Neoclassical toroidal viscosity for low-density ohmic plasmas in DIII-D

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Presented at

APS-DPP 2007

Orlando, Fl

12-16 November 2007

Abstract

A recent model[a] for field error penetration that includes both resonant and non-resonant perturbed 3-D magnetic fields has for the first time obtained quantitative agreement with empirical scaling studies of the error-field penetration threshold with electron density. The relevance of the new model relies on the error-field induced neoclassical toroidal viscosity [NTV] being comparable to cross-field diffusive viscosity near a resonant surface of interest (e.g., $q = 2$).

The strength and harmonic structure of NTV for low density ohmic plasmas on DIII-D are determined from intrinsic vacuum error-field data. Preliminary analysis has shown that NTV in DIII-D is dominated by non-resonant modes. We neglect the plasma response in this initial investigation. An effective cross-field momentum transport owing to NTV is determined, for future comparison with possible cross-field momentum transport rates in ohmic discharges.

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[a] A.J. Cole, C.C. Hegna, and J.D. Callen, Phys. Rev. Lett. **99**, 065001 (2007).

Non-resonant ($q \neq m/n$) non-axisymmetric $\vec{B}$ components induce nonambipolar radial ion loss via ripple trapping or banana drift effects: I

- Error-field induced radial nonambipolar particle flux[a] has the form (prime denotes d/dr)

$$\Gamma_{ri}^{na} \propto -\nu_{\|} B_{NR}^2 \left[\frac{p_i'}{p_i} - \frac{Z_i e E_r}{T_i} + \# \frac{T_i'}{T_i} \right] \propto \left\langle \vec{e}_\phi \cdot \vec{\nabla} \cdot \overleftrightarrow{\Pi} \right\rangle ,$$

where $\nu_{\|}$ is a regime-dependent damping rate, and B_{NR}^2 is a sum over products of non-resonant error-field harmonics.

- However, radial force balance for timescales $> \tau_A$,

$$0 \simeq n_i Z_i e \left(E_r + V_\theta B_\phi - V_\phi B_\theta \right) - p_i' \Longrightarrow \boxed{V_\phi = \frac{E_r}{B_\theta} + \frac{B_\phi}{B_\theta} V_\theta - \frac{p_i'}{n_i Z_i e}} ,$$

insures E_r and V_ϕ are interdependent.

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[a] K.C. Shaing, Phys. Plasmas **10**, 1443 (2003).

Non-resonant ($q \neq m/n$) non-axisymmetric $\vec{B}$ components induce nonambipolar radial ion loss via ripple trapping or banana drift effects: II

- Rewrite nonambipolar flux as

$$\Gamma_{ri}^{na} \propto \left(V_\phi - \frac{B_\phi}{B_\theta} V_\theta - \# \frac{T_i'}{T_i} \right) \Longrightarrow (V_\phi - V_*^{NC}) \quad \text{where} \quad V_*^{NC} \simeq \frac{1.17 + \#}{Z_i e B_\theta} T_i'.$$

  - Case i: $V_\phi > V_*^{NC} \Longrightarrow$ radial ion *loss* ($\Gamma_{ri} > 0$) $\Longrightarrow$ NTV causes braking.
  - Case ii: $V_\phi < V_*^{NC} \Longrightarrow$ radial ion *flow towards core* ($\Gamma_{ri} < 0$) $\Longrightarrow$ NTV causes "spin-up" to V_*^{NC}.

- Key feature: NTV brakes plasma to V_*^{NC} (counter to I_p) not zero!!!

Neoclassical viscous torque [NTV] affects toroidal flow equation

$$\frac{\partial V_\phi}{\partial t} \simeq -\frac{1}{\rho_m}\left\langle \vec{e}_\phi \cdot \vec{\nabla} \cdot \overleftrightarrow{\Pi} \right\rangle + \ldots \simeq -\frac{\omega_{ti}^2}{\nu_i}\left|\frac{\tilde{B}_{NR}^{eff}}{B_\phi}\right|^2 \left(V_\phi - V_*^{NC}\right) + \ldots$$

in the $1/\nu$ regime ($\omega_E < \nu_i/\epsilon < \sqrt{\epsilon}\,\omega_{ti}$) where $V_*^{NC} \simeq 3.5/\left(Z_i e\, B_\theta\right)[dT_i/dr]$. [a,b]

$$\left|\frac{\tilde{B}_{NR}^{eff}}{B_\phi}\right|^2 \propto q^2(r)\epsilon^{3/2} \sum_{nmm'} n^2 B_{nmm'} \int_0^1 d\kappa^2 \frac{F_{nmc}(\kappa) F_{nm'c}(\kappa)}{\mathsf{E}(\kappa) - (1-\kappa^2)\mathsf{K}(\kappa)}.$$

Here[a] $B_{nmm'} \equiv (b_{nmc} b_{nm'c} + b_{nms} b_{nm's})$,

$F_{nmc}(\kappa) = \oint d\Theta \sqrt{\kappa^2 - \sin^2(\Theta/2)}\,\cos[(m-nq)\Theta]$, κ is a normalized pitch angle variable, and the b_{nmc}, b_{nms} are the non-resonant harmonic decomposition of the perturbed total $|B|$ defined by $B = B_0\left(1 + \sum_{m,n}[b_{nmc}\cos\alpha + b_{nms}\sin\alpha]\right)$, with $\alpha = (m\Theta - n\zeta)$.

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[a] K.C. Shaing, Phys. Plasmas **10**, 1443 (2003).

[b] Helically trapped particles are collisional, toroidally trapped particles are collisionless.

Salient features of neoclassical toroidal viscosity

- Non-local: acts throughout plasma—unlike resonant electromagnetic torque which acts only on resonant surface.

