla}\psi\right|^3\right)}\delta(\psi-\psi_{mn})\vec{B} \Rightarrow \left(\delta\vec{B}\cdot\hat{n}\right)_{mn}$$

Jump of tangential fields — *Parallel singular current preventing magnetic islands from opening* — *Total resonant normal field driving magnetic islands*





I.III. Boundary condition on the boundary

- DCON gives M neighboring perturbed equilibria with the associated total field on the plasma boundary

$$\vec{\xi}_i(\psi,\theta,\varphi),\ \left(\vec{\xi}_i\cdot\hat{\boldsymbol{n}}_b\right)(\theta,\varphi),\ \delta\vec{\boldsymbol{B}}(\psi,\theta,\varphi)=\vec{\nabla}\times(\vec{\xi}\times\vec{\boldsymbol{B}}),\ \ldots\ldots \Longrightarrow \delta\vec{\boldsymbol{B}}\cdot\hat{\boldsymbol{n}}_b,\ \ \hat{\boldsymbol{n}}_b\times\delta\vec{\boldsymbol{B}}$$

- The normal field is continuous across the boundary and gives the unique vacuum field outside

$$\delta\vec{\boldsymbol{B}}\cdot\hat{\boldsymbol{n}}_b \Longrightarrow \hat{\boldsymbol{n}}_b\times\delta\vec{\boldsymbol{B}}^v$$

- If an wall at infinity is taken as a boundary condition, then the jump of the tangential field on the boundary is precisely the representative surface current on the control surface

$$\mu_0\vec{\boldsymbol{K}}=\hat{\boldsymbol{n}}_b\times\delta\vec{\boldsymbol{B}}-\hat{\boldsymbol{n}}_b\times\delta\vec{\boldsymbol{B}}^v$$

- The surface current gives the external field

$$\mu_0\vec{\boldsymbol{K}}=\vec{\nabla}\times\delta\vec{\boldsymbol{B}}^x \Longrightarrow \delta\vec{\boldsymbol{B}}^x\cdot\hat{\boldsymbol{n}}_b$$

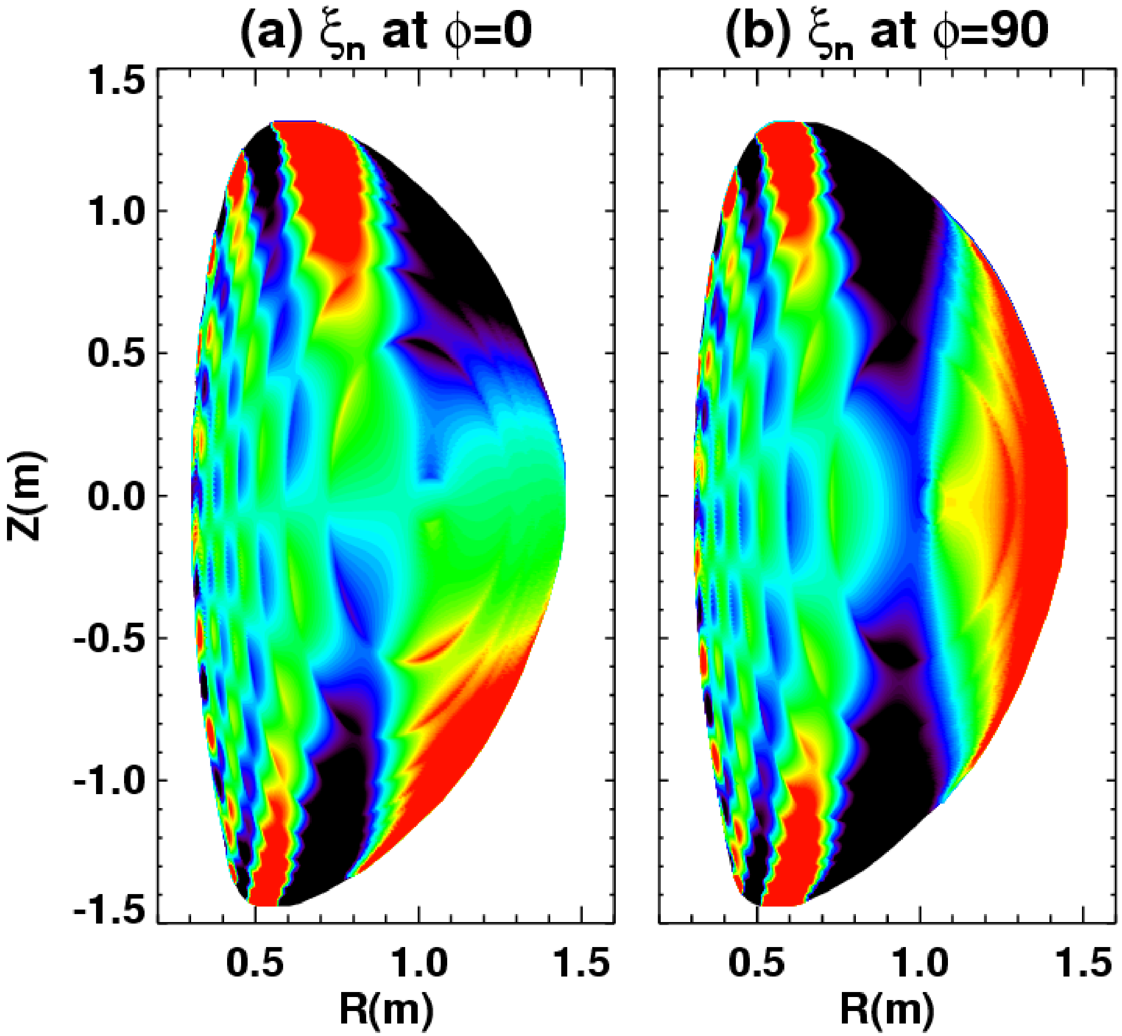
$$(\delta\vec{\boldsymbol{B}}\cdot\hat{\boldsymbol{n}}_b)=\hat{\Lambda}\left[\vec{\boldsymbol{K}}\right]$$

