PERTURBED PLASMA EQUILIBRIA

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1. Plasma shape perturbations and δW
2. Required external magnetic field
3. Magnetic islands and surface currents
4. Persistence of surface currents at rational surfaces
5. General plasma equilibria $\vec{f} = \vec{j} \times \vec{B}$ with $\vec{f} = \vec{\nabla} p + \delta\vec{f}$
6. Applications

1. Plasma shape perturbations and δW

Ideal MHD (δW) gives pert. equilibria, $\vec{\nabla}\delta p = \delta\vec{j}\times\vec{B} + \vec{j}\times\delta\vec{B}$, with $\delta\vec{B} = \vec{\nabla}\times(\vec{\xi}\times\vec{B})$ and $\delta p = -\vec{\xi}\cdot\vec{\nabla}p$. Keeps $q(\psi)$ and $p(\psi)$ fixed.

Standard analysis (DCON & CAS3D) reduces problem to a set of coupled homogenous 2nd order ordinary diff. equations, $\partial^2\xi_{mn}/\partial\psi^2 + \ldots$, for the Fourier components of $\vec{\xi}\cdot\vec{\nabla}\psi = \sum \xi_{mn}e^{i(n\varphi - m\theta)}$.

A set of J modes chosen by choosing largest ξ_{mn} as $\psi \to 0$.

For each of the J modes, the ξ_{mn} at the plasma boundary defines an equilibrium with the boundary perturbed by a distance $\vec{\xi}$.

2. Required external magnetic field

Each of the J perturbed equilibria calculated by DCON or CAS3D defines a vacuum magnetic field, $\delta\vec{B}_j = \vec{\nabla}\delta\Phi_j$, with $\delta\vec{B}_j \cdot \hat{n}$ continuous at plasma surface and $\delta\Phi_j(\vec{x} \to \infty) = 0$. Assumes no perturbing currents away from plasma surface.

At plasma surface have two $\delta\vec{B}_j$'s one inside and one outside. Surface current $\vec{K}_j = \hat{n} \times (\delta\vec{B}_{out} - \delta\vec{B}_{in})$ gives normal field $\delta\vec{B}_j^{(x)} \cdot \hat{n}$.

$\delta\vec{B}_j^{(x)} \cdot \hat{n}$ gives externally produced normal field required to support j^{th} perturbed equilibrium.

Actual external field $\delta\vec{B}_x \cdot \hat{n} = \sum c_j \delta\vec{B}_j^{(x)} \cdot \hat{n}$, so actual $\vec{\xi} = \sum c_j \vec{\xi}_j$.

3. Magnetic islands and surface currents

Ideal MHD codes have a jump in $\hat{n}\square/\vec{B}$ across rational magnetic surfaces, $q=m/n$, which means a surface current $\vec{K}_{mn}$.

$\vec{K}_{mn}$ surface current gives a normal magnetic field $\square\square\vec{B}^{(d)}_{mn}\cdot\hat{n}$.

$\square\vec{B}^{(d)}_{mn}\cdot\hat{n}$ is the field trying to drive an island at $q=m/n$ due to currents away from the rational surface. $\vec{K}_{mn}$ shields out the island.

The modified ideal MHD code IPEC by J-K Park [Phys. Plasmas**14**, 052110 (2007)] gives the drive for islands in the plasma $\square\vec{B}^{(d)}_{mn}\cdot\hat{n}$ of an externally produced magnetic perturbation $\square\vec{B}_x$. The distortion of the path of the equilibrium plasma current is included and is often dominant. *Superposition of equilibrium and vacuum fields is generally a bad approx.*

4. Persistence of surface currents at rational surfaces

Surface currents sound unphysical but can be maintained in steady-state by plasma transport effects, $\int \vec{v} \cdot (\vec{j} \times \vec{B}) d^3x = -\int \eta j^2 d^3x$.

Current channel or island must have width of order ρ_i to interact with ions. Toroidal torque required to shield out island is then

$$\frac{\mu_0 N_{req}}{R(\delta \vec{B}_{mn}^{(d)} \cdot \hat{n})^2} \approx \left(\frac{a}{\rho_i}\right)^2 \frac{1}{S\sqrt{\beta}} \frac{(\rho_i / a) C_s}{V_\phi}$$

where $\beta = C_s^2 / V_A^2$ and $S \equiv a V_A \mu_0 / \eta \approx 10^9$.

Empirically $\delta \vec{B}_{mn}^{(d)} \cdot \hat{n} < 5\text{G}$ shielded out.

5. General plasma equilibria

$$\vec{f} = \vec{j} \times \vec{B} \text{ with } \vec{f} = \vec{\nabla} p + \vec{\nabla}\cdot\vec{\tilde{f}}$$

Boozer and C. Nührenberg, Phys. Plasmas **13**, 102501 (2006)

Given a set of magnetic surfaces $\vec{x}(\psi,\theta,\varphi)$ and q(ψ) one can simply solve $\vec{f} = \vec{j} \times \vec{B}$ exactly except $\left(\vec{f} - \vec{j} \times \vec{B}\right)\cdot \frac{\partial \vec{x}}{\partial \psi} = \delta\tilde{f}_{\psi}(\psi,\theta,\varphi)$.

Using standard arguments $\delta W = \int\left(\delta\tilde{f}_{\psi}\xi^{\psi} - \frac{1}{2}F_{\psi}\left[\xi^{\psi}\right]\xi^{\psi}\right)d^3x$, which is minimized by $F_{\psi}\left[\xi^{\psi}\right] = \delta\tilde{f}_{\psi}$, an inhomog. eq. for $\xi^{\psi} \equiv \vec{\xi}\cdot\vec{\nabla}\psi$.

Can be used to find general equilibria near scalar pressure equilibria or to improve accuracy of scalar pressure equilibria.

Drive for islands given by jump in perturbed field across $q=m/n$.

6. Applications

a. Tokamak sensitivity to external field errors (Park et al, PRL 2007)

b. Determination of importance of islands produced by intrinsic shaping of stellarators (C. Nührenberg & Boozer, submitted PRL)

Found $\Box\tilde{f}_{\Box}$ for NCSX using VMEC surfaces.

c. Shielding of variations magnetic perturbations that give non-ambipolar transport (Phys. Plasmas **3**, 3375 (1996); Park & Boozer in progress).

Calculate perturbed tokamak equilibria with $p_{\parallel} \Box p_{\Box} \propto (\Box' \Box\Box_A')$ given by non-axisymmetry of B. Max. torque pert. can exert on plasma is $\oint\Box R(\Box\vec{B}_p \cdot \hat{n})(\Box\vec{B}_x \cdot \hat{n})da$. Otherwise shielding.

Topics Discussed

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    b. Determination of importance of islands produced by intrinsic stellarator shaping

    c. Shielding of variations magnetic perturbations that give non-ambipolar transport