- Acts to maintain plasma flowing at V_*^{NC}, not lock to stationary coil/wall frame as T_{EM} does.

First step: calculate $\tilde{B}^{eff}_{NR}/B_\phi$ for vacuum fields on DIII-D

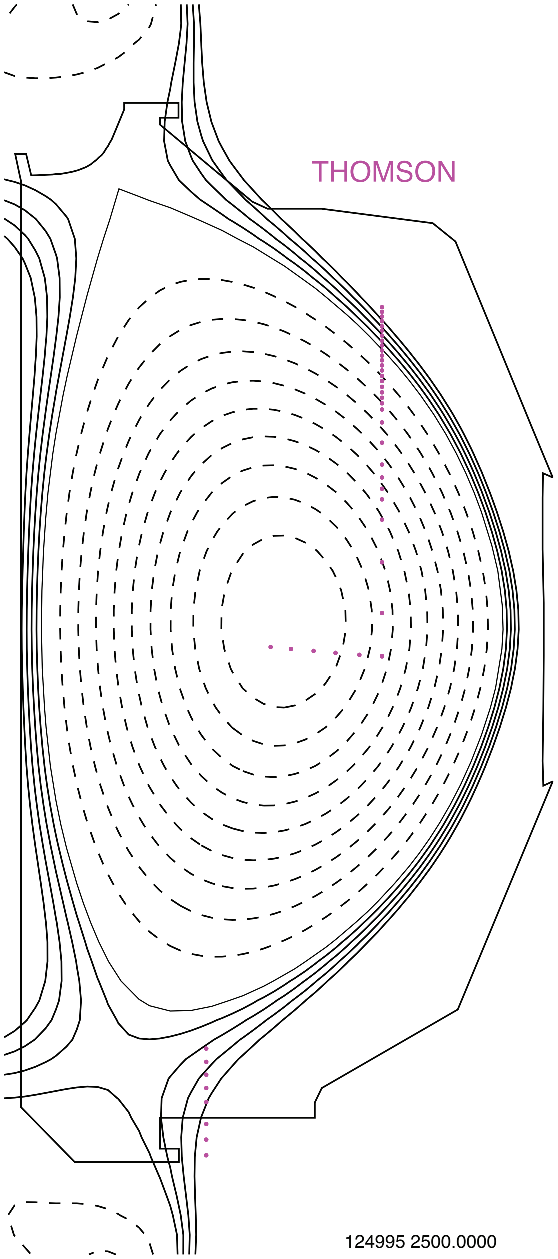
- We need $\tilde{B}^{eff}_{NR}/B_\phi$ on a flux surface:

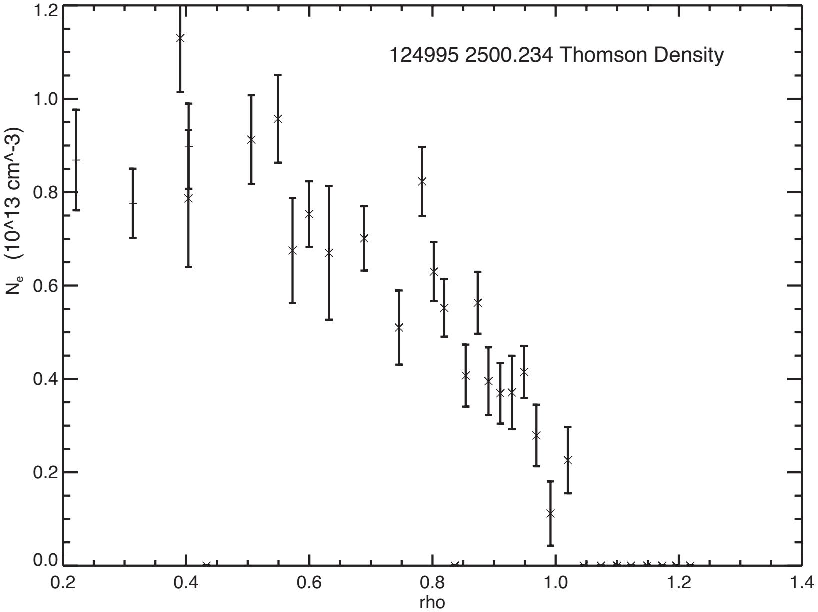
$$B_\lambda \equiv \sum_n \sum_{mm'} n^2 \underbrace{(b_{nmc}b_{nm'c} + b_{nms}b_{nm's})}_{\text{Data input: M. Schaffer}} \overbrace{\int_0^1 d\kappa^2 \frac{F_{nmc}(\kappa)F_{nm'c}(\kappa)}{\mathsf{E}(\kappa) - (1-\kappa^2)\mathsf{K}(\kappa)}}^{\text{Pitch angle integral: Scilab}}$$