$$(\delta\vec{\boldsymbol{B}}^x\cdot\hat{\boldsymbol{n}}_b)=\hat{\boldsymbol{L}}\left[\vec{\boldsymbol{K}}\right]$$

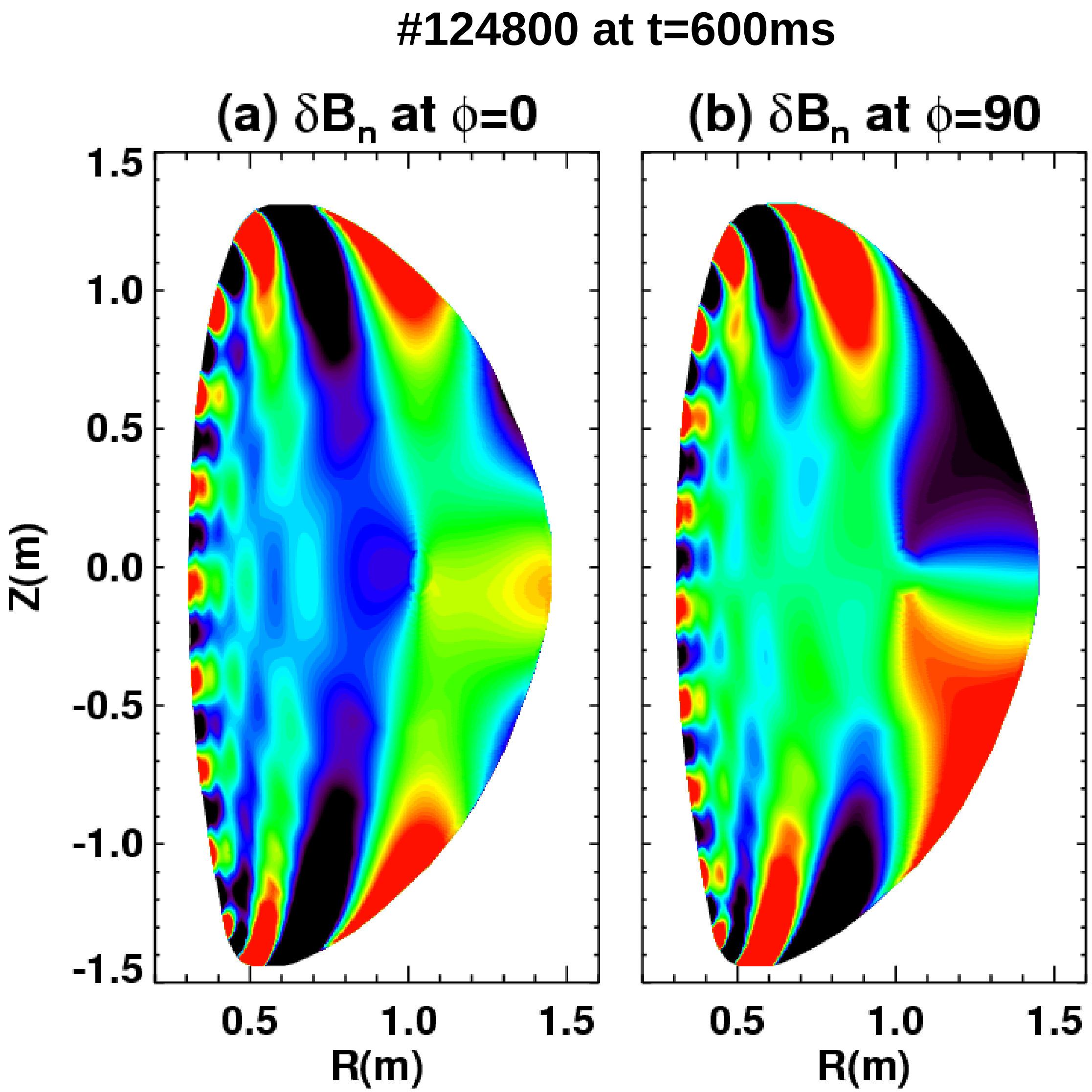
$$(\delta\vec{\boldsymbol{B}}\cdot\hat{\boldsymbol{n}}_b)=\hat{\boldsymbol{P}}\left[(\delta\vec{\boldsymbol{B}}^x\cdot\hat{\boldsymbol{n}}_b)\right]$$

