

Toroidal Flow Evolution: Effects, How To Test

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- Theses:

1) Many effects influence toroidal flow evolution in tokamak plasmas — momentum sources, poloidal flow damping, neoclassical transport due to non-axisymmetric \vec{B} (field errors, NTMs, RWMs, MHD inst.), resonant field error locking, anomalous diffusion, intrinsic flows, pinches.

2) A coherent picture that includes all these effects is needed; fluid momentum balance and neoclassical fluid model provides useful framework.

3) Error-field “drag” and anomalous diffusion effects on toroidal flow are different — and should be experimentally distinguishable and testable.

- Outline:

Key poloidal, toroidal flow physical processes in cylindrical-type geometry

Error-field effects can be tested separately

Summary

Physics: Key Poloidal And Toroidal Flow Processes I

- Radial force balance is established on Alfvén time scale ($\sim \mu\text{s}$); it must be satisfied on all longer time scales (subscripts $t = \text{toroidal}$, $p = \text{poloidal}$):

$$0 = n_i e_i (E_r + V_p B_t - V_t B_p) - \frac{dp_i}{dr} \quad \Longrightarrow \quad \boxed{V_t = \frac{E_r}{B_p} + \frac{B_t}{B_p} V_p - \frac{1}{n_i e_i B_p} \frac{dp_i}{dr}},$$

which relates toroidal flow V_t to radial electric field E_r — they are not independent!

- Plasma flows on ψ are incompressible on sound wave time scale ($\sim 10 \mu\text{s}$):

$$\vec{\nabla} \cdot \vec{V} \simeq 0 \quad \Longrightarrow \quad U_\theta(\psi) \equiv \frac{V_p}{B_p} \simeq \frac{V_\parallel}{B} + \frac{\vec{V}_\perp \cdot \vec{e}_\theta}{\vec{B} \cdot \vec{e}_\theta} \quad \Longrightarrow \quad \boxed{V_p \simeq \frac{B_p}{B} V_\parallel - \frac{E_r}{B} + \frac{1}{n_i e_i B} \frac{dp_i}{dr}},$$

which relates poloidal flow V_p to parallel flow V_\parallel , $\vec{E} \times \vec{B}$ and diamagnetic flows, and poloidal heat flow q_p to parallel heat flow q_\parallel and diamagnetic heat flow $(1/eB)(dT/dr)$.

- Viscous drag on poloidal untrapped particle flow by particles trapped in variation of $|\vec{B}|$ along field lines from outboard to inboard side of torus ($f_t \sim \sqrt{\epsilon}$) damps poloidal flow V_p on ion collision time scale ($\sim \text{ms}$):

$$\langle \vec{B} \cdot \vec{\nabla} \cdot \vec{\pi} \rangle \simeq mn\sqrt{\epsilon} \nu_i B (V_p + \#q_p) \simeq 0 \quad \Longrightarrow \quad \boxed{V_p \simeq -\#q_{pi} \simeq \frac{1.17}{e_i B} \frac{dT_i}{dr}},$$

which specifies poloidal flow V_p in terms ion diamagnetic heat flow, because $q_{\parallel i} \simeq 0$.

Physics: Key Poloidal And Toroidal Flow Processes II

- Taking account of faster processes, on transport time scale (~ 100 ms), toroidal flow V_t is related to radial electric field E_r and diamagnetic flows:

$$V_t = \frac{E_r}{B_p} - \frac{1}{n_i e_i B_p} \frac{dp_i}{dr} + \frac{1.17}{e_i B_p} \frac{dT_i}{dr} \quad \Longrightarrow \quad \boxed{V_t \simeq \frac{E_r}{B_p}} \text{ plus diamagnetic flow effects.}$$

- There are a number of effects on V_t ($\sim E_r$) on the transport time scale:

Toroidal viscous drag due to toroidal non-axisymmetries of $|\vec{B}|$, including intrinsic flow — caused by \tilde{B}_r field errors and collective plasma responses (NTMs, RWMs, MHD).

Resonant $\tilde{\vec{J}} \times \tilde{\vec{B}}$ interaction at $q = m/n$ via error field penetration into rotating plasma — due to field errors, RFA at high β , MHD inst. int. with resistive walls & field errors.

Radial diffusion and pinch effects on the toroidal ion flow, including intrinsic flow effects — neoclassical, paleoclassical and anomalous transport ($\Pi_\zeta \sim -\rho_m \chi_\zeta \partial V_t / \partial r + \dots$).

Toroidal torques on the plasma by momentum sources \vec{S} — NBI etc.

Physics: Key Poloidal And Toroidal Flow Processes III

- The general form of the evolution equation for the toroidal flow V_t (or radial electric field E_r) is of the form (\vec{e}_t is unit vector in toroidal direction and $\langle \dots \rangle$ represents the flux surface average)

$$\rho_m \frac{\partial V_t}{\partial t} \simeq \underbrace{-\langle \vec{e}_t \cdot \vec{\nabla} \cdot \vec{\pi}_{\parallel i} \rangle}_{\text{toroidal drag}} + \underbrace{\langle \vec{e}_t \cdot \tilde{\vec{J}} \times \tilde{\vec{B}} \rangle}_{\text{res. wall int.}} - \underbrace{\langle \vec{e}_t \cdot \vec{\nabla} \cdot \vec{\pi}_{\perp i} \rangle}_{\perp \text{ transport}} + \underbrace{\langle \vec{e}_t \cdot \vec{S} \rangle}_{\text{mom. sources}} \implies$$