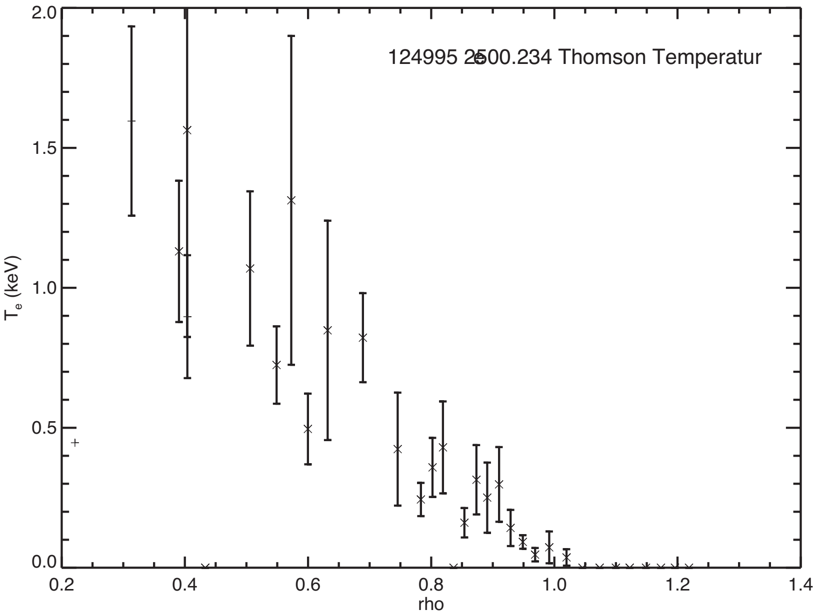
- Constructed Scilab[a] routine to handle normalized pitch angle integral given matrix inputs of b_{nmc}, b_{nms} data. In principle, code can handle any input magnetic field data.
- Initial study: use sample DIII-D equilibrium from EFIT to construct flux surfaces,
- using Surfmn code (M. Schaffer) Fourier decompose $|B|$ on flux surfaces for 3 test cases:
  - Known intrinsic DIII-D error-field, calculated from source models calibrated against empirical data,
  - known intrinsic error + 'Optimum' I-Coil correction,
  - and known intrinsic error + 'Optimum' C-Coil correction.

[a] Matlab clone, see www.scilab.org.