I.IV. (1) Perturbed normal displacement

#124800 at t=600ms

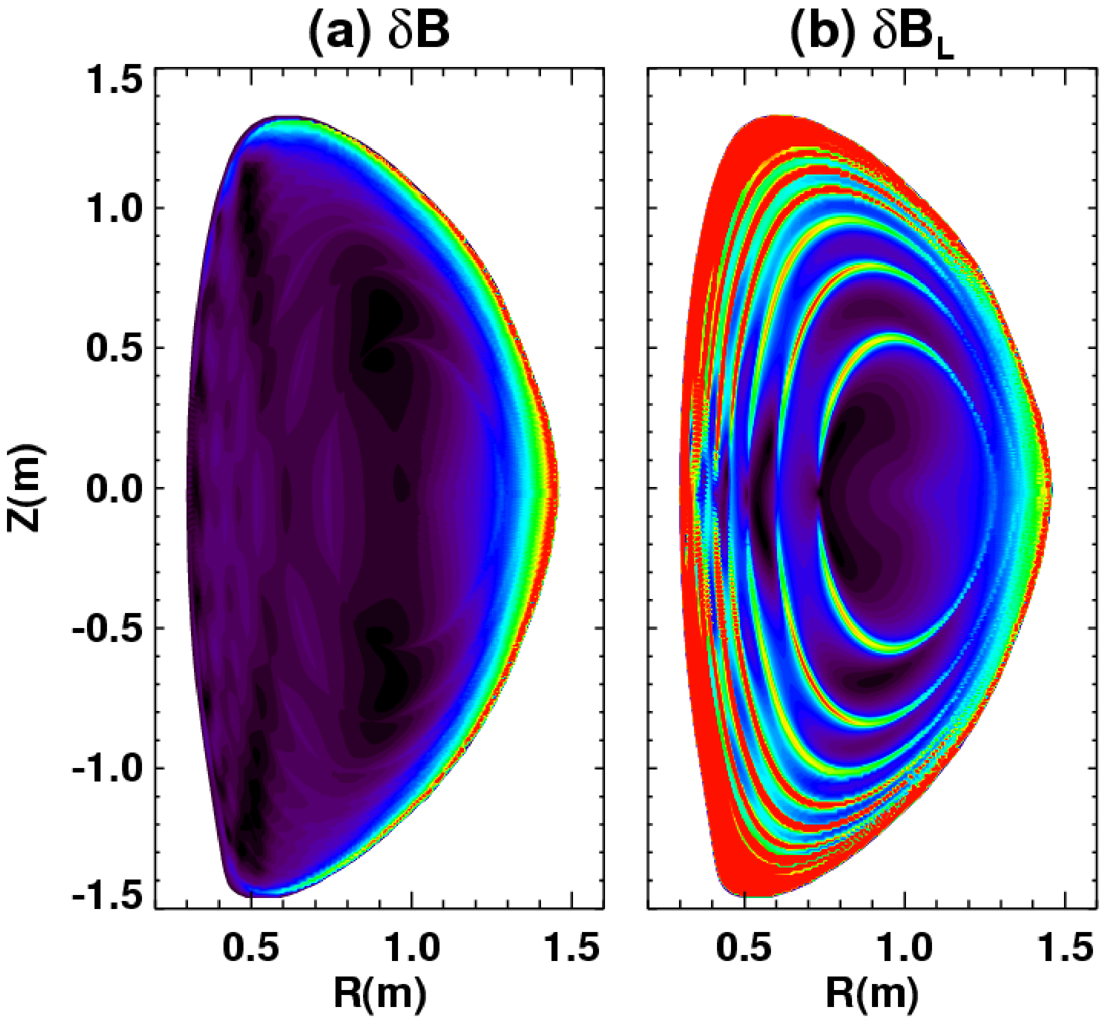


I.IV. (2) Perturbed normal field



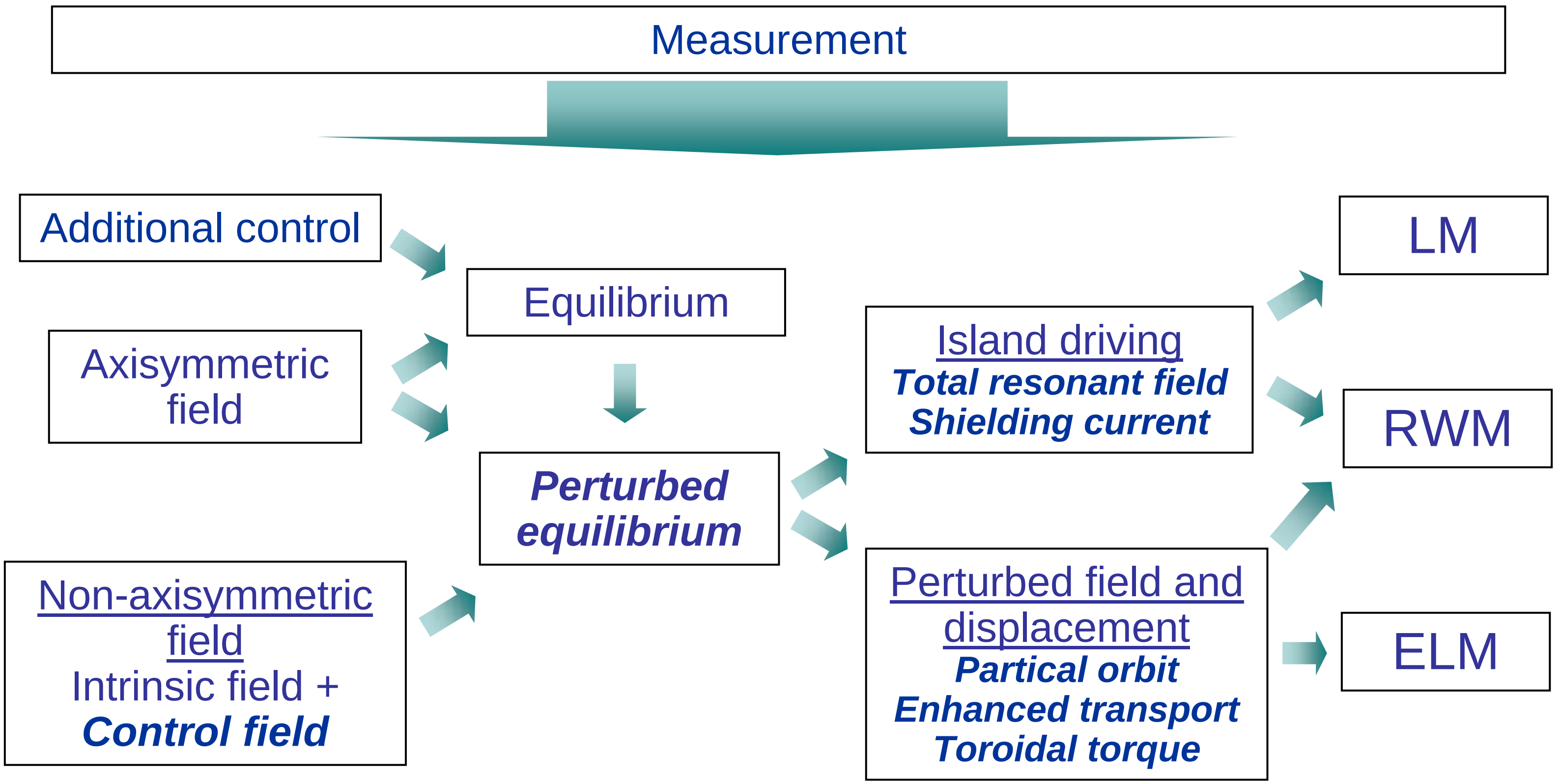
I.IV. (3) Perturbed |B|

#124800 at t=600ms



I.V. 3D equilibrium reconstruction in tokamaks

- IPEC can be used for three-dimensional equilibrium reconstruction in tokamaks when non-axisymmetric external field and current are specified