$$\boxed{\frac{\partial V_t}{\partial t} \simeq -\mu_{\parallel} \frac{\tilde{B}_{\text{eff}}^2}{B^2} (V_t - V_*^{\text{nc}}) - \check{\omega}_A \frac{\tilde{B}_{r mn}^2}{B^2} V_t + \frac{1}{r} \frac{\partial}{\partial r} (r \Pi_{\zeta}) + \frac{\langle \vec{e}_t \cdot \vec{S} \rangle}{\rho_m}},$$

with $\check{\omega}_A \sim \frac{r \delta(r - r_{mn})}{\tau_{A\theta}} \frac{\Omega_t \tau_R}{\Delta'^2 + (\Omega_t \tau_R)^2}$ (for a $\vec{J} \times \vec{B}$ resonant linear interaction) on the $q = m/n$ rational surface, including “rotation shielding” in layer due to $\Omega_t \simeq V_t/R \neq 0$.

- Non-axisymmetric magnetic perturbation effects (\tilde{B}_r and \tilde{B}_{mn}) are often neglected in electron and ion heat and particle transport studies; they are important here because they introduce new types of toroidal flow effects — toroidal drag (or E_r effects) and resonant interactions at $q = m/n$.

Physics: Key Poloidal And Toroidal Flow Processes IV

- Non-resonant (NR) \tilde{B} field errors cause toroidal flow damping (to V_*^{nc}):

Non-resonant (i.e., $q \neq m/n$ in plasma) non-axisymmetric \vec{B} components are caused by magnetic field ripple, coil irregularities, active control coils (e.g., C-Coil, I-coils) and nonlinear collective magnetic fields in the plasma (e.g., due to RWMs, MHD effects).

These field errors cause radial ion losses via ripple trapping or banana drift effects.

An $E_r < 0$ field builds up to hold them back, which thus causes a decrease in V_t .

Since energetic tail ions drift radially more rapidly than bulk ions, the resultant intrinsic rotation is $V_*^{\text{nc}} \propto dT_i/dr < 0$ in the “counter” (to I_p) direction — to reduce V_t & E_r .

Overall, the NR field error effects tend to produce $\mu_{\parallel} \sim v_{Ti}/Rq \sim 10^5/\text{s}$ and $V_*^{\text{nc}} \simeq c_{\#}(1/e_i B_p)(dT_i/dr)$ with $c_{\#} \sim 1$ and try to relax the toroidal flow V_t to V_*^{nc} .

- A resonant field error causes toroidal torque drag at $q = m/n$ that scales with Alfvén frequency times \tilde{B}_{rmn}^2/B_0^2 and tries to lock plasma to the wall (lab coordinates) and thereby induce a magnetic island in the plasma.

- Perpendicular momentum transport has various components:

Axisymmetric neoclassical — $\Pi_{\zeta} \sim -0.1 \nu_i \varrho_i^2 (dV_t/dr - \dots)$, negligible.

Paleoclassical — $\Pi_{\zeta} \sim -D_{\eta}(dV_t/dr + \dots)$, probably small; E_r in H-mode pedestal?

Anomalous — $\Pi_{\zeta} \sim -\chi_{\zeta}(dV_t/dr) + nV_{\text{pinch}} + \text{other}$, important & being developed.

Ways To Test Components Of Toroidal Flow Evolution

- *Global scaling*: Since the various effects (global and locally resonant torques due to field errors plus diffusion etc.) can affect various plasma regions differently, it seems difficult to test them via global scaling studies.
- *Drag*: Neoclassical toroidal viscosity (NTV) torques are global; they are tested by measuring V_t response to increasing non-resonant field errors.
- *Resonant*: Resonant field error penetration and mode locking can be tested by following flow evolution response in vicinity of resonant surface, with and without significant non-resonant field errors.
- *Net transport*: The net plasma transport properties (effective diffusion coefficient) can be tested by measuring flow profile response to changes in the momentum source.
- *Transport Components*: The various components (diffusion, pinches, intrinsic rotation) of radial transport of toroidal flow can be tested via responses to transient responses to source changes, preferably radially localized ones.

Summary

- The toroidal flow in a tokamak plasma is influenced by many effects:
 - faster time scale constraints — radial force balance, poloidal ion flow damping, drag torques — induced by nonaxisymmetric \vec{B} due to field errors, NTMs RWMs etc., resonant error field interactions at rational surfaces — leads to mode locking, islands, radial transport (diffusion, pinches and intrinsic rotation) of the toroidal flow — classical/neoclassical, paleoclassical, anomalous.
- Fluid momentum balances of electrons & ions provide useful paradigm:
 - lowest order axisymmetric equilibrium — plus small perturbations, parallel momentum balances — \parallel neoclassical Ohm's law and ion poloidal flow damping, radial particle fluxes — collisional (classical, neo, paleo, nonaxisymmetric), anomalous, the only collisional nonambipolar particle flux is due toroidal asymmetries, which represents toroidal torque on the plasma from neoclassical transport induced by field errors.
- The various effects on the toroidal flow should be experimentally distinguishable and separately testable:
 - non-resonant field errors — direct torque drag throughout the plasma, resonant field errors — locking of resonant surface to wall, local island-induced torque, radial transport — diffusion, pinch and intrinsic rotation effects.