Equilibrium 124995.02500 used for all surface Fourier decompositions



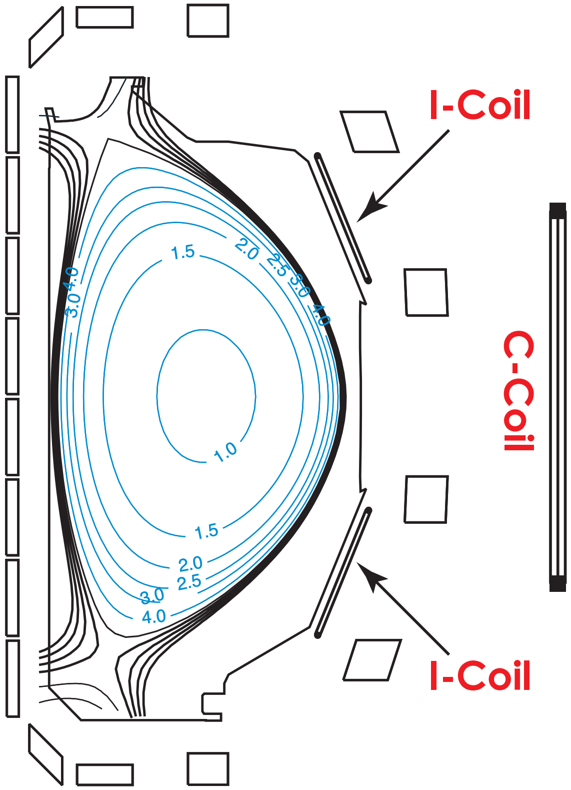


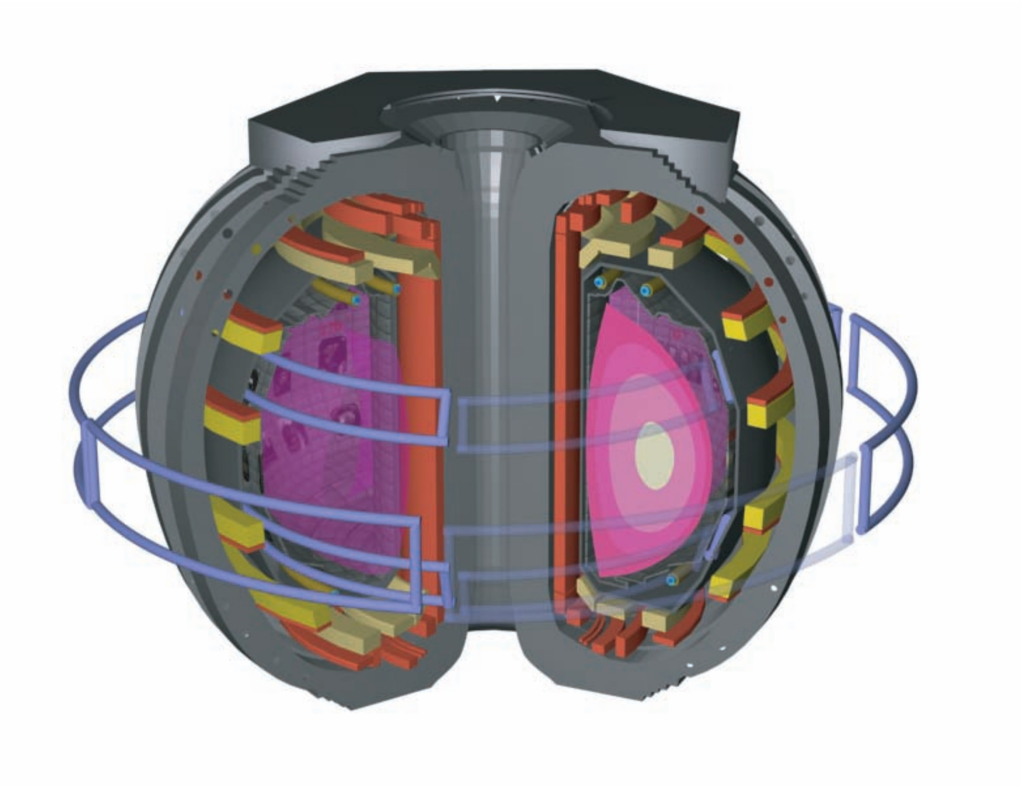


Case i: Known intrinsic field error in DIII-D

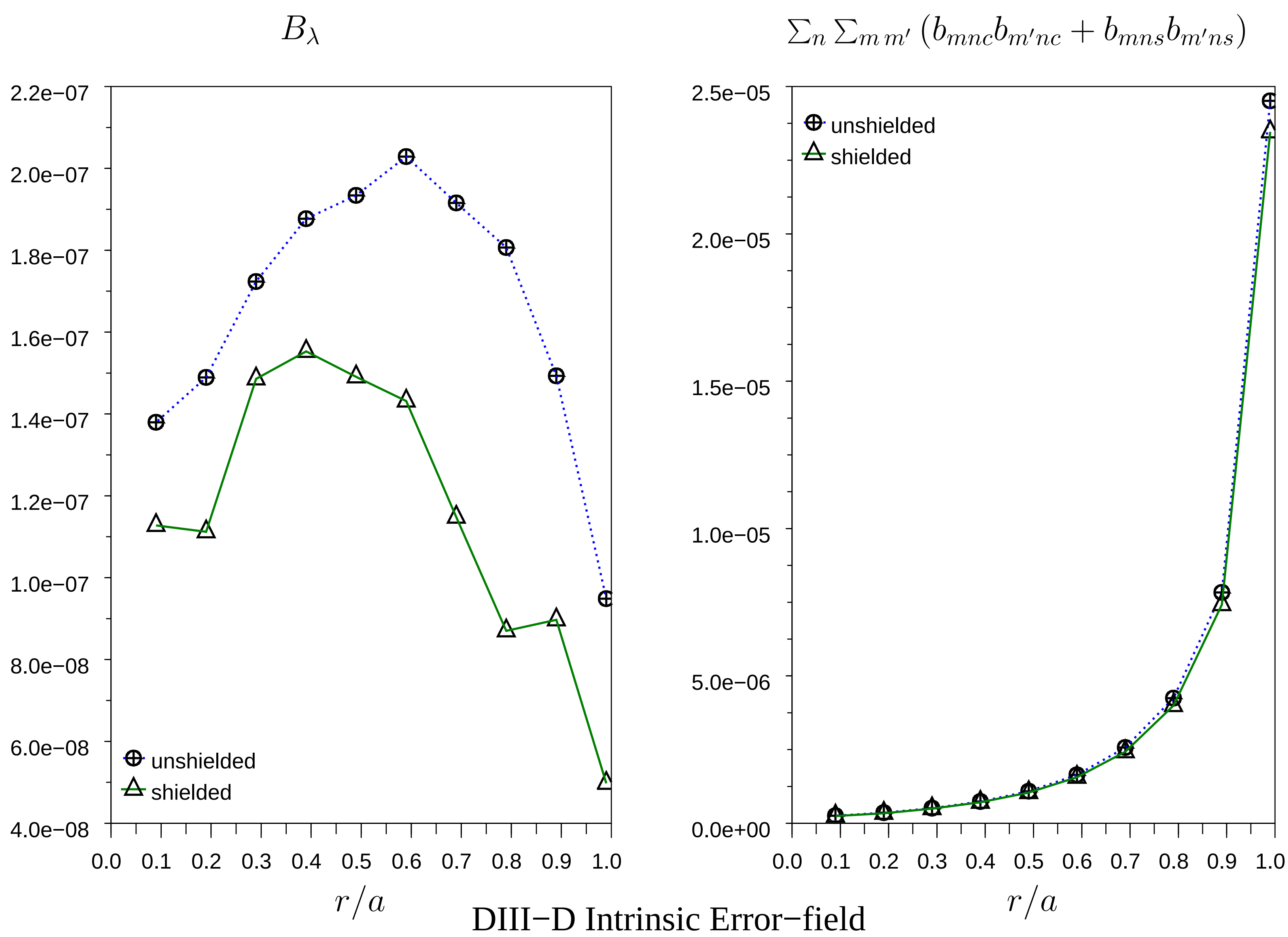
Known sources of intrinsic error-field since 2006 campaign:

- Misalignment between each of the 18 F-coils (poloidal field coils) and the B-coil (TF coil).
- Remaining 'old style' B-coil current feed, and the 10 times smaller error field from the improved current feed where the beam line was moved.