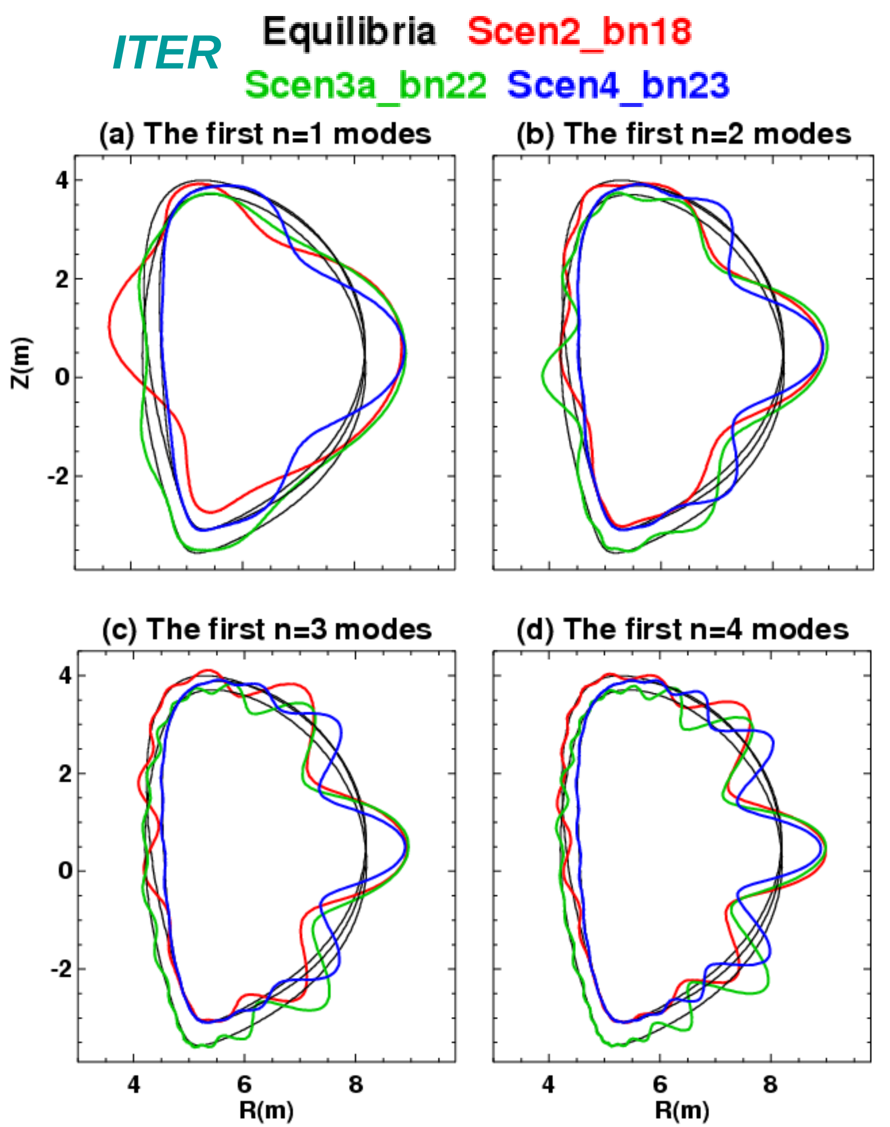
II.I. LM: Error field correction scheme

- The i^{th} important external field can be defined by i^{th} singular eigenvector of coupling matrix $\overleftrightarrow{\mathbb{C}}$

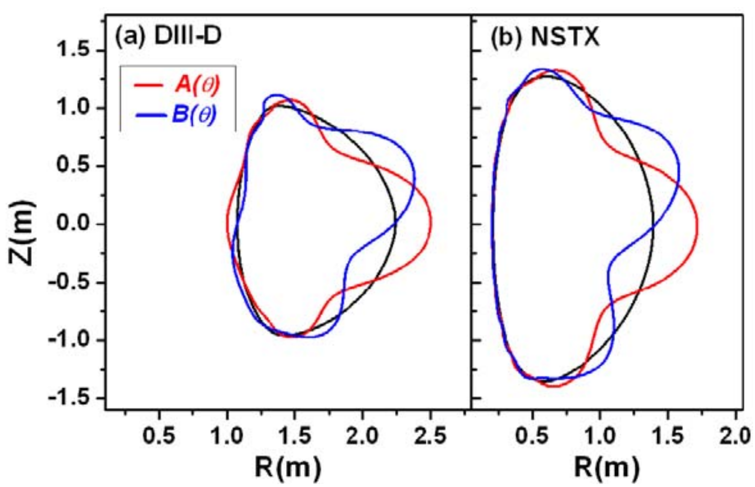
$$\left(\delta\vec{B}\cdot\hat{n}\right)_{mn}$$

$$\left(\delta\vec{B}^{x}\cdot\hat{n}_{b}\right)(\theta,\varphi)=Re\left(\sum_{m}\Phi^{x}_{mn}w(\theta)e^{i(m\theta-n\varphi)}\right)$$

$$\vec{B}=\overleftrightarrow{\mathbb{C}}\cdot\vec{\Phi}^{x}$$



$$\left(\delta\vec{B}^{x}\cdot\hat{n}_{b}\right)=A(\theta)\cos\phi+B(\theta)\sin\phi$$



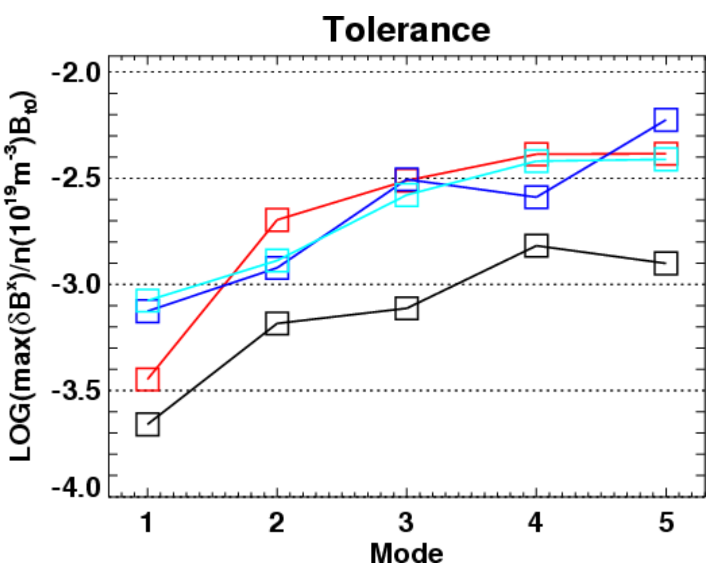
II.II. LM: Error field correction in ITER

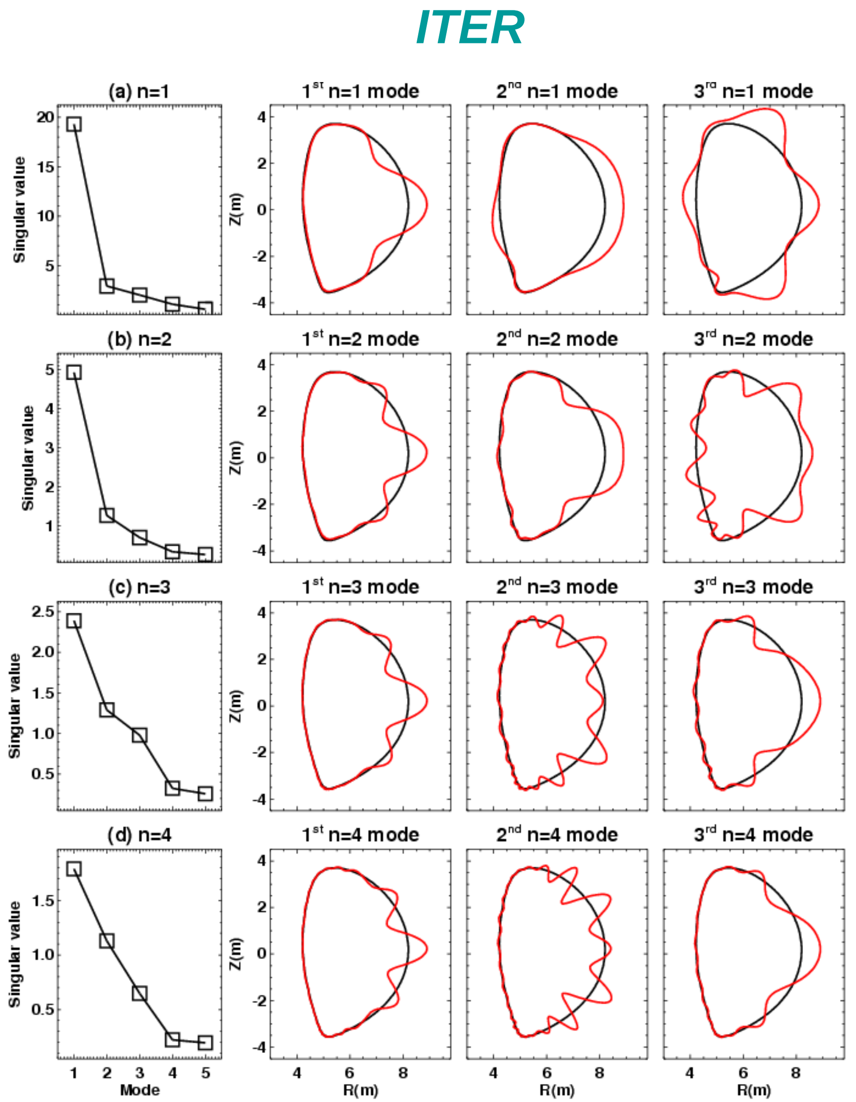
- When plasma evolves between possible scenarios, then combined coupling matrix can be used to identify ith important external field

$$\vec{B}^{combined} = \overleftrightarrow{\mathbb{C}}^{combined} \cdot \vec{\Phi}^{x}$$

- Tolerances guided by the combined coupling matrix are:

using $\left(\delta\vec{B}\cdot\hat{n}\right)_{mn}^{critical}(Gauss) \cong n_e(10^{18}m^{-3})$





III.I. RWM: Plasma response to external field

- Plasma response to external field is generally determined by energy and torque required to produce perturbations

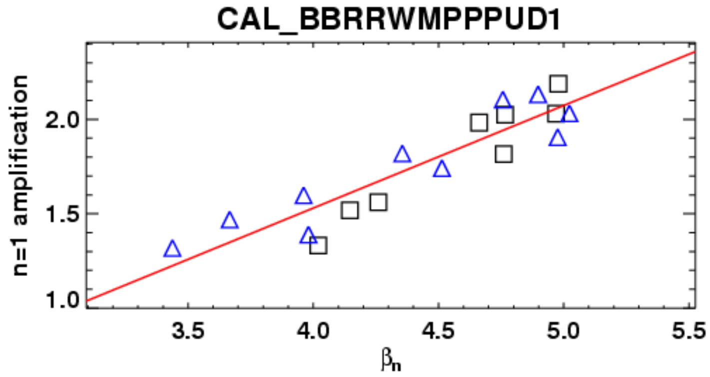
$$\delta W = \int \vec{j} \cdot \vec{A} dx^3 = \frac{1}{4}\left(\vec{I}^{\dagger} \cdot \vec{\Phi} + \vec{\Phi}^{\dagger} \cdot \vec{I} \right)$$