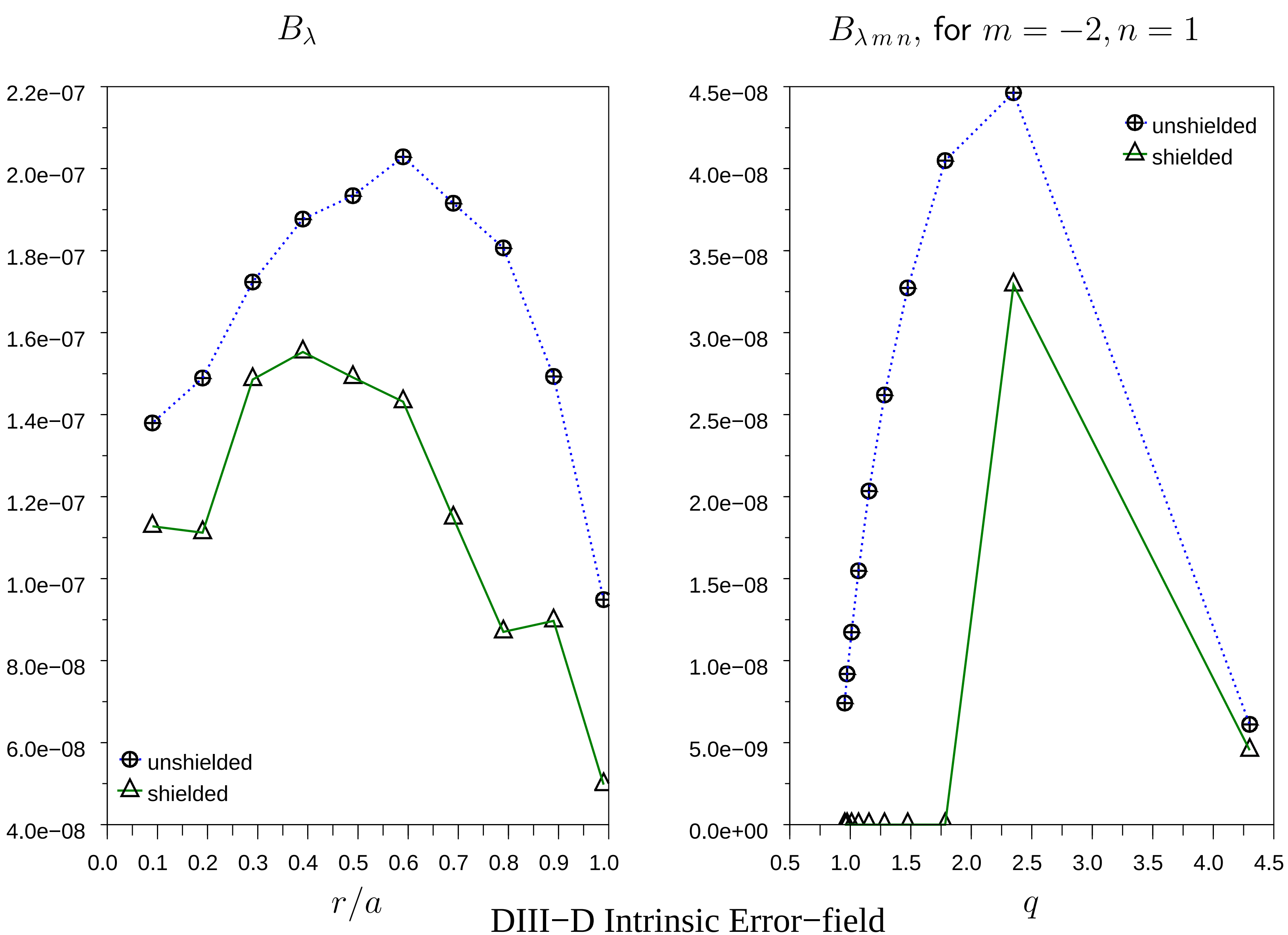




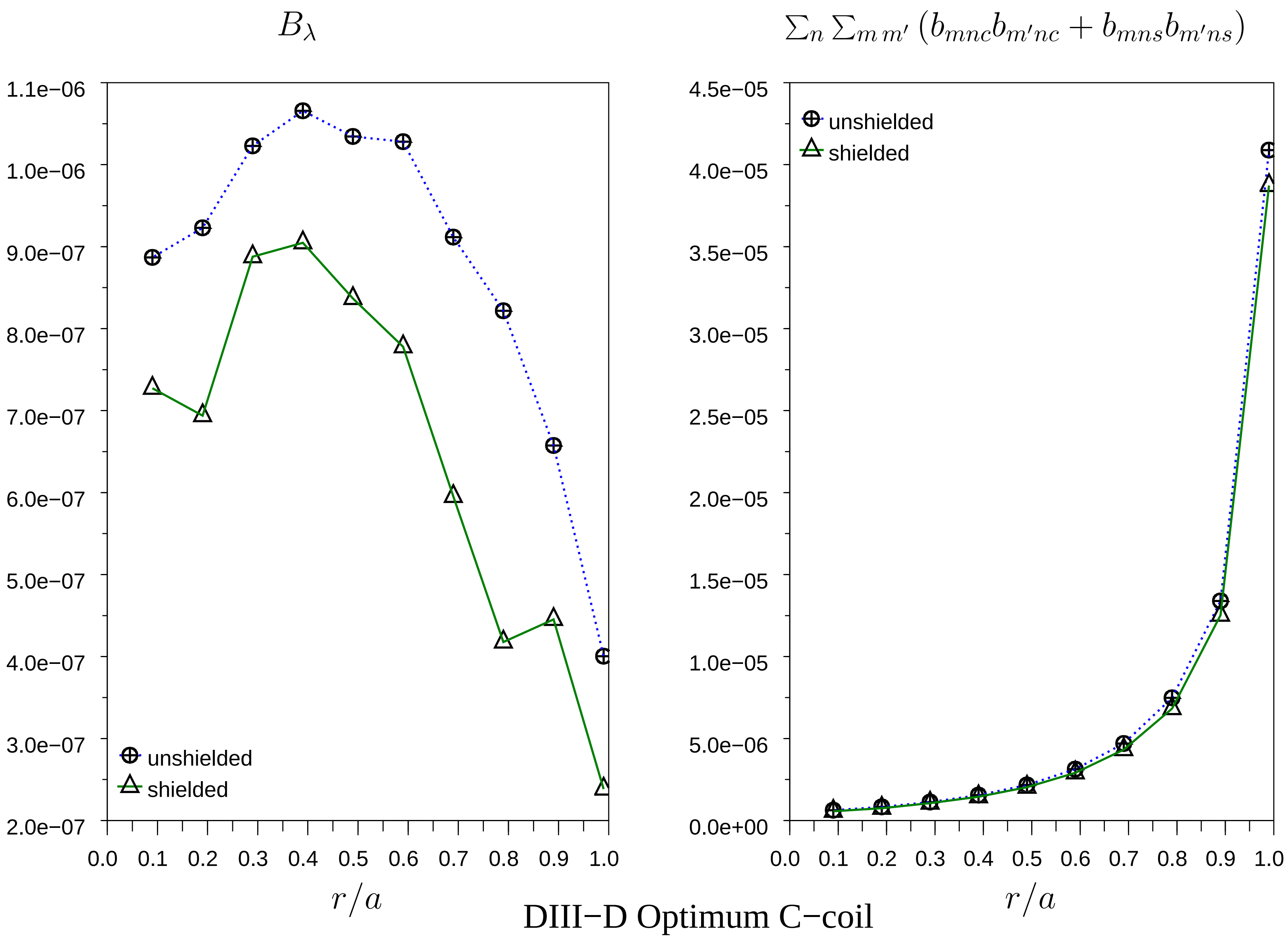
Plot of B_λ for known intrinsic error-field i