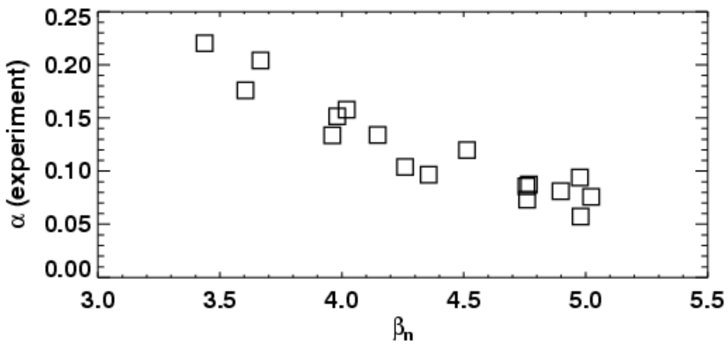
$$\tau_{\varphi} = \int \left(\vec{j} \times \vec{B} \right) \cdot \frac{\partial \vec{x}}{\partial \varphi} dx^3 = i\frac{n}{2}\left(\vec{I}^{\dagger} \cdot \vec{\Phi} - \vec{\Phi}^{\dagger} \cdot \vec{I} \right) \longrightarrow \vec{\Phi} = \vec{P} \cdot \vec{\Phi}^{x} \longrightarrow \vec{\Phi} \approx -\frac{1}{s - i\alpha} \vec{\Phi}^{x}$$

$$\vec{\Phi}^{x} = \overleftrightarrow{L} \cdot \vec{I}$$

- Torque parameter α can be estimated by measuring amplification of applied field in experiment and by calculating s in IPEC, assuming $\alpha << 1$

$$\frac{\Phi_w}{\Phi_w^x} = -\frac{\gamma_w}{v_g + i\omega_r} = \frac{(1-c)\left\{(d-s)^2 + \alpha^2\right\}}{\left\{s(d-s) - \alpha^2\right\} - id\alpha} \quad ,where \;\; d = c/(1-c)$$





III.II. RWM: Torque by neoclassical theory

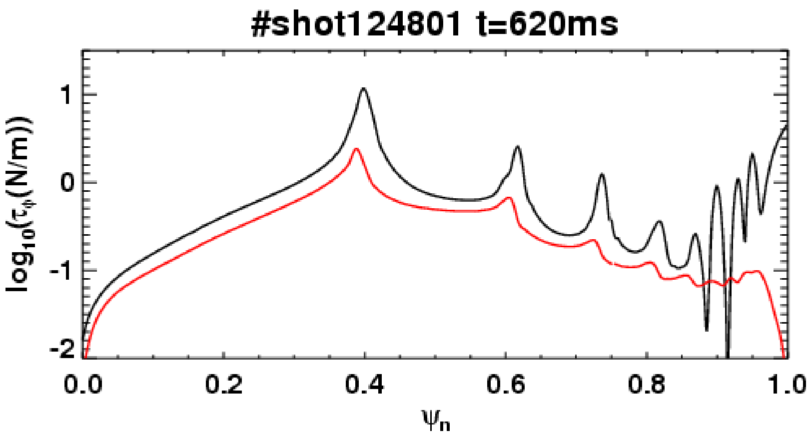
- Torque parameter can be calculated by

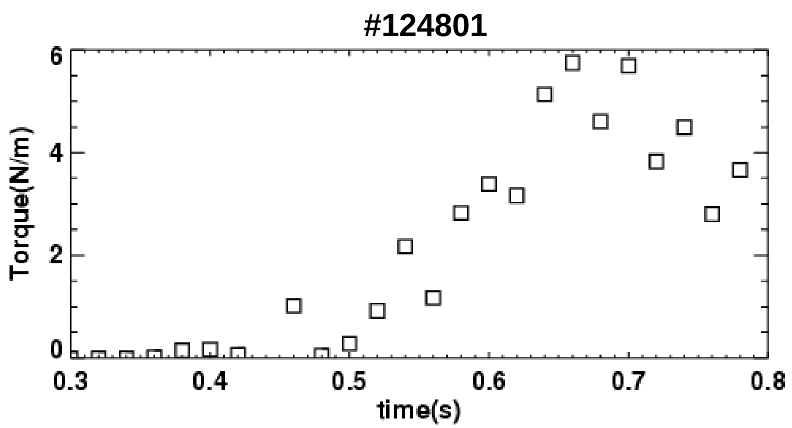
$$\tau_\varphi = \int d\psi \langle R \rangle \left\langle \frac{dV}{d\psi} \right\rangle \left\langle C_{eq1} \frac{p_i}{V_{Ti}} \Omega_\phi q \sum_{n,m\neq 0} \left(n^2 |\delta B_{nm}|^2 \frac{\mu_{ps1}}{\frac{2\sqrt{\pi}}{3} \frac{\nu_i}{V_{Ti}/R_0 q} + \mu_{ps1} |m - nq|} \right) + C_{eq2} \frac{p_i}{\nu_i} \Omega_\phi \sum_{n,m\neq 0} \left(n^2 |\delta B_{nm}|^2 W_{nm} \right) \right\rangle$$