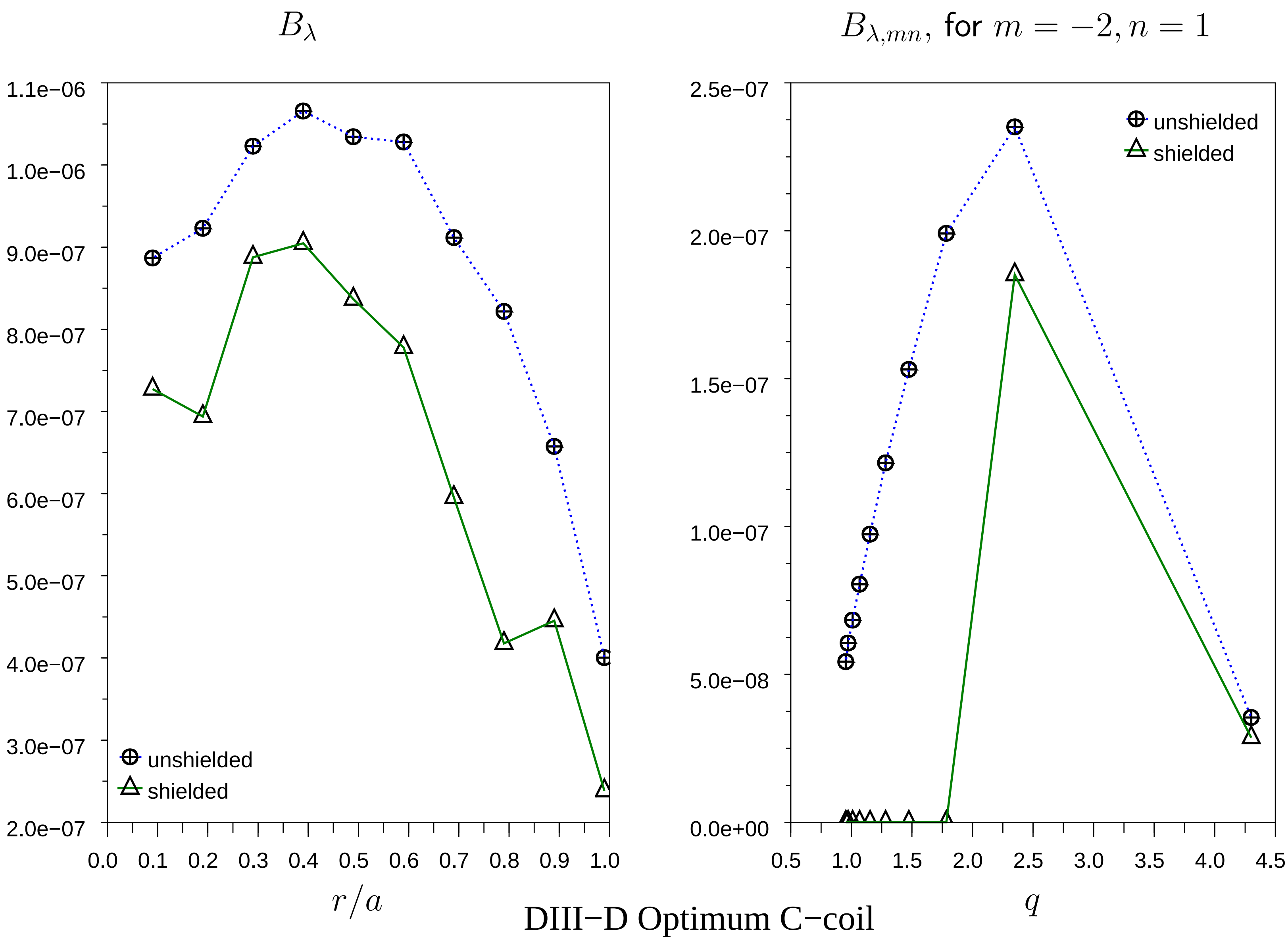
Plot of B_λ for known intrinsic error-field ii