- The variation of field strength is calculated on perturbed flux surfaces

$$B\delta B_L = \left(\vec{B} \cdot \delta \vec{B} \right)_L = \left(\vec{B} \cdot \delta \vec{B} \right) + B(\vec{\xi} \cdot \vec{\nabla} B) \gg B\delta B = \left(\vec{B} \cdot \delta \vec{B} \right) \gg B\delta B^x = \left(\vec{B} \cdot \vec{b}^x \right)$$

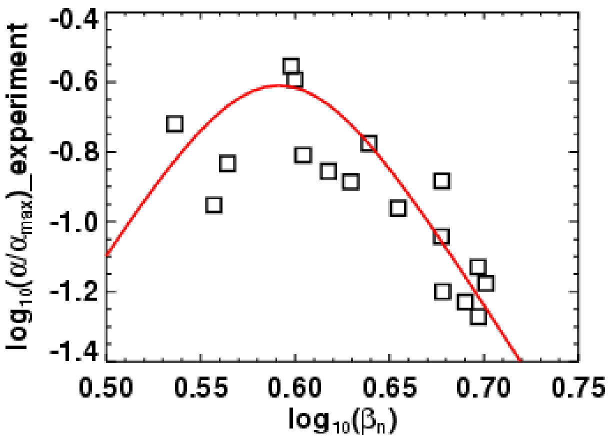
- Torque can not be larger than the maximum, $\oint R(\delta \vec{B}^p \cdot \hat{n}_b)(\delta \vec{B}^x \cdot \hat{n}_b) da$
- Theory and experiment gives different α / α_{max}

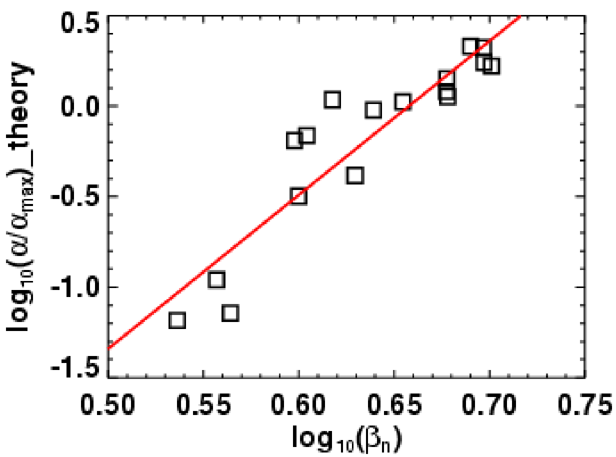




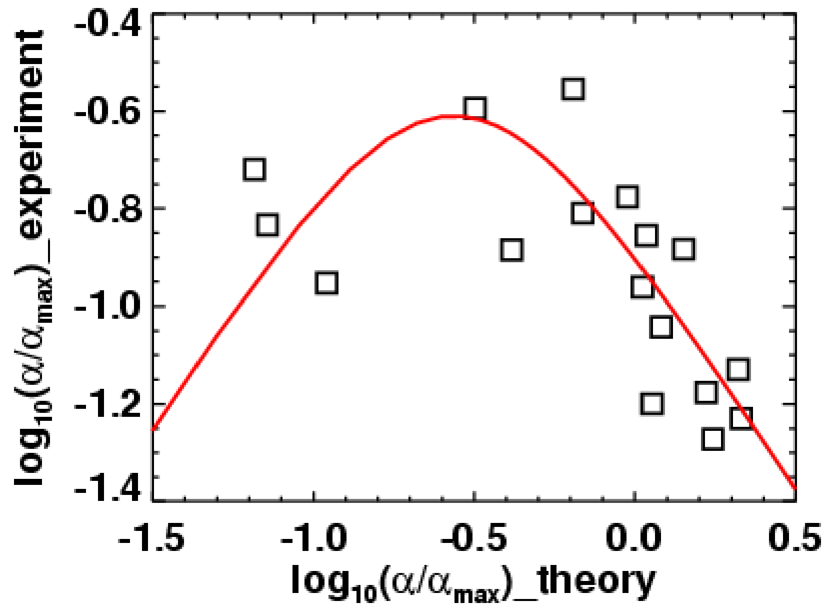
III.III. RWM: Self-shielding effect

- Anti-correlation between theory and experiment in high density regime indicates plasma self-shielding effect, that is, shielding of external perturbation by amplified viscous force





$$\alpha_{exp} / \alpha_{max} = \frac{\boldsymbol{a}\left(\alpha_{the} / \alpha_{max}\right)}{1 + \boldsymbol{b}\left(\alpha_{the} / \alpha_{max}\right)^2}$$



Summary

I. IPEC (Ideal Perturbed Equilibrium Code) has been developed modifying DCON and VACUUM, and constructing interface between - external and total field.
This code can be used to reconstruct perturbed equilibria in tokamaks

II. Locked Mode (LM) in tokamaks can be effectively mitigated by resolving i^{th} important mode minimizing total resonant field on rational surfaces.
The new approach is applied to ITER

III. Plasma response to external perturbation can be calculated precisely in ideal MHD, and is important to describe Resistive Wall Mode (RWM).
Calculation of perturbed energy and torque in theory and experiment shows self-shielding effect of plasma by amplified viscous force