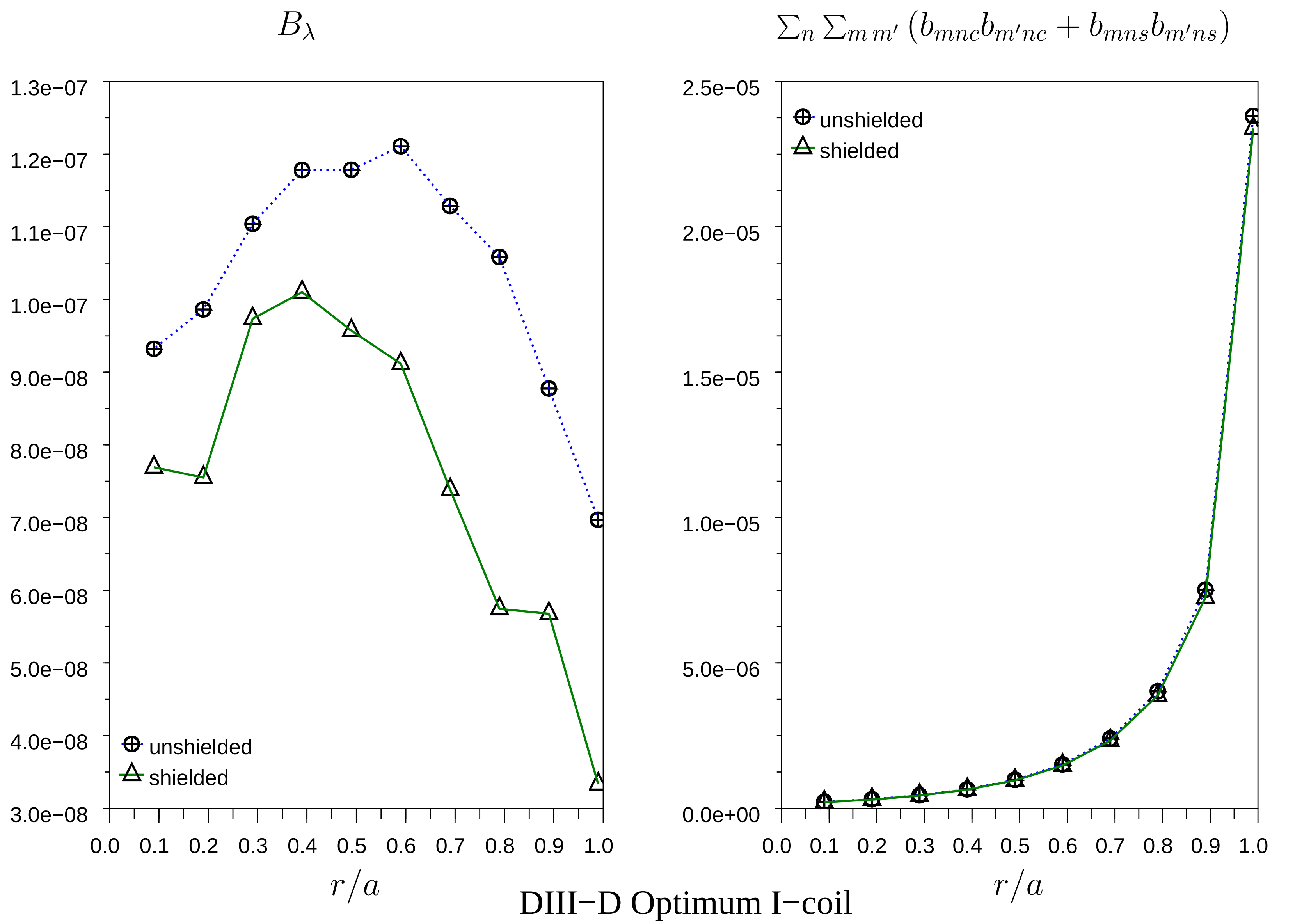
Plot of B_λ for optimum C-coil error correction i



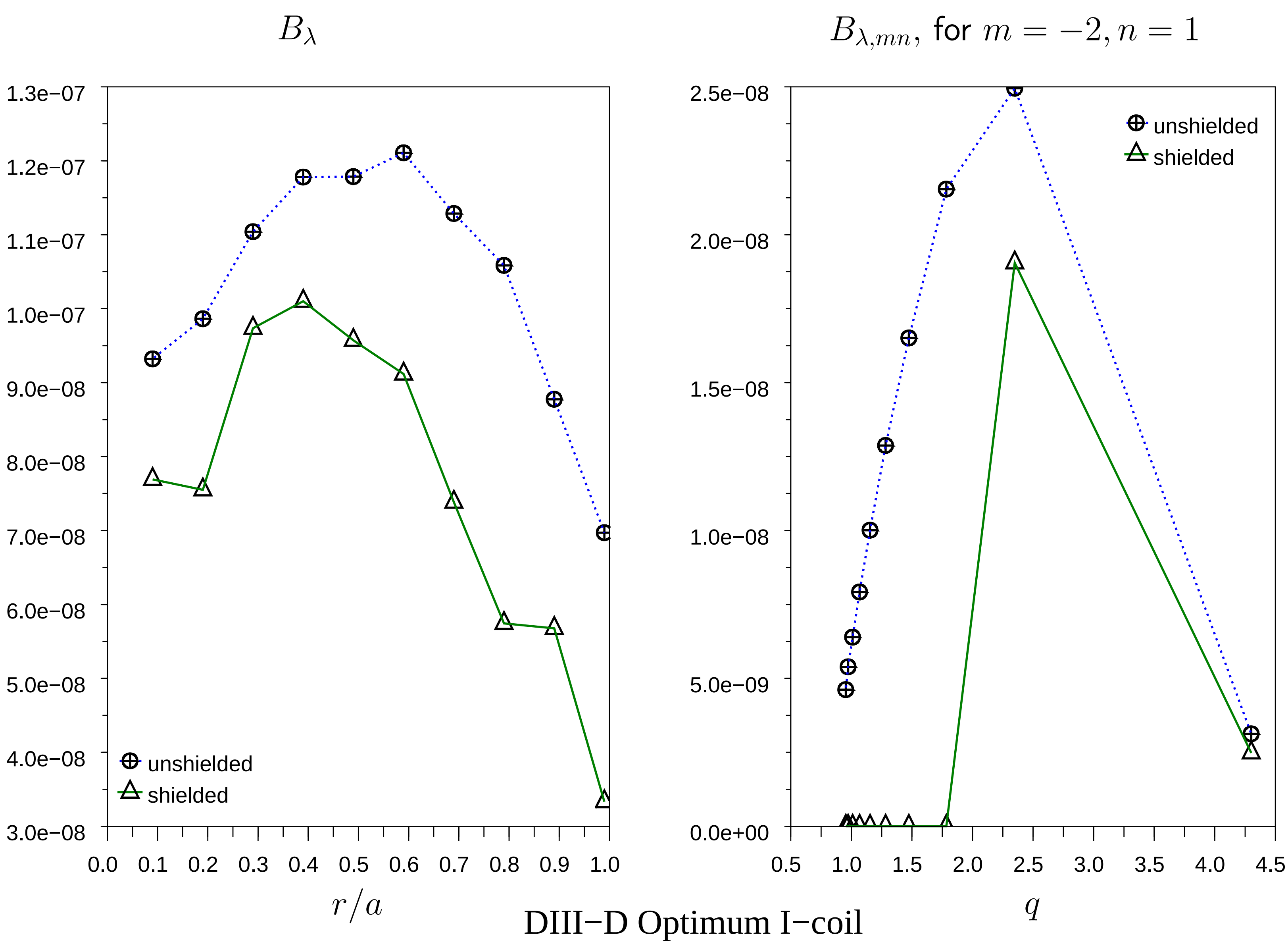
Plot of B_λ for optimum C-coil error correction ii



Plot of B_λ for optimum I-coil error correction i



Plot of B_λ for optimum I-coil error correction ii



Relevance of NTV model depends on condition $\Gamma_s = \sqrt{\nu_\| \tau_V} b(r_s) > 1$ near $q = 2$ $(r/a = .84)$ surface

- For $1/\nu$ this is equivalent to asking

$$\nu_\| r_s^2 q^2 \epsilon^{3/2} \times B_\lambda > \mu_i(r_s)/\rho_m(r_s)$$

- From Thomson data this is approximately

$$\nu_\| r_s^2 q^2 \epsilon^{3/2} \times B_\lambda \simeq 2.25 \times 10^5 \times B_\lambda \ \ \text{m}^2/\text{sec}$$

- Details

Assuming Deuterium ($M_D \simeq 3.34 \times 10^{-27}$ kg) and using the EFIT and Thomson data at time: 2500, find $r_s \simeq .84 \times a \simeq .512$ m, $n(r_s) \simeq 0.6 \times 10^{19}$ m^{-3}, $\rho_m(r_s) \simeq 2 \times 10^{-8}$ kg m^{-3}. Now, $V_{ti} \simeq 6.92 \times 10^3 \times [T_i(\text{eV})]^{1/2}$ m/ sec, which, assuming $T_i \simeq 0.5 T_e \simeq .25$ keV gives $V_{ti} \simeq 1.094 \times 10^5$ m/ sec or ~ 110 km/ sec. Hence $\omega_{ti} \simeq V_{ti}/(1.66 \times 2)$, or $\omega_{ti} \simeq 3.3 \times 10^4$ /s. Also, recalling $\nu_{i,i} \simeq 3.39 \times 10^{-8} n_i(\text{cgs}) \ln \Lambda [T_i(\text{eV})]^{-3/2}$ sec^{-1} ($\ln \Lambda \sim 17$), I find $\nu_{i,i} \simeq 874$ /s. Altogether this gives $\nu_\| \simeq 1.25 \times 10^6$ /s.

Effective momentum diffusivity χ_ϕ near $q = 2$ surface

- For intrinsic DIII-D field error $B_\lambda \simeq 9.0 \times 10^{-8}$ gives

$$\chi_\phi^{NTV} \simeq \left(2.25 \times 10^5\right) \times \left(9.0 \times 10^{-8}\right) \simeq .02 \ \ \text{m}^2/\text{sec}$$

- For DIII-D optimum C-coil correction $B_\lambda \simeq 4.0 \times 10^{-7}$ gives

$$\chi_\phi^{NTV} \simeq \left(2.25 \times 10^5\right) \times \left(4.0 \times 10^{-7}\right) \simeq .09 \ \ \text{m}^2/\text{sec}$$

- For DIII-D optimum I-coil correction $B_\lambda \simeq 6.0 \times 10^{-8}$ gives

$$\chi_\phi^{NTV} \simeq \left(2.25 \times 10^5\right) \times \left(6.0 \times 10^{-8}\right) \simeq .014 \ \ \text{m}^2/\text{sec}$$

- For comparison

$$\begin{aligned}
\chi_{\phi,c} &\sim \rho_i^2 \nu_{i,i} \simeq 4.54 \times 10^{-3} \ \ \text{m}^2/\text{sec} \\
\chi_{\phi,B} &\sim \rho_i^2 \omega_{ci}/16 \simeq 15.6 \ \ \text{m}^2/\text{sec} \\
\chi_{\phi,GB} &\sim \rho_i^3 \omega_{ci}/r_s \simeq 1.1 \ \ \text{m}^2/\text{sec}
\end{aligned}$$

Summary and future directions

- Have ability to calculate neoclassical toroidal viscosity [NTV] in tokamak devices. (Need harmonic decomposition of $|B|$ on flux surface.)
- Preliminary findings are that NTV is larger than classical diffusion but much less than Bohm and gyro-Bohm diffusion:

$$\chi_\phi^{NTV}/\chi_{\phi,c} \sim 4.4, \qquad \chi_\phi^{NTV}/\chi_{\phi,B} \sim 1 \times 10^{-3}, \qquad \chi_\phi^{NTV}/\chi_{\phi,GB} \sim 2 \times 10^{-2}.$$

- To proceed further, need cross-field viscosity, μ_i, in ohmic plasma discharges.
- Develop tools to quantify the plasma response: i.e., shielding, toroidal coupling, finite β ... etc.